Name ________________

Solutions to Final Exam
May 5, 2008

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered. Some possibly useful formulas appear below

Focal Length
$$f = \tfrac{1}{2}R$$
$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

2-Slit Interference
$$\sin\theta_{\text{bright}} = \frac{m\lambda}{d}$$
$$I = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Diffraction Grating
$$mN > \lambda/\Delta\lambda$$

Thin Film
$$2t_{\text{Weak}} = \lambda m$$

Position on a screen
$$x = L\tan\theta \approx L\sin\theta$$

Magnification
$$M \equiv h'/h = -q/p$$
$$m \equiv \theta/\theta_0$$

Diffraction
$$\sin\theta_{\text{dark}} = \frac{m\lambda}{d}$$
$$I = I_{\max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

X-Ray Scattering
$$2d\cos\theta = m\lambda$$

Brewster's Angle
$$\tan\theta_P = n_2/n_1$$

Telescope Magnification
$$m = f_o/f_e$$

Relativity
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$
$$u' = \frac{u - v}{1 - uv/c^2}$$

Relativistic Doppler Effect
$$f = f_0\sqrt{\frac{1 + v/c}{1 - v/c}}$$

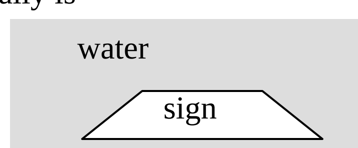
Constants
$$e = 1.602\times10^{-19}\,\text{C}$$
$$\varepsilon_0 = 8.854\times10^{-12}\,\text{C}^2/\text{N/m}^2$$
$$\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$$

Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

1. Suppose you have two nearly identical waves, but they differ by a phase. The two waves are then added together. For what phase difference will we get maximum destructive interference, so the waves tend to cancel out? Write the phase in radians.
A) 0 B) ½π **C) π** D) 2π E) 4π

2. If you placed a sign 1 m under water, so the surface of the sign is parallel to the surface of the water, and the water were perfectly smooth, the sign would appear
A) Magnified in size, but neither closer nor farther than it actually is
B) Magnified in size, and also farther away than it actually is
C) Magnified in size, and also closer than it actually is
D) Its usual size, but farther than it actually is
E) Its usual size, but closer than it actually is



3. If an object without visual aids has an angular size of 1°, but with visual aids has an angular size of 2°, the angular magnification is m =
   A) 4 **B) 2** C) 1 D) ½ E) ¼

4. When light goes through a diffraction grating, what sort of pattern does it make?
   A) It comes out at a set of discrete angles that depend on wavelength
   B) It makes a pattern of alternating dark and light regions
   C) It makes a pattern with a bright band in the middle, and then several surrounding bands that are each half as wide as the central band
   D) In the limit of many grating lines, the light goes straight through
   E) If placed at the correct distance, the light will be focused to a point like a lens

5. One way to make EMF is with a battery. Another is to
   A) Have a constant magnetic field through a stationary loop (only)
   B) Have a constant magnetic field through a moving or rotating loop (only)
   C) Have a changing magnetic field through a stationary loop (only)
   D) B and C are both correct
   E) A, B, and C are all correct

6. If you diffract light through a small hole, how will the diffraction pattern change if you make the hole smaller?
   A) It will be smaller and brighter
   B) It will be smaller and less bright
   C) It will be smaller but the same brightness
   D) It will be bigger and brighter
   E) It will be bigger and less bright

7. Which of the following is <u>not</u> a consequence of special relativity?
   A) Objects moving at high speed seem to get shorter
   B) Clocks moving at high speed seem to run slower
   C) The Doppler shift formula needs to be modified
   D) Events that seem to be simultaneous according to one observer may not be simultaneous as viewed by another observer
   E) Actually, all of these <u>are</u> consequences of special relativity

8. A converging lens in air is always
   A) Convex on the front face
   B) Concave on the front face
   C) Convex on the back face
   D) Thicker in the middle than on the edges
   E) Thicker on the edges than in the middle

9. If you put polarized light into an optically active material, like sugar water, the polarization
   A) Will be completely blocked in one direction and mostly transmitted in the other direction
   B) Will gradually rotate to be at a different angle
   C) Will slowly become randomly polarized due to scattering
   D) Will gradually reverse, so the electric field points the opposite direction
   E) Will be irrelevant, since “polarization” is defined only in air

