

Name \_\_\_\_\_

## Test 2 October 14, 2009

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. The equations below may be helpful with some problems.

<u>Constants</u>
$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$
$\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$
$k_B = 1.3807 \times 10^{-23} \text{ J/K} = 8.6173 \times 10^{-5} \text{ eV/K}$
$k = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$
$e = 1.602 \times 10^{-19} \text{ C}$
$\alpha = \frac{ke^2}{\hbar c} = 7.29735 \times 10^{-3} \approx \frac{1}{137}$

<u>Black Bodies</u>
$U = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3}$
$\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$

<u>Rutherford Scattering</u>
$b = \frac{kqQ}{m_\alpha v^2} \cot\left(\frac{\theta}{2}\right)$
$R = \frac{2Ze^2 k}{E}$

<u>Compton Effect</u>
$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$
$\frac{h}{mc} = 2.426 \times 10^{-12} \text{ m}$

<u>Wave Relationships</u>
$\lambda = \frac{2\pi}{k}$
$\frac{\omega}{2\pi} = f = \frac{1}{T}$

<u>Hydrogen-Like Atoms</u>
$E = -\frac{k^2 e^4 \mu Z^2}{2\hbar^2 n^2} = -\frac{(\mu c^2) \alpha^2 Z^2}{2n^2}$
$E = \frac{-(13.6 \text{ eV}) Z^2}{n^2}$

**Part I: Multiple Choice [20 points]**

For each question, choose the best answer (2 points each)

1. According to the particle data booklet, a highly unstable  $W$  boson requires 80.4 GeV of energy; however, it is not uncommon to produce one with about 2 GeV too little or too much energy. How can this be possible?
  - A) Quantum mechanically, the creation of any particle involves some uncertainty in its position, so some of the energy may be coming from/going to an unknown source
  - B) Any production of elementary particles is likely to be accompanied by additional incoming/outgoing photons, which carry additional energy
  - C) Since it lives such a short time, there will be a large uncertainty in its energy, according to  $\Delta E \Delta t \geq \frac{1}{2} \hbar$ .
  - D) Since one of the decay products of the  $W$  may be invisible, this particle is probably carrying off the missing energy
  - E) Particle physics is incredibly difficult, and these must simply represent measurement errors

2. The Bohr model worked well on hydrogen, and other atoms with one electron, but not at all on most other atoms, with one notable exception, which was
- Helium atoms containing an extra electron
  - The visible light spectrum from the outermost electrons of other atoms
  - Noble gasses
  - The scattering of X-rays from electrons of atoms
  - The X-ray spectrum from the innermost electrons of other atoms
3. If the wave function for a particle is given by  $\psi$ , the probability density of finding the particle somewhere is given by
- $\psi^*$
  - $\psi^2$
  - $\psi^* \psi$
  - $(\psi^*)^2$
  - None of these is correct
4. If  $z = \sqrt{12+5i}$ , then  $|z|^2 =$
- 169
  - 13
  - 7
  - $\sqrt{7}$
  - $\sqrt{119}$
5. The great contribution of deBroglie to quantum mechanics was
- The discovery of the electron
  - The realization that electromagnetic energy comes in chunks, now called photons
  - The suggestion that electrons might circle the nucleus, akin to a mini-solar system
  - The suggestion that physical particles, such as electrons, might have wave properties
  - The discovery of the nucleus
6. When Rutherford scattered alpha-particles off of very thin gold foil, he discovered, much to his surprise, that they sometimes would scatter at large angles, even backwards sometimes. From this he concluded
- The mass and positive charge must be concentrated in a tiny region he called the nucleus
  - The positive charge and mass must be spread out through the atom, like plum pudding
  - Alpha particles must be very light compared to atoms, rather than very heavy
  - Electrons were actually heavier than alpha particles, and could deflect them a lot
  - Many atoms must cluster into especially dense pockets, with large gaps between the atoms that were mostly empty space

7. Ordinary hydrogen  ${}^1\text{H}$  and heavy hydrogen  ${}^2\text{H}$  both have exactly the same nuclear charge, and identical electrons orbiting them, but heavy hydrogen has a nucleus that is twice as heavy. Why is it that the spectra are very slightly different for these two atoms?
- A) One nucleus is effectively a point, while the other is more spread out
  - B) The magnetic interactions of the nucleus with the electron are different
  - C) It is actually the nucleus, not the electron, that makes the transitions in heavy hydrogen
  - D) The actual mass that comes into the computation of the energy is the “reduced mass”, which depends (weakly) on the mass of the nucleus
  - E) It is the interaction of the electron WITH the nucleus that makes it heavier, and hence also changes the spectrum
8. The importance of the Millikan oil drop experiment was that
- A) It was the first to demonstrate that oil, like water, was made of atoms
  - B) It was the first demonstration that electrons actually have charge
  - C) It was the first sign that electrons really did orbit the nucleus
  - D) It was the first indication that charges come in chunks
  - E) It allowed him to measure the elementary charge  $e$ .
9. In the Compton effect, X-rays that scatter from atoms come out with a slightly longer wavelength/lower frequency because
- A) The longer frequency is necessary to scatter coherently from different layers of atoms in crystals
  - B) The X-ray photons have transferred energy into kinetic energy of the electrons, and the photons have lower energy and hence lower frequency
  - C) The phase velocity for the photons have been converted to group velocity, which is lower
  - D) The X-ray photons have been divided into pairs of photons, each of which has (naturally) less energy than the original
  - E) Only integer numbers of wavelengths can scatter off of atoms
10. Another formula for  $e^{i\theta}$  is
- A)  $\cos \theta + i \sin \theta$
  - B)  $\cos \theta - i \sin \theta$
  - C)  $\sin \theta + i \cos \theta$
  - D)  $\sin \theta - i \cos \theta$
  - E) None of these is correct

