

Name \_\_\_\_\_  
Test 3  
November 8, 2004

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

**Part I: Multiple Choice [20 points]**

For each question, choose the best answer (2 points each)

- Suppose I were to measure the total internal angular momentum (spin) of a single electron,  $\vec{S}^2$ . What value would I get?
  - $\frac{1}{2}\hbar$
  - $\frac{1}{4}\hbar^2$
  - $\frac{3}{4}\hbar^2$
  - It depends on the spin of that particular electron. It is different for different electrons.
  - It depends on what shell and subshell it is in. It is different for different electrons.
- Under what circumstances can one use the time-independent Schrödinger equation?
  - Only when the potential  $V$  is independent of both space and time
  - Only when the potential  $V$  is independent of space, though it may depend on time
  - Only when the potential  $V$  is independent of time, though it may depend on space
  - Only on Sundays and holidays
  - The time-independent Schrödinger equation can always be used
- What about the Hydrogen atom problem allowed us to break the problem into a radial part and an angular part?
  - Because we knew all the hydrogen atom wave functions would be spherically symmetric
  - Because the hydrogen atom has an inverse square law force
  - Because the proton is approximately spherical
  - Because the electron is moving much slower than the speed of light
  - Because the potential depends only on the distance  $r$ , not on  $\theta$  and  $\phi$ .
- If I measure the expectation value of the Hamiltonian, the result will give me the expectation value of the
  - momentum
  - total energy
  - kinetic energy
  - potential energy
  - total angular momentum

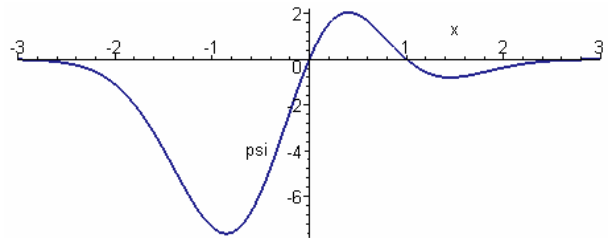
5. Which operator, in quantum mechanics, is proportional to  $\partial/\partial x$  ?
- A)  $x$ , the position operator
  - B)  $p$ , the momentum operator
  - C)  $\Delta x$ , the uncertainty in the position
  - D)  $H$ , the Hamiltonian
  - E)  $L_z$ , the  $z$ -component of the angular momentum
6. A 5d electron in hydrogen has
- A)  $n = 5$  and  $m = 3$
  - B)  $n = 5$  and  $m = 4$
  - C)  $l = 5$  and  $m = 3$
  - D)  $n = 5$  and  $l = 4$
  - E)  $n = 5$  and  $l = 3$
7. What does the magnetic quantum number  $m$  (or  $m_l$ ) tell you?
- A) The angular momentum around the  $z$ -axis,  $L_z$
  - B) The total angular momentum,  $\vec{L}^2$
  - C) The internal angular momentum (spin) around the  $z$ -axis,  $S_z$
  - D) The total internal angular momentum,  $\vec{S}^2$
  - E) The energy  $E$ .
8. How many electrons fit into the 2p subshell of hydrogen?
- A) One
  - B) Three
  - C) Five
  - D) Six
  - E) None of the above
9. If you put an electron in an infinite square well, which of the following is not true?
- A) There is a pressure associated with this confinement, even if the temperature is zero
  - B) The wave function must be continuous at the two boundaries of the square well
  - C) The energy of the electron comes from the electric repulsion of the walls acting on the electron
  - D) The energy of the ground state is greater than zero
  - E) Light particles, like electrons, have more energy than heavy particles, like neutrons, in the same quantum state

10. What is the minimum energy of the harmonic oscillator (formulas are at the start of part III, below)?
- A)  $-\hbar\omega$
  - B)  $-\frac{1}{2}\hbar\omega$
  - C) 0
  - D)  $\frac{1}{2}\hbar\omega$
  - E)  $\hbar\omega$

**Part II: Short answer [20 points]**

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).

11. Plotted at right is a wave function. What is the least likely place(s) to find the particle, and what is the most likely place(s) to find the particle?



12. Give a qualitative description, possibly including a diagram, of what happens when an electron of energy  $E < V_0$  encounters a step of height  $V_0$  potential. In particular, tell me what the wave function looks like to the right of the step.
13. Give a list of the four quantum numbers that completely describe an electron in hydrogen (just the letters is fine), and give me a list of restrictions on these numbers.

**Part III: Calculation: [60 points]**

Choose **three** of the following four questions and perform the indicated calculations (20 points each). You may find the following formulas helpful:

$$\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$$

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$$

$$T \approx 16 \frac{E}{V_0} \left(1 - \frac{E}{V_0}\right) \exp(-2\alpha L)$$

14. An electron of mass  $m = 9.109 \times 10^{-31} \text{ kg}$  has kinetic energy  $E = 1.000 \times 10^{-19} \text{ J}$  and is impacting on a barrier that is  $V_0 = 2.000 \times 10^{-19} \text{ J}$  high and 1.00 nm thick. What is the probability that the electron makes it out to the other side? Show your work.

15. The one-dimensional square well has potential and ground state wave functions given by

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise} \end{cases} \quad \text{and} \quad \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

These waves exist only in the allowed region  $0 < x < L$ .

- (a) Show that this wave functions satisfies the time-independent Schrödinger equation, and determine the energy  $E$ .
- (b) Use this solution to find a solution  $\Psi(x, t)$  to the time-dependent Schrödinger equation.
16. At the end of this problem can be found a set of possibly useful integrals. A wave function is of the form

$$\psi(x) = \frac{\sqrt{2a^3/\pi}}{x^2 + a^2}$$

- (a) What is the expectation value of  $x$ , that is, compute  $\langle x \rangle$ .
- (b) What is the expectation value of  $x^2$ , that is, compute  $\langle x^2 \rangle$ .
- (c) What is the uncertainty in the position  $\Delta x$  ?

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2} &= \frac{\pi}{a} & \int_{-\infty}^{\infty} \frac{xdx}{x^2 + a^2} &= 0 & \int_{-\infty}^{\infty} \frac{x^2 dx}{x^2 + a^2} &= \infty \\ \int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^2} &= \frac{\pi}{2a^3} & \int_{-\infty}^{\infty} \frac{xdx}{(x^2 + a^2)^2} &= 0 & \int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)^2} &= \frac{\pi}{2a} \end{aligned}$$

17. Listed below are some of the angular and radial wave functions for hydrogen.

- (a) Write explicitly the wave function  $\psi_{210}(r, \theta, \phi)$  of the electron
- (b) What are the values of  $L^2$ ,  $L_z$ , and  $E$  for this electron?
- (c) What are the most likely angles  $\theta$  compared to the  $z$ -axis where this electron is likely to be found? Recall that  $0 \leq \theta \leq \pi$ .

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}} \exp(-r/a_0) \quad R_{20}(r) = \frac{a_0 - r/2}{\sqrt{2a_0^5}} \exp(-r/2a_0)$$

$$R_{21}(r) = \frac{r}{\sqrt{24a_0^5}} \exp(-r/2a_0)$$

$$Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos \theta \quad Y_{20}(\theta, \phi) = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$$

$$Y_{21}(\theta, \phi) = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$