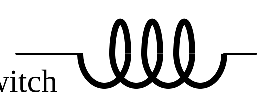
10. When light impinges on a boundary between regions at different indices of refraction at Brewster’s Angle, what happens?
   A) All of the light is reflected
   B) All of the light is refracted
   C) The light that is reflected will be completely polarized
   D) The light that is refracted will be completely polarized
   E) None of the above

11. In class, I dropped a small magnet down a copper tube. It fell, but very slowly. Why?
   A) According to Lenz’s law, the copper tube didn’t like changing magnetic flux, so it created currents that resulted in magnetic fields that resisted the motion of the magnet
   B) Copper is a magnetic material, so the magnetic field of the magnet created a corresponding magnetic field in the copper, which the magnet was attracted to
   C) The moving magnet created an electric field, which in turn attracted charges that held the magnet nearly in place
   D) The moving magnet created electric fields, which caused charge to build up on the magnet, which deflected it sideways as it fell through the tube
   E) Like Magneto from “X-men”, anything that has control of magnetism controls all metals, so it used its power to slow its fall

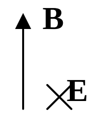
12. When working with a lens or a mirror, a real image is one that is
   A) In front in both cases
   B) In back in both cases
   C) In front for a lens, in back for a mirror
   D) In back for a lens, in front for a mirror
   E) None of the above

13. If a resistor with resistance 5 Ω had 30 V across it, what would be the current through the resistor?
   A) 1/150 A B) 1/6 A C) 150 A **D) 6 A** E) None of the above

14. At right is the circuit symbol for a
   A) Battery B) Resistor **C) Inductor** D) Capacitor E) Switch



15. Suppose the electric and magnetic field of an electromagnetic wave are as shown at right. In which direction is the wave traveling?
A) Up B) Down **C) Right** D) Left E) Into the page



16. Gauss's Law for electric fields says that the total electric flux out of a region of space is proportional to
A) The total charge in that region
B) The total current in that region
C) The rate of change of magnetic flux
D) The rate of change of current in that region
E) The total electric potential on the surface of that region

17. Place the following three colors in correct order from longest to shortest wavelength
A) Orange, Blue, Green
B) Orange, Green, Blue
C) Blue, Green, Orange
D) Blue, Orange, Green
E) Green, Blue, Orange

18. When you point X-rays at a crystalline arrangement of atoms, what pattern do you see reflected?
A) You see bright reflections, but only at certain special angles
B) You see bright reflections, except at certain special angles
C) You see bright reflections in all directions
D) You don't see reflections at all, but you do see spots caused by refraction
E) X-rays are unaffected by crystals, so you don't see reflection or refraction effects

19. Basically, which of the following does a "step-up" transformer do to an AC source?
A) It increases the voltage and increases the current
B) It increases the voltage and decreases the current
C) It decreases the voltage and increases the current
D) It decreases the voltage and decreases the current
E) A transformer has no effect on AC, it only affects DC

20. If a capacitor has capacitance C and potential difference ΔV, its charge Q is
A) $\frac{1}{2}C(\Delta V)^2$ B) $(\Delta V)^2/2C$ C) $\Delta V/C$ D) $C/\Delta V$ **E) $C\Delta V$**

Part II: Short answer, old material: [20 points]

Choose **two** of the following three questions and give a short answer (2-3 sentences) (10 points each)

21. For each of the following five quantities, tell me the name of the metric unit and the appropriate symbol

a) Inductance b) Capacitance c) Current d) Resistance e) Frequency

Quantity	**Inductance**	**Capacitance**	**Current**	**Resistance**	**Frequency**
Unit	Henry	Farad	Amp/Ampere	Ohm	Hertz
Abbreviation	H	F	A	Ω	Hz

22. Describe qualitatively the motion of a charged particle in a constant magnetic field

If the charged particle is moving in a plane perpendicular to the magnetic field, it will move in a circle. If it is parallel, it will move in a straight line. If it is moving in some combination of these, it will move in a helical pattern, circling in the perpendicular plane while advancing steadily in the parallel direction.

