Name ___________________

Test 3

November 7, 2005

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. Some possibly useful formulas can be found below.

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$

Reflection off a step:

$$R = \begin{cases} \left(\dfrac{\sqrt{E}-\sqrt{E-V_0}}{\sqrt{E}+\sqrt{E-V_0}}\right)^2 & \text{if } E > V_0 \\ 1 & \text{if } E < V_0 \end{cases}$$

1D square well:

$$E_n = \frac{\pi^2\hbar^2n^2}{2mL^2}$$
$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$$
$$n = 1,2,3,\ldots$$

Harmonic Osc.

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right)$$
$$n = 0,1,2,\ldots$$

$$H = \frac{p^2}{2m} + V(x)$$

Barrier penetration:

$$T \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)\exp\left(-2L\sqrt{2m(V_0-E)}/\hbar\right)$$

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. How does the energy formula for hydrogen from the quantum mechanical theory differ from the energy formula derived from the Bohr model of the atom?
   A) The quantum mechanical formula gives energies twice as large
   B) The quantum mechanical formula gives energies half as large
   C) The quantum mechanical formula gives energies just as big, but with the opposite sign
   D) They give the same value
   E) Because of the uncertainty principle, quantum mechanics makes no such prediction.

2. A particle is placed in a spherically symmetric potential $V(r)$, and solutions are then found of the form
   $$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$$
   Which of these functions will depend on the exact form of $V(r)$?
   A) R will depend on $V(r)$, but not Y.
   B) Y will depend on $V(r)$, but not R.
   C) Both R and Y will depend on $V(r)$.
   D) Neither R nor Y will depend on $V(r)$.
   E) There is insufficient information to answer this question

3. Which of the following is an impossible suborbital for an electron in hydrogen?
   A) 1s B) 2d C) 4d D) 2s E) 3p
4. Which of the following more or less describes what spin is?

A) It is the way my head feels after a Modern Physics test.
B) It is the energy associated with one object orbiting another
C) It is the energy associated with an object rotating on its axis
D) It is the angular momentum associated with one object orbiting another
E) It is the angular momentum associated with an object rotating on its axis

5. If l is the total angular momentum quantum number, what is the formula for the square of the total angular momentum L^2?
A) $\hbar l$ B) $\hbar^2 l^2$ C) $\hbar^2 \left(l^2 + l\right)$ D) $\hbar^2 \left(l^2 - l\right)$ E) $\pm\hbar l$

6. Why is the quantum mechanical pressure for electrons so much more than it is for other particles, like protons or neutrons?
A) Electrons have a smaller mass
B) Electrons repel each other due to electrostatic repulsion
C) Electrons are intrinsically bigger than other particles
D) Quantum mechanics only applies to electrons, not to other particles
E) I have no idea; please mark this one wrong

7. A particle with energy E encounters a thick barrier with height $V_0 > E$. According to quantum mechanics, can the particle go through the barrier?
A) no, never
B) rarely, though the probability will be extremely small
C) often, the probability will not be small
D) usually, though there is a tiny probability that it will be reflected
E) yes, always

8. Which of the following is <u>not</u> true about the three-dimensional Schrödinger equation?
A) Derivative terms occur in all three dimensions
B) The wave function is a function of all three variables, x, y, and z.
C) The norm squared of the wave function $|\psi|^2$ is still the probability density for the particle to be at a particular position
D) There is only one energy E, not three for the three dimensions
E) All of these actually <u>are</u> true about the three-dimensional Schrödinger equation.

9. Which of the following is the energy of a particle of mass m confined in one dimension to a space of size L, if the particle is in the <u>first excited state</u> (the second lowest energy state)?
A) $\pi^2\hbar^2/4mL^2$ B) $\pi^2\hbar^2/2mL^2$ C) $\pi^2\hbar^2/mL^2$ D) $2\pi^2\hbar^2/mL^2$ E) $4\pi^2\hbar^2/mL^2$

10. Electrons all have the same spin quantum number s, but other particles have different spins. Which of the following is *not* a possible value for the spin quantum number s?
A) 0   B) $\frac{3}{2}$ C) -1   D) 2   E) $\frac{5}{2}$

Part II: Short answer [20 points]

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).

11. Give a complete list of every possible value for the set of quantum numbers (n, l, m, m_S) describing Hydrogen for $n = 2$. Your answer should be a list of quadruples of numbers.

12. Explain what the Hamiltonian is, giving a formula for it. If I calculate the expectation value of the Hamiltonian $\langle H \rangle$, what does this tell me?

13. Besides satisfying Schrödinger's equation, the wave function ψ must satisfy some additional constraints. Give at least three of them.

Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each).

14. The ground state wave function of the harmonic oscillator is given by

$$\psi(x) = \left(\frac{\hbar}{\pi m\omega}\right)^{1/4} \exp\left(-\frac{\hbar}{2m\omega}x^2\right)$$

(a) Where is the most likely place to find this particle? Is there any place where we are certain the particle is not?
(b) What is the energy of this solution? You are *not* expected to solve Schrödinger's equation to answer this.
(c) Using this solution of the time-independent Schrödinger equation, find a solution of the time-dependent Schrödinger equation.

15. The wave function at a particular time is given by

$$\psi(x) = \begin{cases} \sqrt{105/8a^7}\left(a^2x - x^3\right) & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the expectation value of p; that is, compute $\langle p \rangle$.
(b) What is the expectation value of p^2; that is, compute $\langle p^2 \rangle$.
(c) What is the uncertainty in the momentum Δp ?

16. An electron with velocity $v = 1.00 \times 10^{7}$ m/s and mass $m = 9.109 \times 10^{-31}$ kg impacts on a potential barrier of height $V_0 = 4.00 \times 10^{-17}$ J
(a) What is the probability that it is reflected?
(b) Suppose the barrier is increased to $V_0 = 8.00 \times 10^{-17}$ J. Now what is the probability that it is reflected?
(c) The potential is now reversed, $V_0 = -8.00 \times 10^{-17}$ J. What is the probability that it is reflected?

17. A particle in one dimension has wave function

$$\psi(x) = \frac{N}{\sqrt{x^2 + a^2}}$$

Some possibly useful integrals and other math facts can be found below.
(a) What is the correct normalization N?
(b) What is the probability that the particle will lie in the region $0 < x < a$?

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right), \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \ln(\infty) = \infty,$$

$$\ln(0) = -\infty, \quad \tan^{-1}(\pm\infty) = \pm\tfrac{\pi}{2}, \qquad \tan^{-1}(\pm 1) = \pm\tfrac{\pi}{4}.$$