Name __________________

Final Exam
December 12, 2008

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered. Some helpful equations and a short table of isotopes can be found on the last page

Part I: Multiple Choice (mixed new and review questions) [50 points]
For each question, choose the best answer (2 points each)

1. The purpose of giant interferometers that very carefully monitor the distance in two perpendicular directions (LIGO) is to
   A) Look for "frame dragging" effects caused by the rotation of the Earth
   B) Look for "time dilation" effects caused by time slowing down due to gravity
   C) Look for "precession of the perihelion" in Earth's orbit
   D) Look for "gravitational deflection of light" caused by light passing near the Sun
   E) Look for "gravity waves," distortions in space time caused by distant astronomical events

2. The central source of power for active galactic nuclei is generally believed to be
   A) A giant black hole in the center of the galaxy
   B) A giant star that recently went supernova
   C) Concentrated magnetic fields
   D) Matter-antimatter annihilation
   E) Relativistic electric currents

3. To find the probability of finding a particle with wave function $\psi(x)$ in the region $0 < x < a$, which of the following integrals should you perform?
   A) $\int_0^a \psi(x)\,dx$
   B) $\int_0^a \psi^2(x)\,dx$
   C) $\int_0^a \psi^*(x)\,dx$
   D) $\int_0^a |\psi(x)|\,dx$
   E) $\int_0^a |\psi(x)|^2\,dx$

4. According to the general theory of relativity, the orbit of Mercury around a source of gravity like the Sun should be:
   A) A circle that stays constant indefinitely
   B) A circle, but one that gradually shrinks towards zero over time
   C) An ellipse which it follows repeatedly
   D) Pretty close to an ellipse, but the long axis will slowly rotate (precess) around the Sun
   E) Pretty close to an ellipse, but the plane of the orbit will slowly wobble (nutate) up and down

5. If an object is moving in the *x*-direction, how will a stationary observer see its dimensions change?
   A) It will shrink, but only in the direction of motion
   B) It will expand, but only in the direction of motion
   C) It will shrink in all three dimensions
   D) It will expand in all three dimensions
   E) It will shrink in the direction of motion and expand in the other dimensions

6. The Schwarzschild solution, the formula that describes the metric around a black hole, can also be used
   A) For any spherically symmetric source of gravity, both inside and outside the object
   B) For any spherically symmetric source of gravity, but only outside the object
   C) For any spherically symmetric source of gravity, but only inside the object
   D) For neutron stars and other high mass objects, but not for planets or the Sun
   E) The Schwarzschild solution is useful only for black holes

7. The differential form of the distance formula, $c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ is superior to the form we originally learned, because
   A) You can use it to calculate the proper time for curved paths, not just straight ones
   B) It can be used at relativistic speeds, not just non-relativistic speeds
   C) In this form it already has curvature in it, an important part of general relativity
   D) It can be used for tachyons, not just slower-than-light particles
   E) It is smaller, and with today's economy, we all need to cut back

8. The main evidence for dark matter in spiral galaxies is
   A) Dust and other objects that obscure our view of the center
   B) High speed stars orbiting the center, indicating giant black holes
   C) Flat rotation curves showing that rotation velocity doesn't decrease as you get far from the center of a galaxy
   D) Shift in the wavelength of the 21 cm line indicating gravitational red shift from all the mass
   E) Huge clouds of cool hydrogen – "molecular clouds" – which is presumably the source of the gravity

9. Rutherford found that α-particles fired at very thin gold foil usually went almost straight through, but occasionally deflected at large angles, even straight backwards. From this he concluded that
   A) The alpha particle must be composed of a highly elastic material
   B) The electrons orbited the nucleus, rather than the other way around
   C) The atom must contain protons, in addition to electrons
   D) Most of the mass and positive charge of an atom must be concentrated in a small region he named the nucleus
   E) The alpha particle must be much lighter than the electrons, since they could bounce it backwards

10. What is the minimum and maximum energy that an object with a rest mass of *m* can have?
    A) 0, mc^2
    B) 0, infinity
    C) mc^2, infinity
    D) mc^2, mc^2 (i.e., it's always mc^2)
    E) None of the above

11. According to quantum mechanics, which types of objects listed below have wave properties?
    A) Particles of light (photons), but not electrons
    B) Electrons, but not photons
    C) Electrons and photons, but not atoms
    D) Electrons, photons, and atoms, but not molecules
    E) Electrons, photons, atoms, and molecules