23. An inductor resists changes in what? Give any corresponding formulas.

Inductors oppose any sudden change in current. It only changes current when there is a voltage across it, according to the formula

$$\mathcal{E} = -L\frac{dI}{dt}$$

Part III: Short answer, new material: [20 points]

Choose **two** of the following three questions and give a short answer (2-3 sentences) (10 points each)

24. Often, if you look closely at soap bubbles, you can see colorful patterns in the reflected light. What causes these colors?

Light that crosses the thin layer of a soap bubble has two chance to reflect: one as it enters the soapy water, and one when it leaves it. The two reflected parts of the light will interfere destructively whenever $2t = \lambda m$, where *m* is an integer, and interfere constructively whenever $2t = \lambda\left(m + \frac{1}{2}\right)$. Since this will be different for different colors, the effect is that some colors are strongly reflected and others are not, turning white light into colors.

25. For a telescope of fixed diameter, what ultimately limits how fine detail you can see on some distant astronomical object, using visible light? Give at least one formula.

Because the light must pass through the primary objective of the telescope, it will be diffracted by an angle of approximately $\theta_{\min} = \lambda/D$, (or, more accurately, $\theta_{\min} = 1.22\,\lambda/D$) where *D* is the diameter of the telescope. Hence it is this diameter of the primary lens or mirror that determines how fine of detail you can see.

26. Explain briefly what spherical aberration and chromatic aberration are.

Both of these are causes of imperfect imaging. Spherical aberration comes about primarily because the lenses and mirrors we use are spherical, and the small angle approximation is not valid as you move far from the center of such lenses and mirrors. Chromatic aberration occurs because lenses (but not mirrors) refract light of different wavelengths differently, so that the focal length for each wavelength is a little different.

Part IV: Calculation, old material: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

27. The electrostatic potential in a certain region of space is given by

$$V(x,y,z) = 20\left(2z^2 - y^2 - x^2\right)$$

where *V* is in volts and (*x*,*y*,*z*) are the coordinates of a point in space, all in meters. A single proton of charge *e* = 1.602×10^{-19} C and mass *m* = 1.673 × 10^{-27} kg is at the position (*x*,*y*,*z*) = (10 m, 0, 0).

(a) What is the potential energy of this proton at this position?

The potential energy is given by

$$U = eV = e20\left(2\cdot 0^2 - 0^2 - 10^2\right)\text{ V} = -2000\text{ eV}$$
$$= -\left(2000\text{ V}\right)\left(1.602\times 10^{-19}\text{ C}\right) = -3.204\times 10^{-16}\text{ J}$$

(b) What is the electric field at this position?

The electric field can be found from the potential as

$$E_x = -\frac{\partial V}{\partial x} = -20\frac{\partial}{\partial x}\left(2z^2 - y^2 - x^2\right) = 40x = 40(10) = 400\text{ V/m}$$
$$E_y = -\frac{\partial V}{\partial y} = -20\frac{\partial}{\partial y}\left(2z^2 - y^2 - x^2\right) = 40y = 0$$
$$E_z = -\frac{\partial V}{\partial z} = -20\frac{\partial}{\partial z}\left(2z^2 - y^2 - x^2\right) = -80z = 0$$

In summary, the electric field is in the +*x* direction and has magnitude 400 V/m.

(c) What is the electric force on the proton, and its acceleration at this position?

The electric force is simply given by

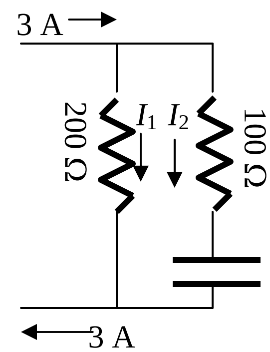
$$\mathbf{F} = q\mathbf{E} = \left(1.602\times 10^{-19}\text{ C}\right)\left(400\text{ V/m}\right)\hat{\mathbf{i}} = 6.408\times 10^{-17}\hat{\mathbf{i}}\text{ N}$$

The acceleration is then obtained from **F** = *m***a**, so we have

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{6.408\times 10^{-17}\hat{\mathbf{i}}\text{ N}}{1.673\times 10^{-27}\text{ kg}} = 3.83\times 10^{10}\hat{\mathbf{i}}\text{ m/s}^2$$

28. In the circuit at right, there are 3 A flowing in from the top and out of the bottom of the circuit at all times.

(a) Write an equation for I_1 and I_2 using Kirchoff's First law.



