Physics 741 – Graduate Quantum Mechanics 1

Homework Set α
Due Wednesday, November 19

1. Two spin-1/2 non-interacting fermions are in the $n = 0$ and $n = 1$ state of a 1D harmonic oscillator;

$$H = \sum_{i=1}^{2}\left(\frac{P_i^2}{2m} + \frac{m\omega^2}{2}Q_i^2\right)$$

Naively, we might write the quantum state as $|01\rangle$, but this is wrong because (i) it's not antisymmetrized, and (2) we didn't include spin states

(a) Taking account spin states, write four correctly antisymmetrized wave functions, written so they are eigenstates of $\mathbf{S}^2$ and S_z, where **S** is the total spin of the two electrons. You can write your states in whatever basis is most convenient; I recommend $|n_1 n_2; s m_s\rangle = |n_1 n_2\rangle \otimes |s m_s\rangle$ and/or $|n_1 n_2; m_1 m_2\rangle = |n_1 n_2\rangle \otimes |\pm\pm\rangle$.

(b) For each of the four states, calculate $\langle\psi|(Q_1 - Q_2)^2|\psi\rangle$

(c) Suppose the particles had an additional repulsive force between them, so the fermions prefer to be far apart. Which of the two states would have lower energy?

2. A certain quantum system consists of a spin ½ particle and a measuring device, and is described has basis $\{|m_s, D_i\rangle\}$, where m_s describes the spin state of the particle and D_i describes the quantum state of the measuring device, initially in the state $|D_0\rangle$. Different spins are fed into the system, and it is found that when fed $|\pm_z\rangle$, the resulting state is

$$|+_z, D_0\rangle \to |+_z, D_+\rangle,$$
$$|-_z, D_0\rangle \to |-_z, D_+\rangle.$$

What will be the resulting quantum state if it is instead fed $|+_\theta, D_0\rangle$, where $|+_\theta\rangle$ is $|+_\theta\rangle = \cos\left(\tfrac{1}{2}\theta\right)|+_z\rangle + \sin\left(\tfrac{1}{2}\theta\right)|-_z\rangle$