12. The difference between wave velocity and group velocity is:
    A) Wave velocity is how fast the peaks move, group velocity is how fast the troughs move
    B) Wave velocity is how fast the troughs move, group velocity is how fast the peaks move
    C) Wave velocity is how fast the the individual peaks and troughs move, and group velocity is how fast whole wave as a "lump" moves
    D) Wave velocity is how fast the whole wave as a "lump" moves, and group velocity is how fast the individual peaks and troughs move
    E) Wave velocity and group velocity are different names for the same thing

13. In special relativity, which of the following is an accurate comment about conservation of momentum?
    A) It still always applies, and in exactly the same way it did before special relativity
    B) It still always applies, but you have to use a different formula for momentum
    C) It only sometimes applies, but the formula for momentum is still the same
    D) It only sometimes applies, and you have to use a different formula for momentum
    E) There is no conservation of momentum rule in special relativity

14. Which of these might be the actual mass of a ^{209}Bi atom?
A) 83.020 u B) 125.987 u C) 208.980 u D) 209.984 u E) 292.016 u

15. Which of the following accounts for the phenomenon of quantum tunneling, where a particle makes it through a barrier that is taller than the energy of the particle?
A) Thermal fluctuations occasionally cause the particle to have extra energy, making it over the barrier
B) Due to the uncertainty principle, the particle's position is uncertain, so occasionally it just winds up on the other side of the barrier
C) Due to quantum fluctuations, the barrier height occasionally fluctuates low enough for the particle to jump over it
D) The wave function in the classically forbidden region dies exponentially, which means the probability is small, but not zero
E) Virtual particle-antiparticle pairs appear out of nowhere and give the particle the extra energy to jump the barrier, before disappearing

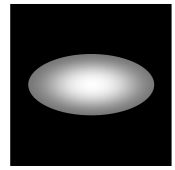
16. For heavy nuclei, the most stable nuclei tend to have about what fraction of neutrons?
A) 90% B) 60% C) 50% D) 40% E) 10%

17. According to special relativity, how does the speed of light in vacuum differ as viewed by different observers?
A) Only a special observer at rest will see it moving at *c*, all other observers will see it moving at different speeds
B) Only observers moving at low velocities will see it moving at *c*, all other observers will see it moving at different speeds
C) Only observers moving at relativistic velocities will see it moving at *c*, all other observers will see it moving at different speeds
D) Because it is moving so fast, it is impossible for any observer to measure its speed
E) All observers will see it moving at the same speed *c*

18. Which was an important aspect of the Bohr model of the atom that had not been considered previously?
A) Photons come in only discrete chunks of energy
B) Atoms can have electrons orbiting the nucleus in ellipses, not just circles
C) The frequency of light coming from an atom is determined by the energy difference from the electron jumping from one level to another
D) The frequency of light coming from an atom is determined by the frequency at which an electron orbits the atom
E) Electrons should be described as waves, not just photons

19. How many neutrons are there in ^{42}Ca (element # 20)?
A) 42 B) 20 C) 62 D) 22 E) insufficient information

20. If a picture of a galaxy looked like the sketch at right, how should you most likely classify it?



A) Sb B) SBc C) E0 D) E5 E) Irr

21. Which types of particles are affected by the strong force, or strong nuclear force?
   A) Electrons (only)
   B) Protons (only)
   C) Neutrons (only)
   D) Protons and electrons, but not neutrons
   E) Protons and neutrons, but not electrons

22. In which of the following decays do you <u>not</u> get any neutrinos or anti-neutrinos out?
   A) α-decay (only)
   B) β^+-decay (only)
   C) electron capture
   D) α-decay and electron capture
   E) α-decay and β^+-decay

23. The energy levels of the simple harmonic oscillator is given in back. If a particle in a simple harmonic oscillator moves from level n = 3 to level n= 1, how much energy would be released?
   A) $\hbar\omega$ B) $2\hbar\omega$ C) $3\hbar\omega$ D) $\frac{7}{2}\hbar\omega$ E) $5\hbar\omega$

24. Where in a galaxy like our own is one likely to find globular clusters?
   A) The bulge B) The halo C) The disk D) The nucleus E) Hershey, PA

25. One version of the uncertainty principle relates which of the following two properties?
   A) Momentum and position
   B) Momentum and energy
   C) Momentum and angular momentum
   D) Energy and position
   E) Energy and momentum

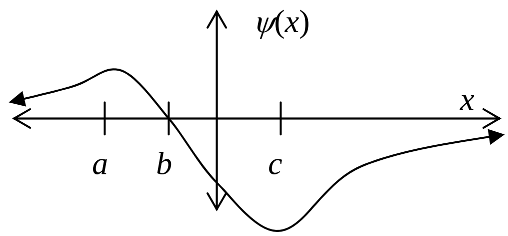
Part II: Short answer (review material) [20 points]

Choose **two** of the following three questions and give a short answer (1-3 sentences) (10 points each).