Looking at the vertex at the top, we insist that the current in (3 A) match the current out (the two currents I_1 and I_2), so

$$3 = I_1 + I_2$$

(b) Write an equation for I_1 and I_2 using Kirchoff's Second law. Denote the voltage across the capacitor at any given time as ΔV, with the top at a higher voltage than the bottom

The total voltage change around the loop must total zero. For the 200 ohm resistor, you are going upstream, so the voltage increases, whereas for the 100 ohm resistor, it is downstream. Finally, the capacitor represents a decrease in voltage, since the top is assumed to be at higher voltage, so we have

$$0 = 200I_1 - 100I_2 - \Delta V$$

(c) If at $t = 0$, the capacitor has ΔV=0 on it, solve the equations you found in the previous two parts for the currents I_1 and I_2.

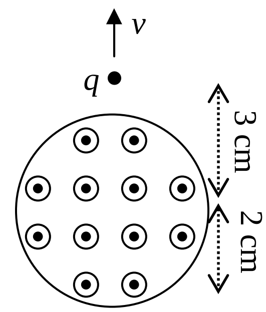
From part (b), we see that if ΔV=0, we have $0 = 200I_1 - 100I_2$, or $I_2 = 2I_1$. Plugging this into the equation from part (a), we have

$$3 = I_1 + I_2 = I_1 + 2I_1 = 3I_1, \quad \text{so} \quad I_1 = 1\text{ A} \quad \text{and} \quad I_2 = 2I_1 = 2\text{ A}.$$

(d) After a long time, the capacitor reaches a steady state (no longer ΔV=0). What are the currents I_1 and I_2 at this time?

When the capacitor reaches a steady state, there can no longer be any current flowing through it. Therefore, we have $I_2 = 0$. Using the equation from part (a), we see that we now have $I_1 = 3$ A.

29. A cable of radius 2 cm has 100 A running through it. The current is distributed uniformly over the cross-sectional area of the cable, and as sketched at right, the current is moving directly out of the plane of the paper towards you.



(a) Find the magnitude and direction of the magnetic field 3 cm from the center of the cable, at the point charge *q*.

This problem is set up for Ampere's Law. By the symmetry of the problem, it is clear that the magnetic field must be looped around the cable in a symmetric way. Consider an Ampere loop of radius R = 3 cm around the cable. If we let B be the magnitude of the magnetic field in the counter-clockwise direction, then $\mathbf{B}\cdot d\mathbf{s} = B\,ds$, and B will be constant around the cable. By Ampere's Law, we must have

$$\mu_0 I_{\text{in}} = \oint \mathbf{B}\cdot d\mathbf{s} = B\oint ds = B(2\pi R)$$

Solving for the magnetic field, we have

$$B = \frac{\mu_0 I_{\text{in}}}{2\pi R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(100\ \text{A})}{2\pi(0.03\ \text{m})} = 6.67\times10^{-4}\ \text{T} = .667\ \text{mT}$$

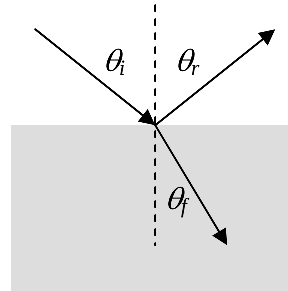
The magnetic field points in the counter-clockwise direction, or to the left at the point charge *q*.

(b) A small charge of magnitude *q* = - 3 μC at a distance of 3 cm from the center is moving directly away from the cable at a velocity of *v* = 100 m/s. Calculate the magnitude and direction of the force on the charge *q*.

To find the force, we use the formula $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$. The velocity is perpendicular to the magnetic field, so the magnitude of the force is just

$$F = |q|vB = (3\times10^{-6}\ \text{C})(100\ \text{m/s})(6.67\times10^{-4}\ \text{T}) = 2.00\times10^{-7}\ \text{N}$$

Since **v** is up and **B** is to the left, $\mathbf{v}\times\mathbf{B}$ will be out of the plane of the paper. But since *q* is negative, the direction is actually reversed, or into the plane of the paper.



