Name ___________________

Final Exam
December 14, 2011

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered.

Part I: Multiple Choice (mixed new and review questions) [50 points]

For each question, choose the best answer (2 points each)

1. Which of the following is <u>not</u> a prediction of general relativity?
   A) Light is deflected by the Sun or other astrophysical objects
   B) Time slows down when you are near a gravitational source
   C) If a massive object is shrunk enough, it should collapse to a black hole
   D) Rotating objects can twist space, causing a phenomenon called "frame dragging"
   E) Actually, all of these are predictions of General Relativity

2. Which of the following shows that gravitational waves are almost certainly real, based on current observations?
   A) Large scale interferometers have actually detected the fluctuations between mirrors separated by several kilometers
   B) Sets of gyroscopes in gravity probe B changed the direction they spin as they were affected by gravitational waves
   C) The orbits of satellites have indicated that they are moving randomly as pushed by gravitational waves
   D) The orbits of pairs of neutron stars have been shown to be slowly spiraling inwards, indicating a loss of energy from gravitational waves
   E) No measurement, direct or indirect, has yet indicated the reality of gravitational waves

3. An electron at rest has total energy 0.511 MeV. If we accelerate it to nearly the speed of light, what will the limit of its energy be?
   A) 0.255 MeV B) 0.511 MeV C) 0.766 MeV D) 1.022 MeV E) Infinity

4. In which type of decay is there no readily detectable particle that leaves the nucleus?
   A) α decay B) β^- decay C) β^+ decay D) γ decay E) electron capture

5. Suppose one observer measures the total momentum and energy of a group of particles. On which of the following quantities will all observers agree?
   A) The total momentum of the particles
   B) The total energies of the particles
   C) The invariant mass of the set of particles
   D) The center of mass velocity of the set of particles
   E) Observers will not agree on any of these

6. Below are listed the mass, in u, of various isotopes of phosphorous. Which one is ^{30}P?
   A) 29.9783 B) 30.9738 C) 31.9738 D) 32.9717 E) 33.9736

7. According to quantum mechanics, suppose a particle with energy E encounters a barrier of height V_0 which is smaller than E. What will happen?
   A) The wave is completely reflected
   B) The wave is completely transmitted
   C) The wave is completely absorbed
   D) The wave is partially transmitted and partially reflected
   E) The wave is partially transmitted and partially absorbed

8. What aspect of Mercury's orbit about the sun was explained successfully by Einstein's general theory of relativity?
   A) The fact that the orbit was more oval than elliptical
   B) The fact that the closest point of the orbit of Mercury around the Sun (the perihelion) changed direction, very slowly (precessed)
   C) The fact that the Sun-Mercury distance seemed to be slowly decreasing
   D) The fact that Mercury orbited slightly faster than predicted by Newton's laws
   E) The fact that clocks on spacecraft near Mercury seemed to slow down, compared to those on Earth

9. The proton contains
   A) Three quarks
   B) Three anti-quarks
   C) A quark and an anti-quark
   D) Two leptons
   E) None of the above; protons are believed to be elementary.

10. When an object is moving at high velocity, its length in the direction of motion is ______while in directions perpendicular to the direction of motion it is ______
    A) decreased, decreased
    B) decreased, increased
    C) decreased, stay the same
    D) increased, stay the same
    E) stay the same, decreased

11. Which of the following is true about the quantum mechanical wavelength of an electron?
    A) It is proportional to the energy
    B) It is inversely proportional to the energy
    C) It is proportional to the momentum
    D) It is inversely proportional to the momentum
    E) None of the above is correct

12. Quarks in a baryon are held together by the exchange of which of the following particles?
A) photons B) gluons C) pions D) W-particles E) Higgs bosons

13. Which of the following might be the size of the nucleus of an atom?
A) 10^{-10} m B) 10^{-12} m C) 3×10^{-15} m D) 3×20^{-18} m E) 0

14. The probability density of a particle being at the point *x* is given by
A) $\psi^*\psi$ B) ψ^* C) ψ^2 D) ψ^{*2} E) $\psi+\psi^*$

15. What is the difference between the phase velocity and group velocity of a wave?
A) Phase velocity is how fast the individual peaks move, group velocity is how fast the entire packet moves
B) Phase velocity is how fast the entire packet moves, group velocity is how fast the individual peaks move
C) Phase velocity is how fast a specific wavelength moves; group velocity is the average over all wavelengths
D) Phase velocity is the average over all wavelengths; group velocity is how fast a specific wavelength moves
E) Phase velocity is the speed of the peaks; group velocity is the speed of the troughs

16. The percentage of protons in a stable nucleus is typically about
A) 50 percent for all masses of nuclei
B) 50 percent for light nuclei, 60 percent for heavy nuclei
C) 50 percent for light nuclei, 40 percent for heavy nuclei
D) 60 percent for light nuclei, 50 percent for heavy nuclei
E) 40 percent for light nuclei, 50 percent for heavy nuclei

