Name ___________________

Test 1
September 18, 2009

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Possibly useful formulas:

$$\vec{F} = q\vec{E} + q\vec{u}\times\vec{B}$$
$$p = qRB$$

$$f = \frac{f_0}{\gamma\left(1 - v\cos\theta/c\right)}$$

$$x' = \gamma\left(x - vt\right)$$
$$t' = \gamma\left(t - vx/c^2\right)$$
$$y' = y$$
$$z' = z$$

$$E' = \gamma\left(E - vp_x\right)$$
$$p'_x = \gamma\left(p_x - vE/c^2\right)$$
$$p'_y = p_y,$$
$$p'_z = p_z$$

$$\left(1+\varepsilon\right)^n = 1 + n\varepsilon + \tfrac{1}{2}n\left(n-1\right)\varepsilon^2 + \cdots$$

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}$$
$$u'_y = \frac{u_y}{\gamma\left(1 - vu_x/c^2\right)}$$
$$u'_z = \frac{u_z}{\gamma\left(1 - vu_x/c^2\right)}$$

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. What distinguishes “good” coordinate changes, like the Lorentz boost and rotation, from “bad” ones, like a rescaling coordinate change?
   A) They leave the (four dimensional) distance unchanged
   B) The origin of coordinates does not accelerate under good coordinate choices
   C) Angles always remain the same under good coordinate choices
   D) Good coordinate changes do not mix time with distance
   E) Good coordinate changes always involve angles of rotation

2. The Michelson-Morley experiment demonstrated that
   A) The Earth moved around the Sun (only)
   B) The Earth moved compared to the background ether (only)
   C) The Solar System orbited around the center of the galaxy (only)
   D) A, B, and C are all correct
   E) Actually, it demonstrated no motion of the Earth compared to anything.

3. Particle colliders like the Large Hadron Collider make charged particles (protons) move in a giant circle by pushing on them using
   A) Photonic fields
   B) Gravitational fields
   C) Electric fields
   D) Magnetic fields
   E) Little elves

4. Which of the following <u>is</u> a conservation law in special relativity?
   A) Energy (only)
   B) Momentum (only)
   C) Mass (only)
   D) Energy and momentum, but not mass
   E) Energy, momentum, and mass

5. The force and work formulas from non-relativistic physics are $\vec{F} = m\vec{a}$ and $W = \vec{F} \cdot \vec{d}$. Which of these are valid in special relativity?
   A) $\vec{F} = m\vec{a}$ only B) $W = \vec{F} \cdot \vec{d}$ only C) Both D) Neither
   E) I'm not going to do the Work on this one, and you can't Force me to.

6. According to the conventions I used in class, the mass of an object:
   A) Is equal to the sum of the masses of each part
   B) Is the same at all speeds; it is independent of speed
   C) Is equal to E/c^2
   D) Will not change if you add, say, chemical energy to the object
   E) None of these are true

7. The frequencies from a moving source are red-shifted by the greatest amount when it is moving
   A) Directly away from you
   B) Directly towards you
   C) Perpendicular to your line of sight
   D) It doesn't matter the direction, only the speed
   E) Frequencies can't be red-shifted

8. Clock A is at rest, and clock B moves past it at high velocity. Observers accompanying each clock observe the other clock, and conclude
   A) Clock B is slower than clock A
   B) Clock A is slower than clock B
   C) Each person will conclude the other clock runs slowly
   D) Each person will conclude the other clock runs quickly
   E) Each person sees the other clock running at normal speed

9. A cube of steel is heated from a temperature of 300 K to 600 K. As a consequence, its mass will
   A) Stay the same
   B) Decrease substantially
   C) Decrease slightly
   D) Increase substantially
   E) Increase slightly

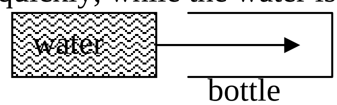
10. Which of the following is the best estimate of γ at low velocities?