**Part II: Short answer [20 points]**

Choose **two** of the following questions and give a short answer (2-3 sentences) (10 points each).

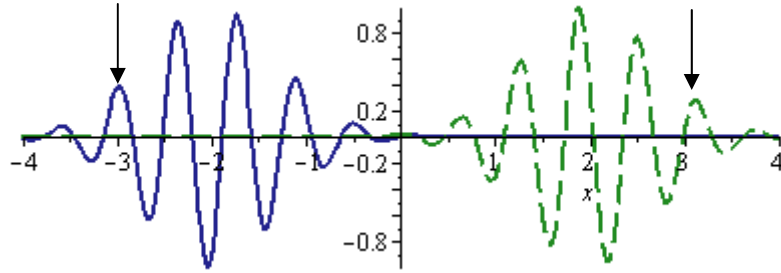
11. What were the three assumptions Bohr used in trying to develop his model of the hydrogen atom?
12. Under what circumstances does a lot of diffraction occur? In particular, how come you can easily hear around a corner, but you don't see around a corner, nor do material objects (like people) diffract around corners?
13. According to classical physics, the electron would prefer to have momentum zero and be exactly at the proton, where it would have infinite negative energy. Explain quantum mechanically why this doesn't happen. An equation or inequality might be in order.

**Part III: Calculation: [60 points]**

Choose **three** of the following four questions and perform the indicated calculations (20 points each).

14. An important era of the early universe was *recombination*. At the time, the universe was at a temperature of approximately 2975 K, and was filled with a thermal distribution of radiation.
  - (a) At what wavelength, in nm, would this radiation have been strongest?
  - (b) What would have been the energy of one typical photon at this wavelength?
  - (c) What was the energy density, in  $\text{J/m}^3$ , at this time?
15. Nuclear forces require that a proton be inside the nucleus, a region typically 4 fm in radius ( $4 \text{ fm} = 4 \times 10^{-15} \text{ m}$ ).
  - (a) If this is the radius, according to Carlson's rule, what is the uncertainty in its position?
  - (b) What is the corresponding uncertainty in the momentum due to this motion?
  - (c) Predict the approximate kinetic energy of a proton, based on the fact that it is in the nucleus? Your answer may be in J or MeV. The proton has a mass of  $m_p = 1.672 \times 10^{-27} \text{ kg} = 938 \text{ MeV}/c^2$ .

16. A wave packet is observed to move from the initial position (solid lines) to the final position (dashed lines) at right over a period of time  $t = 12$  s. The

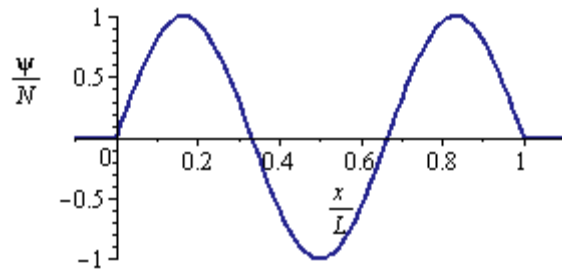


particular wave that starts at the first arrow is observed to move to the position of the final arrow. The  $x$ -axis is marked in meters.

- Estimate the wavelength  $\lambda$  of the wave (in m) and the wave number  $k$ .
- What is the phase velocity  $v_p$  and the group velocity  $v_g$  for this wave?
- What is the angular frequency  $\omega$  for this wave?

17. One of the solutions of Schrödinger's time-independent equation for an infinite square well is

$$\psi(x) = \begin{cases} N \sin(3\pi x/L) & \text{if } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$



This wave function is sketched at right.

- In the region  $0 < x < L$ , is there any place that the particle cannot be?
- In the region  $0 < x < L$ , what is (are) the most likely place (places) to find the particle?
- What is the correct normalization constant  $N$ ? Some possibly useful integrals are given below.

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) dx = \begin{cases} 2L/n\pi & \text{if } n \text{ is odd,} \\ 0 & \text{if } n \text{ is even,} \end{cases} \quad \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = \int_0^L \cos^2\left(\frac{n\pi x}{L}\right) dx = \frac{1}{2}L$$