17. Which of the following contains a succinct summary of general relativity?
A) The gravitational force between two objects is proportional to the product of their masses and inversely proportional to the square of their separation
B) Objects follow straight lines when forces are acting on them, in which case they follow geodesics
C) Time slows down in the vicinity of masses
D) Matter tells space how to curve, and space tells matter how to move
E) To err is human; to forgive, divine

18. If an electron has *l* = 2, then the angular momentum $\vec{L}^2$ will be
A) $\hbar^2$ B) $2\hbar^2$ C) $4\hbar^2$ D) $6\hbar^2$ E) $8\hbar^2$

19. In hydrogen, how do the energy of an electron in the 3d and 3p state differ?
   A) The 3p is significantly lower energy
   B) The 3d is significantly lower energy
   C) The energies are essentially identical
   D) It depends on which specific orbital the electron goes in
   E) It depends on the spin state of the electron

20. What does the uncertainty principle say about the uncertainty in the position of a particle Δx and/or the uncertainty in the momentum Δp?
   A) There is a fundamental limit on the position Δx
   B) There is a fundamental limit on the momentum Δp
   C) There is a fundamental limit on the ratio of these, $\Delta p / \Delta x$
   D) There is a fundamental limit on the product of these, $\Delta p \cdot \Delta x$
   E) The uncertainty relation says nothing about these quantities

21. Suppose we have the coordinates of two events in space-time, *A* and *B*. Under what circumstances can we determine which of the two events is actually first, as viewed by any observer?
   A) Only when they are space-like separated
   B) Only when they are time-like (or light-like) separated
   C) Only when they are at the same location but different times
   D) Only when they are at the same time but different locations
   E) Under none of these conditions will all observers agree on which came first

In questions 22 – 25, you will be given a reaction, and you must state what type of interaction it is. A mimi-table of particles with their spin, mass, strangeness, and type is listed at right. All particles have charge 0. Masses are in MeV/c².

Name	Spin	Mass	St.	Type
n^0	½	940	0	baryon
Λ^0	½	1116	-1	baryon
Σ^0	½	1193	-1	baryon
Δ^0	3/2	1234	0	baryon
π^0	0	139	0	meson
$\bar{K}^0$	0	495	-1	meson
γ	1	0	0	other

22. $\Lambda^0 \to n^0 + \pi^0$
   A) impossible B) strong C) weak D) electromagnetic E) gravity

23. $\Lambda^0 \to n^0 + \bar{K}^0$
   A) impossible B) strong C) weak D) electromagnetic E) gravity

24. $\Sigma^0 \to \Lambda^0 + \gamma$
   A) impossible B) strong C) weak D) electromagnetic E) gravity

25. $\Delta^0 \to n^0 + \pi^0$
   A) impossible B) strong C) weak D) electromagnetic E) gravity

Part II: Short answer (review material) [20 points]

Choose **two** of the following three questions and give a short answer (1-3 sentences) (10 points each).

26. Explain qualitatively how a pole vaulter whose pole is the same length as a barn can have the far end of his pole hit the back of the barn, even though (to a stationary observer) it looks like the pole is shorter than the barn.

27. Why is it that we so rarely notice quantum effects when everyday events occur; for example, why don't we see diffraction when people walk through doors?

28. What does spin mean? What is the spin of the electron? What are the possible values of the z-component of the spin of the electron, S_z?

Part III: Short answer (new material) [30 points]

Choose **three** of the following four questions and give a short answer (1-3 sentences) (10 points each).

29. Explain qualitatively why very heavy nuclei tend to be unstable and undergo α-decay.

30. In the movie "Roxanne," Darryl Hannah, playing a scientist describing quarks, says, "There are at least six different types: Up, Down, Strange, Charmed, Bottom and Top. Top and Bottom Quarks are most common." Explain what is right or wrong about her statements.

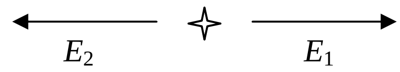
31. What are the four forces of nature? Which three are described by the standard model of particle physics? Of these three, which ones must conserve electric charge? Which ones must conserve strangeness?

32. Explain in your own words the distinction between curved coordinates and curved spacetime.

Part IV: Calculation (review material) [40 points]

Choose **two** of the following three questions and perform the indicated calculations (20 points each)

33. According to rumors, there is some indication that the Higgs particle may have been discovered at the LHC by its decay to pairs of (massless) photons, $H \to \gamma\gamma$. Suppose the photons come out back to back, as illustrated below, and have energies of $E_1 = 42.0 \text{ GeV}$ and $E_2 = 93.0 \text{ GeV}$.



(a) What is the momentum of each of the photons, in GeV/c? Remember that momentum is a vector!
(b) What is the total energy and momentum of the initial Higgs?
(c) What is the mass of the Higgs, in GeV/c^2?
(d) What was the initial velocity of the Higgs, as a fraction of the speed of light?

34. A laser with wavelength 237.0 nm is shone on a sample of cesium, which has a work function of $\phi = 2.14 \text{ eV}$.