30. A beam of light with frequency f = 5.86 × 10^{14} Hz passes from air to glass at an angle of incidence of θ_i = 57.0°. Assume glass has an index of refraction of n = 1.500

(a) At what angle θ_f does the light continue into the glass? Some of the light reflects from the glass. What is the reflection angle θ_r?

The angle for refraction can be found by Snell's Law, which says

$$n_i \sin\theta_i = n_f \sin\theta_f \,, \quad \text{so} \quad \sin\theta_f = \frac{n_i \sin\theta_i}{n_f} = \frac{\sin(57°)}{1.500} = 0.559$$

Taking the arcsin, we see that $\theta_f = 35.0°$. For the reflected wave, we just use $\theta_i = \theta_r$, so $\theta_r = 57.0°$.

(b) For the incident light, what is the speed v and the wavelength λ?

Air has an index of refraction of 1, so the speed is $v = c = 3.00 \times 10^8 \text{ m/s}$. The wavelength can then be found from $v = f\lambda$, so

$$\lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.86 \times 10^{14} \text{ s}^{-1}} = 5.12 \times 10^{-7} \text{ m} = 512 \text{ nm}$$

(c) For the transmitted light, what is the frequency f, the speed v, and the wavelength λ? Also, what are these values for the reflected light?

The frequency of a wave does not change as it enters a medium, so we still have $f = 5.86 \times 10^{14} \text{ Hz}$. The velocity, however, is now

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.500} = 2.00 \times 10^8 \text{ m/s}$$

The wavelength is also changed

$$\lambda = \frac{v}{f} = \frac{2.00 \times 10^8 \text{ m/s}}{5.86 \times 10^{14} \text{ s}^{-1}} = 3.41 \times 10^{-7} \text{ m} = 341 \text{ nm}$$

For the reflected wave, it is still in air, so nothing changes. We have f = 5.86 × 10^{14} Hz, $v = c = 3.00 \times 10^8 \text{ m/s}$, and $\lambda = 512 \text{ nm}$.

(d) The critical angle for a glass/air transition can be shown to be 41.8°. Does this mean the light is totally reflected in this case?

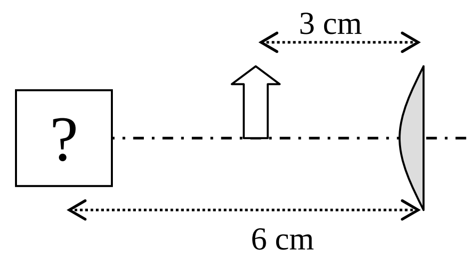
Total internal reflection only occurs when you move from a high index of refraction material to a low index material. Since this isn't occurring here, this is irrelevant.

Part V: Calculation, new material: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

31. Below is sketched a lens imaging system, consisting of a single plano-convex lens. It is discovered that when an object of height 2 cm is placed 3 cm in front of the lens, an image appears 6 cm in front of the lens.

(a) What is the magnification for this optical system? How big will the image be? Will it be upright or inverted?



By convention,whan the object is in front of the lens, p is positive, so we have $p = + 3$ cm. However, for the image, when it is in front of the lens, q is negative, so we have $q = - 6$ cm. The magnification is then given by

$$M = -\frac{q}{p} = \frac{6\text{ cm}}{3\text{ cm}} = +2$$

Since it is positive, the image is upright. Since the original object has height 2 cm, and magnification is defined as $M = h'/h$, so

$$h' = Mh = 2(2\text{ cm}) = 4\text{ cm}.$$

(b) What is the focal length for this optical system?

The focal length is related to the object and image distances by

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{3\text{ cm}} + \frac{1}{-6\text{ cm}} = \left(\tfrac{1}{3} - \tfrac{1}{6}\right)\text{ cm}^{-1} = \tfrac{1}{6}\text{ cm}^{-1} \qquad \text{so} \qquad f = 6\text{ cm}.$$

(c) If the lens is made from glass with an index of refraction of *n*= 1.6, what is the radius of curvature of the lens?