26. In a variation of the barn-pole paradox, a runner runs completely through the barn (in the front, out the back). The runner claims both ends of the pole simultaneously sticks out of the barn, while the farmer claims both ends of the pole are simultaneously inside the barn. Explain how special relativity helps resolve this apparent paradox.

27. In the Compton effect, why is it that the wavelength of an X-ray changes when it scatters off of a stationary electron?

28. The graph at right illustrates a wave function. Explain where the particle is <u>most</u> likely to be and where it is <u>least</u> likely to be. You don't have to perform any computations.



Part III: Short answer (new material) [30 points]

Choose **three** of the following four questions and give a short answer (1-3 sentences) (10 points each).

29. How does γ-decay change the value of Z and A? How dangerous is it, compared to other type of radiation? What types of nuclei undergo γ-decay?

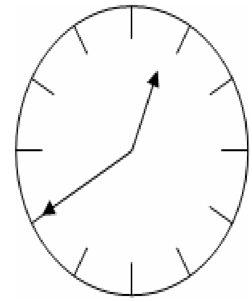
30. How can we tell that the source of power in some active galaxies must be very small? Give any relevant formulas

31. Explain qualitatively what a geodesic is. Under what circumstances, according to general relativity, do objects follow geodesics?

32. Iron (Fe) is element 26. Would you expect ^{56}Fe to be a fairly stable nucleus, and why?

Part IV: Calculation (review material) [40 points]

Choose **two** of the following three questions and perform the indicated calculations (20 points each)



33. A circular clock is set to 12:00, and then accelerated to a constant high speed. Less than an hour later, it looks like the sketch at right, such that a stationary observer sees the clock having an *apparent* size of 10 cm tall and 8 cm wide, as sketched at right.
    (a) Which way is the clock moving, *i.e.*, is it moving up/down or left/right? What is the actual size of the clock?
    (b) What is γ for this clock, and what is the actual velocity?
    (c) You can tell how much time has passed from the sketch, according to the moving clock. According to an observer at rest, how many minutes have passed?

34. A laser produces light with a wavelength of 393.0 nm
    (a) Find the frequency f, angular frequency ω, and wave number k for this wave.
    (b) Find the energy (in eV) and momentum (in any units you want) for this photon.
    (c) The laser is shone on a variety of metals, listed at right. In which cases would electrons be emitted? In those cases where the electron is emitted, tell me the kinetic energy of the resulting electrons.

Metal	ϕ (eV)
Cs	2.14
K	2.29
Li	2.93
Mg	3.66
Sn	4.42

35. A Li^{++} ion has exactly one electron in it. This single electron is in the $n = 3$ state.
    (a) What is the binding energy of this electron (in eV)?
    (b) The total angular momentum squared L^2 and z-component of the angular momentum L_z are measured. What are the possible outcomes? Explain any constraints relating these *i.e.* you might say something like "if L^2 is $-17\hbar^2$ then L_z is $\pm\sqrt{3}\hbar$"
    (c) The total spin squared S^2 and z-component of the angular momentum S_z are measured. What are the possible outcomes? Explain any constraints relating these.
    (d) If the electron suddenly shifted to the $n = 4$ state, would energy be emitted or absorbed? How much energy (in eV)?