A) $\gamma = 1 + \frac{v^2}{c^2}$ B) $\gamma = 1 - \frac{v^2}{c^2}$ C) $\gamma = 1 + \frac{v^2}{2c^2}$ D) $\gamma = 1 - \frac{v^2}{2c^2}$ E) $\gamma = 1$

Part II: Short answer [20 points]

Choose **two** of the following questions and give a short answer (1-3 sentences, or a simple sketch) (10 points each).

11. The Relativity Water Bottling company finds a new way to get money from consumers. They take more than a liter of water, accelerate it to a substantial fraction of the speed of light (basically, firing the water into a VERY sturdy water bottle), so that it is Lorentz contracted, and then put the cap on very quickly, while the water is still moving. They sell the water, advertising "more water per liter!" Would this (in principle) work? If impossible, explain why. If possible, explain how.



12. When a rocket sends us radio signals, the frequency of the received radio waves is measured *very* carefully. Why do we do this, since presumably the radio builders can simply *tell* us what frequency it was designed to transmit?

13. Two observers are moving at velocity v compared to each other. Describe an experiment that would determine which of them is actually moving, or argue that no such experiment is possible.

Part III: Calculation: [60 points]

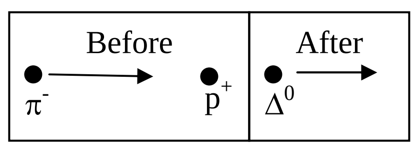
Choose **three** of the following four questions and perform the indicated calculations (20 points each)

14. Due to a low battery in my watch, I discover that it is advancing slowly, so it advances only 50 seconds in every minute. No problem! I will simply run around at very high velocity v so that as viewed by me, all other clocks run slowly as well.
    (a) How fast do I need to run in order to make other clocks appear to run at the same pace as mine?
    (b) According to other observers, how many seconds pass on *my* watch in a minute. Keep in mind that my watch runs slow!
    (c) Arriving at class one minute early, I decide I might as well run around campus at this high speed and get a few errands done. I arrive back when my second hand has advanced 60 seconds. Will I return early, late, or exactly on time?

15. Three events all have coordinates $y = z = 0$, but they have different x and t coordinates, as given in the table at right. For each part of the problem, assume the numbers given at right are exact, and the speed of light is exactly $c = 3 \times 10^8$ m/s
    (a) What is the proper distance (in m) or proper time (in ns) separating A from B? Is one of these events in the absolute future of the other (which one?), is one on the future light cone of the other, or are they "elsewhere" from one another?
    (b) Repeat, this time comparing A to C.
    (c) Repeat, this time comparing B to C.

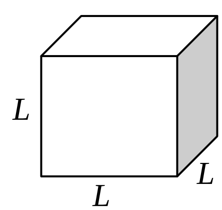
Point	x	t
A	0 m	0 ns
B	5 m	10 ns
C	6 m	20 ns

16. A pion (π^-) of mass m_π = 140 MeV/c^2 is moving with velocity $v = 0.905c$ in the $+x$ direction. It collides with a stationary proton (p^+) of mass m_p = 938 MeV/c^2. The two particles merge to form a Δ^0 particle.



    (a) What is the energy (in MeV) and momentum (in MeV/c) of the pion? How about the proton?
    (b) What is the energy and momentum of the final Δ^0, in the same units?
    (c) What is the mass of the Δ^0 in MeV/c^2? What is its velocity as a fraction of c?

17. A cube of dimensions $L \times L \times L$ is at rest. It is filled with a material with energy density ρ (energy per unit volume) and momentum density 0 (momentum per unit volume). The box is then viewed by an observer moving at speed $v = 0.6c$ in the x-direction.



    (a) What are the dimensions of the box as viewed by the moving observer?
    (b) What are the energy and the momentum of the box in its rest frame? What is the energy and momentum of the box as viewed by the moving observer?
    (c) What is the energy density and momentum density of the box as viewed by the moving observer?