Physics 215 – Elementary Modern Physics

Time in Special Relativity

Because the concept of "simultaneity" or "now" are meaningless in special relativity, it turns out that the concepts of "past" and "future" must be modified as well. Basically, suppose you have two points, P_1 and P_2, with coordinates

$$P_1 = (x_1, y_1, z_1, t_1), \qquad P_2 = (x_2, y_2, z_2, t_2)$$

Then you can't just look at t to tell if one of them is "before" the other, since not all observers will agree on which one is first. To figure out which is first, you first calculate the invariant distance squared, given by

$$s^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - c^2 (t_1 - t_2)^2$$

Roughly speaking, you can only tell unambiguously which one came first if $s^2 \le 0$. In fact, there are five cases in all:

- If $s^2 < 0$ and $t_2 < t_1$, then P_2 is in the <u>past</u> of P_1.
- If $s^2 < 0$ and $t_2 > t_1$, then P_2 is in the <u>future</u> of P_1.

  } time-like separation
- If $s^2 = 0$ and $t_2 < t_1$, then P_2 is on the <u>past light cone</u> of P_1.
- If $s^2 = 0$ and $t_2 > t_1$, then P_2 is on the <u>future light cone</u> of P_1.

  } light-like separation
- If $s^2 > 0$, then P_2 and P_1 are <u>elsewhere</u> compared to each other.

  } space-like separation

The picture at right is a space-time diagram that is supposed to illustrate these ideas. Note that any material object always travels slower than the speed of light, so it must come from the past and will travel into the future. Furthermore, it can be influenced only by objects from the past or the past light cone, and can influence only objects in its future or the future light cone. In particular, light waves that are emitted travel out on the future light cone, and light waves that are absorbed can only come from the past light cone.