(a) What is the frequency of the incident photon?
(b) What is the kinetic energy of those electrons that are liberated by these photons?
(c) Another laser of unknown frequency is then brought to bear, and it is found that the electrons escaping have a kinetic energy of 1.05 eV. What is the frequency of this laser?
(d) What is the minimum frequency and corresponding wavelength that can free an electron?

35. A particle has wave function given by

$$\psi(x) = \begin{cases} x\sqrt{12(a-x)}\big/a^2 & \text{for } 0 \le x \le a \,, \\ 0 & \text{otherwise} \,. \end{cases}$$

This wave function has already been properly normalized.

(a) What is the probability that this particle lies in the region $0 < x < \frac{1}{2}a$?
(b) Find the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ for this wave function.
(c) What is the uncertainty Δx in the position of this particle?

Part V: Calculation (new material): [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

36. Plutonium was once thought to be produced only artificially, but it is now known that a very small amount of ^{244}Pu, with a half-life of 80.0 million years, is probably primordial.
    (a) Suppose you had 1.000 g of ^{244}Pu in a sample of rock when the Earth first formed. How many atoms of ^{244}Pu is this?
    (b) What is the decay rate λ in s^{-1} for this isotope?
    (c) When the sample first existed, how many decays per second would there have been?
    (d) The Earth is approximately 4.55 billion years old. How many atoms of ^{244}Pu would remain from the original 1.00 g?

37. Photocopied with the equation on the next page is a portion of Appendix A from the text. ^{50}V is an unstable nucleus which may have multiple decay modes.
    (a) What would be the resulting isotope if this isotope underwent β^- decay? What if it underwent β^+ decay? What if it underwent α decay?
    (b) What is the Q-value for each of these processes? Which of these modes is allowed or excluded?

38. There is a particle Ξ^{**-} which decays by strong interactions as follows:

$$\Xi^{**-} \rightarrow \Sigma^+ \pi^- X$$

The Ξ^{**-} and Σ^+ are baryons and the π^- is a meson. The mass, spin, strangeness and charge of the other particles are listed above.

All masses in MeV/c^2

Name	Mass	Spin	Strange	Charge
Ξ^{**-}	2025	5/2	-2	-1
Σ^+	1179	½	-1	+1
π^-	135	0	0	-1

    (a) What is the charge and strangeness of the X particle?
    (b) Is it a baryon, anti-baryon, or a meson?
    (c) Is it a fermion or a boson?
    (d) What, if anything, can you conclude about the mass of the X?

39. A neutron star is discovered to have a radius of $R = 15.24\text{ km}$. It is found that the hydrogen-α line, with a normal wavelength of $\lambda_0 = 656.28\text{ nm}$, is observed instead with a wavelength of $\lambda = 700.35\text{ nm}$.
    (a) If we spent 60.0 minutes on the surface of the neutron star, as viewed by a distant observer, how much would we age (assuming we can survive the gravity)?
    (b) If this neutron star's mass were compressed to make a black hole, what would be the Schwarszschild radius R?
    (c) What is the mass of the neutron star, in solar masses ($M_\odot = 1.989\times10^{30}\text{ kg}$).

Equations

Constants:

$h = 6.626\times10^{-34}\ \mathrm{J\cdot s} = 4.136\times10^{-15}\ \mathrm{eV\cdot s}$

$\hbar = 1.055\times10^{-34}\ \mathrm{J\cdot s} = 6.582\times10^{-16}\ \mathrm{eV\cdot s}$

$G = 6.673\times10^{-11}\,\mathrm{m^3/kg/s^2}$

$N_A = 6.022\times10^{23}$

$1\ \mathrm{y} = 3.156\times10^{7}\ \mathrm{s}$

$\mathrm{u} = 931.494\ \mathrm{MeV}/c^2$

$\mathrm{u} = 1.661\times10^{-27}\,\mathrm{kg}$

$2m_ec^2 = 1.022\ \mathrm{MeV}$

$M_{\mathrm{He}} = 4.002602\ \mathrm{u}$

Gravitational time dilation: $\tau = t\sqrt{1-\dfrac{2GM}{c^2r}}$ Red Shift: $\lambda = \lambda_0\left(1-\dfrac{2GM}{c^2r}\right)^{-1/2}$

Photoelectric Effect: $eV_{\max} = hf - \phi$

Mass, energy, momentum: $\vec{p} = \gamma m\vec{u}$, $E = \gamma mc^2$, $E^2 - \vec{p}^{\,2}c^2 = m^2c^4$, $\dfrac{\vec{p}c}{E} = \dfrac{\vec{u}}{c}$

Schwarzschild radius: $R_S = 2GM/c^2$

Expectation values: $\langle\mathcal{O}\rangle = \int_{-\infty}^{\infty}\psi^*(x)\,\mathcal{O}\,\psi(x)\,dx$ Uncertainty: $(\Delta\mathcal{O})^2 = \langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2$

Isotope Masses