The front surface is convex outwards, and hence is considered a positive radius R_1. The back surface is flat, and is therefore considered $R_2 = \infty$. Plugging into the Lensmaker's equation, we have

$$\frac{1}{f} = \left(\frac{n}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.600 - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{0.600}{R_1},$$

$$R_1 = 0.600 f = 0.600(6\text{ cm}) = 3.60\text{ cm}$$

32. Below is sketched an imaging system consisting of a diverging lens. The two foci of the lens are marked. Also sketched in are three light rays: An upper one, initially parallel to the optic axis, a middle one, initially headed for the far focus of the lens, and the lower one, initially headed for the vertex of the lens. I recommend you work directly on this test.

(a) Sketch the path of each of the light rays after they pass through the lens.

The topmost ray is parallel to the optic axis, and by the rules for diverging lenses, it must refract going directly away from the focal point on the left, as sketched below (red lines). The middle ray is headed for the far focus of the diverging lens, and hence must refract to remain parallel to the optic axis (blue lines). The bottom ray passes through the vertex of the lens, and hence continues straight (green lines).

(b) Using the paths just drawn, estimate the position of the image, and sketch it in.

We need to trace the outgoing rays backwards and see where they intersect. The red line already has it back-traced to the focus, and the green line is straight, so all that remains is the blue line, which we simply trace backwards parallel to the optic axis. The three lines intersect, or nearly intersect, at a single point, so that's where the image goes.

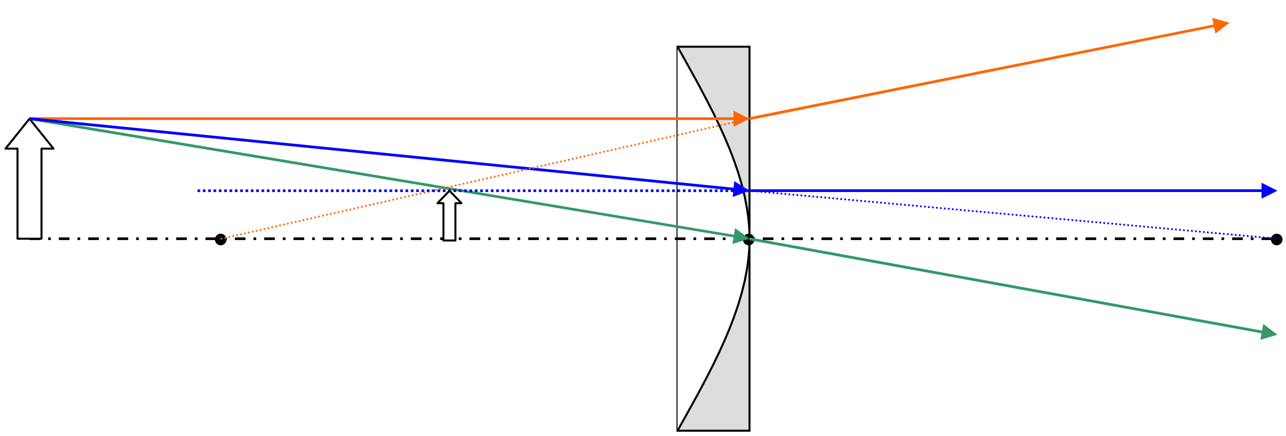
(c) Is the image real or virtual? Upright or inverted?

Since the image is in front of the lens, it is virtual. You can see that it is upright.

(d) Estimate the magnification of the lens, including sign.

The original object has a height of about 1.60 cm, and the image has a size of about 0.66 cm, so the magnification is

$$M = \frac{h'}{h} = \frac{0.66\text{ cm}}{1.60\text{ cm}} = 0.41$$



33. Light of wavelength λ = 432 nm passes through a slit 0.100 mm wide, and then travels a distance of 12.0 m before being projected onto a screen 25.0 cm in total width, such that the center of the pattern is directly at the center of the screen

(a) Calculate all places on the screen where the diffraction pattern is completely dark. Give your answers as distances *x* from the center of the screen. You shouldn't give any answers larger than *x* = 12.5 cm.