Part V: Calculation (new material): [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

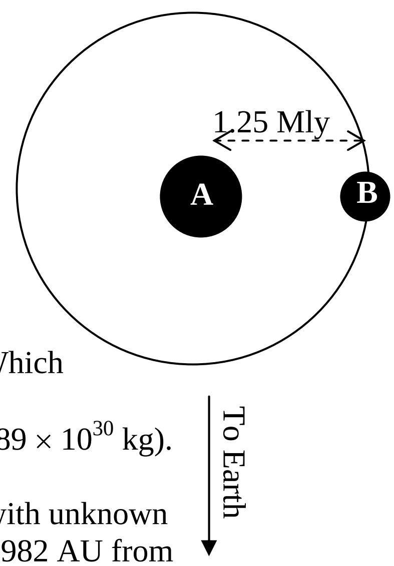
[questions appear on next page]

36. ^{137}Cs is an unstable isotope that decays with a half-life of 30.23 y (y = 3.156×10^7 s). It has been suggested that terrorists might include such an isotope in a "dirty bomb" designed to create high levels of radioactivity for an extended time. Suppose that a dirty bomb of ^{137}Cs is exploded which produces counts initially at the rate of 13 million counts per second.
   (a) What is the decay constant λ in sec^{-1} for ^{137}Cs?
   (b) Approximately how many atoms of ^{137}Cs would be required?
   (c) How much mass (in g) of ^{137}Cs would be required?
   (d) How long will we have to wait until the decay rate drops to 1 million per second?

37. Photocopied with the equation on the next page is a portion of Appendix A from the text. ^{210}Pb, even though it is highly unstable, exists naturally because it is produced by certain nuclear processes. The goal of this problem is to determine which process might *produce* ^{210}Pb. You might want to organize your answer into a table.
   (a) What isotope is the parent if the process that produces ^{210}Pb is α-decay? β^- decay? β^+ decay? Electron capture?
   (b) What is the Q-value for each of these processes?
   (c) Which processes are actually possible sources for ^{210}Pb?

mode	Parent	Q (MeV)	?
α			
β^-			
β^+			
e.c.			

38. A large central galaxy A has a smaller satellite galaxy B orbiting it at a distance of 1.25 Mly (1 Mly = 9.46×10^{21} m). The hydrogen-α line, normally at a wavelength of 656.28 nm, is observed to be at 678.35 nm coming from A and 678.02 nm coming from galaxy B.
   (a) What is the approximate radial velocity of each of these galaxies towards or away from us, in km/s?
   (b) Assume galaxy B is in circular orbit around galaxy A, such that we are viewing the orbit edge on (see sketch at right). Which direction, and at what velocity, is galaxy B orbiting?
   (c) Estimate the mass of galaxy A, in solar masses (1 $M_{\text{Sun}} = 1.989 \times 10^{30}$ kg).



39. A researcher is studying the black hole at the center of a galaxy with unknown mass. He spends two days, as measured by him, at a distance of 982 AU from the center of the black hole.
   (a) When he rejoins his companions far from the black hole, he discovers that his clock is off from other clocks by exactly one hour. Will it be "slow" (i.e., behind them by an hour) or "fast" (i.e., ahead of them by an hour).
   (b) What is the Schwarzschild radius for this black hole, in AU?
   (c) What is the mass of the black hole, in solar masses? (1 AU = 1.496×10^{11} m, 1 $M_{\text{Sun}} = 1.989 \times 10^{30}$ kg).

Equations

Special Relativity Formulas

$$s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 = -(c\tau)^2$$

$$x' = \gamma(x - vt) \quad \text{and} \quad y' = y$$

$$t' = \gamma(t - vx/c^2) \quad \quad z' = z$$

Constants

$$h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}$$

$$\hbar = 1.055 \times 10^{-34}\ \text{J}\cdot\text{s} = 6.582 \times 10^{-16}\ \text{eV}\cdot\text{s}$$

$$G = 6.673 \times 10^{-11}\,\text{m}^3/\text{kg}/\text{s}^2$$

Nuclear Decay

$$\text{u} = 931.494\ \text{MeV}/c^2$$

$$= 1.661 \times 10^{-27}\,\text{kg}$$

$$N_A = 6.022 \times 10^{23}$$

$$2m_e c^2 = 1.02200\ \text{MeV},$$

$$M_{\text{He}} = 4.002602\ \text{u}$$

Hydrogen-like atoms

$$E_n = -\frac{(13.6\ \text{eV})Z^2}{n^2}$$

Simple Harmonic Oscillator

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

Orbital Velocity

$$v = \sqrt{\frac{GM}{r}}$$

General Relativity

$$\tau = t\sqrt{1 - \frac{2GM}{c^2 r}}$$

$$\lambda = \lambda_0\left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}$$

$$R_S = \frac{2GM}{c^2}$$

Isotope Masses