Physics 744 - Field Theory

Homework Set 6

1. In the $\lambda\phi^4$ theory:
   (a) Draw all connected Feynman diagrams at the two loop level with a single ϕ particle with momentum $\mathbf{p}$ entering on the left and leaving on the right. There should be one diagram at the one-loop level, and three at the two loop level. Do *not* take advantage of normal ordering. Include the appropriate symmetry factor.
   (b) Label all intermediate momenta. Indicate which way the momentum is flowing in any intermediate propagators.
   (c) Write the Feynman amplitude in each case. Do *not* attempt to perform the integrals.

2. Suppose you have two particles in the final state for some diagram. You need to know the momentum of the final state particles. Assume the total center of mass energy is *E*.
   (a) Show that the momenta of the final state particles can be written as

$$p = \frac{1}{2E}\sqrt{E^2 + m_1^2 + m_2^2 - 2E^2 m_1^2 - 2E^2 m_2^2 - 2m_1^2 m_2^2}$$

   (b) Simplify this formula in the cases (i) $m_2 = 0$ and (ii) $m_1 = m_2 = m$.

3. The Feynman invariant amplitude for muon decay:

$$\mu^-(\mathbf{p}_\mu) \to e^-(\mathbf{p}_1) + \bar{\nu}_e(\mathbf{p}_2) + \nu_\mu(\mathbf{p}_3)$$

   is given by

$$|i\mathcal{M}|^2 \to 64 G_F^2 (\mathbf{p}_\mu \cdot \mathbf{p}_2)(\mathbf{p}_1 \cdot \mathbf{p}_3).$$

   Compared to the muon, the other particles are so light they can be treated as massless. The constant $G_F = 1.17 \times 10^{-5}\ \text{GeV}^{-2}$ is a constant involved in weak decays called *Fermi's constant*.
   (a) Use conservation of four-momentum to relate the two dot-products appearing in this formula. Working in the rest frame of the muon, write out this matrix element explicitly and show that it depends on only *one* of the final state energies.
   (b) Since the final state particles are massless, the three-momenta have magnitudes equal to their energies. Use this plus conservation of energy to show that none of the final state particles has an energy greater than $\frac{1}{2}m_\mu$. Use this to write *three* inequalities on the energies E_1 and E_2. Sketch the allowed region in E_1-E_2 space.
   (c) Do the final state integrals and determine the decay rate of the muon. The lifetime is the reciprocal of this.
   (d) The mass of the muon is $m_\mu = 0.1057$ GeV. Find the lifetime $\tau = \Gamma^{-1}$ in seconds and compare to the experimental lifetime of $\tau = 2.197\ \mu\text{s}$.