For diffraction, the dark places are given by $\sin\theta_{\text{dark}} = m\lambda/d$, where m is any non-zero integer. But the position on the screen is given by $x = L\tan\theta$. Because the wavelength is much smaller than the size of the slit, we can use the small angle approximation, so

$$x = L\tan\theta \approx L\sin\theta = \frac{mL\lambda}{d} = m\frac{(12.0\text{ m})(4.32\times10^{-7}\text{ m})}{1.00\times10^{-4}\text{ m}} = m(5.18\times10^{-2}\text{ m})$$
$$= 5.18m\text{ cm}$$

Now we just plug in every possible integer such that this doesn't go off the edge of the screen, which works out to $m = \pm 1, \pm 2$. So we have a total of four dark lines at

$$x = \pm 5.18\text{ cm} \qquad \text{and} \qquad x = \pm 10.37\text{ cm}$$

(b) Calculate the ratio of the intensity at the edge of the screen (12.5 cm from the center) compared to the center

For diffraction through a slit, the intensity is given by

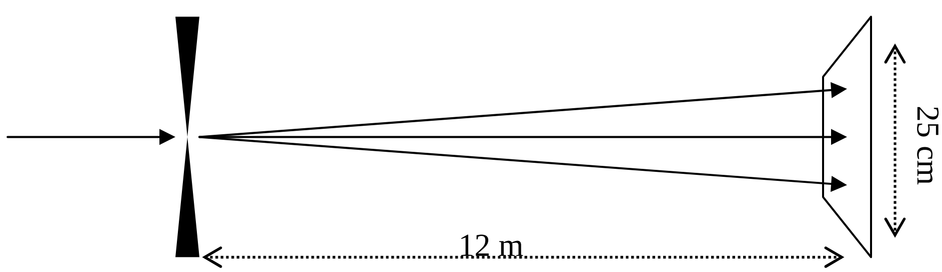
$$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2$$

Again, we can use the small angle approximation to determine that

$$\frac{\pi a\sin\theta}{\lambda} \approx \frac{\pi a\tan\theta}{\lambda} = \frac{\pi a x}{\lambda L} = \frac{\pi(1.00\times10^{-4}\text{ m})(0.125\text{ m})}{(4.32\times10^{-7}\text{ m})(12.0\text{ m})} = 7.575,$$

$$I = I_{\max}\left[\frac{\sin(7.575)}{7.575}\right]^2 = 0.0161I_{\max}.$$

Note that to get this to work, you must be careful to put your calculator into radians. So, the intensity at the edge is about 1.61% of that at the center.



34. A rocket ship of length 150 m passes by us at a velocity of v = 2.1 × 10^8 m/s.

(a) According to us, how long does the rocket ship look?

For most parts of this problem, we will need the Lorentz factor γ, so let's go ahead and find it.

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\left(2.10\times10^8/3.00\times10^8\right)^2}} = 1.40$$

The rocket looks shorter than this to us, so we see it as having length $L = L_P/\gamma = 150 \text{ m}/1.40 = 107 \text{ m}$

(b) According to us, how long does it take from the time the front of the rocket ship passes us until the tail of the rocket ship passes us?

From our perspective, a rocket of length 107 m is going by at v = 2.1 × 10^8 m/s, so the time it takes is

$$\Delta t_P = \frac{L}{v} = \frac{107 \text{ m}}{2.1\times10^8 \text{ m/s}} = 5.10\times10^{-7} \text{ s} = 0.510\ \mu\text{s}$$

It is probably worth mentioning why I have labeled this as the proper time. Imagine a clock sitting at a given location on the ground. Then the clock is at the location of each end of the rocket passing this clock. Since the non-moving clock is at both events (the front end passing and the back end passing us), it is the non-moving clock that measures proper time. In contrast, no clock that is on the moving ship can be at both events, so the moving clock is *not* going to measure proper time.

(c) According to people on the rocket ship, how long does it take them to pass us?

For the people on the rocket ship, the time it takes will be how much time they perceive to have passed on the ground, which is related to the proper time by

$$\Delta t = \gamma\Delta t_P = 1.40\times5.10\times10^{-7} \text{ s} = 7.14\times10^{-7} \text{ s} = 0.714\ \mu\text{s}$$

Another way to get this number is to reason that in their frame, the ground is moving past at v = 2.1 × 10^8 m/s, and it needs to move a distance of 150 m, so we just divide these and get the same answer as before.

(d) Immediately after they have passed, we send them a radio signal, at velocity *c*, of course. According to them, how fast is the radio signal traveling?

This can be done by the subtraction of velocities formula, but it is far easier to simply reason that the velocity of electromagnetic waves is always c, no matter who is observing them. So the answer is c = 3.00×10^8 m/s.