

Name \_\_\_\_\_

Test 1  
September 21, 2007

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

**Possibly useful formulas:**

$$f = \frac{f_0}{\gamma(1 - v \cos \theta / c)}$$

$$\begin{aligned} p'_x &= \gamma(p_x - vE/c^2) \\ E' &= \gamma(E - vp_x) \\ p'_y &= p_y \\ p'_z &= p_z \end{aligned}$$

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - vu_x/c^2} \\ u'_y &= \frac{u_y}{\gamma(1 - vu_x/c^2)} \\ u'_z &= \frac{u_z}{\gamma(1 - vu_x/c^2)} \end{aligned}$$

$$p = qRB$$

$$(1 + \varepsilon)^n = 1 + n\varepsilon + \frac{1}{2}n(n-1)\varepsilon^2 + \dots$$

$$\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$$

**Part I: Multiple Choice [20 points]**

For each question, choose the best answer (2 points each)

- Two incoming particles collide which are moving in the  $x$ -direction. Two particles come out afterwards, now moving in an arbitrary direction. Which of the following will *not* generally be conserved?
  - Mass
  - Energy
  - Momentum in the  $x$ -direction
  - Momentum in the  $y$ -direction and the  $z$ -direction
  - Actually, all of these are conserved
- What is meant by the term *world line* (misspelled *worldline* in the lectures) in special relativity?
  - A line representing a set of points all at the same time, like the  $x$ -axis
  - A line representing a set of points all at the same place, like the  $t$ -axis
  - A line representing any axis of some observer moving compared to us
  - A line tilted at a 45-degree angle on a space-time diagram
  - The path of some material object through space and time

3. The Lorentz boost formulas for coordinates (called by the book Lorentz transformations) tell you how to change coordinates when
  - A) Comparing two observers that are at an angle compared to each other in space
  - B) Comparing two observers that are at the same speed and angle, but one has shifted their space origin of coordinates
  - C) Comparing two observers that are at the same speed and angle, but one has shifted their time origin of coordinates
  - D) Comparing two observers that are oriented in opposite directions compared to time
  - E) Comparing two observers that are moving compared to each other
  
4. When an object is moving very close to the speed of light, its linear dimensions, as viewed by us, will
  - A) Shrink in the direction of motion, but not in perpendicular directions
  - B) Expand in the direction of motion, but not in perpendicular directions
  - C) Shrink in perpendicular directions, but not in the direction of motion
  - D) Expand in perpendicular directions, but not in the direction of motion
  - E) Neither shrink nor expand in any dimension as viewed by us
  
5. In the Large Hadron Collider, currently under construction in Geneva, protons are forced to go around a very large circular track. The method used to keep them going in circles is probably
  - A) Collisions with the walls of the collider
  - B) Gravitational force
  - C) Electric fields
  - D) Magnetic fields
  - E) Nuclear fields
  
6. In the classic version of the twin paradox, where one observer goes to a star and returns, while the other stays here, the reason there is an absolute difference between the two observers is that
  - A) One observer was really moving, while the other wasn't
  - B) One observer was moving compared to other objects (like the Sun), while the other wasn't
  - C) One observer always moved at constant velocity, while the other didn't
  - D) One observer was always near a gravitational source, while the other wasn't
  - E) One observer moved only in the time direction, while the other also moved in the space direction

7. Under what condition(s) is it perhaps advisable to expand the Lorentz factor  $\gamma$  or its inverse using the binomial expansion?
- A) When the object is moving nearly at the speed of light
  - B) When the object is moving much slower than the speed of light
  - C) When calculating time dilation, but not Lorentz contraction
  - D) When calculating Lorentz contraction, but not time dilation
  - E) When calculating Lorentz contraction or time dilation, but not Lorentz transformations
8. Suppose we mix two chemicals, and they react in such a way that energy is released (which then escapes from the mixture). How will the total mass of the final combination be different from the sum of the masses of the constituents?
- A) It will be a lot smaller
  - B) It will be very slightly smaller
  - C) It will be exactly the same
  - D) It will be very slightly larger
  - E) It will be a lot larger
9. Why does energy appear to get “mixed up” with momentum in the formulas above for Lorentz transformations of energy and momentum?
- A) Changing speeds involves emitting light rays, which can convert energy into momentum and vice versa
  - B) As you change speed, you add energy, and therefore you change the mass which changes the momentum
  - C) Energy and momentum are different parts of the same four-dimensional vector, and therefore naturally mix together when you perform a Lorentz boost
  - D) Any method for changing speeds involves electric and magnetic fields, which actually perform this change for you
  - E) I have no idea; please mark this one wrong
10. As an object of finite mass  $m$  approaches the speed of light  $c$ , what happens to the object’s momentum and energy?
- A) They both go to large but finite values
  - B) Momentum goes to infinity, but energy remains finite
  - C) Energy goes to infinity, but momentum remains finite
  - D) They both go to infinity
  - E) There is insufficient information to answer this question

**Part II: Short answer [20 points]**

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).

11. Dr. Carlson has just devised an instantaneous communication device! It consists of a 1 light-year long steel rod, together with two people who know Morse code. One person pushes/pulls the rod at one end to send the message, and the other one watches her end to see what happens. Tell me what you think of Dr. Carlson's invention.
12. Qualitatively, how does the frequency of absorbed radiation from a distant source change if it is moving (a) towards you, (b) away from you, or (c) perpendicular to you. You don't have to use formulas, but you have to tell me if it increases or decreases or stays the same, and perhaps something about the size of the effect.
13. What makes transformations like rotations and Lorentz boosts good, while transformations like the "skew" transformation (that distorted my picture) and the Galilean boost bad, according to special relativity? Use of an equation in your answer is strongly encouraged.

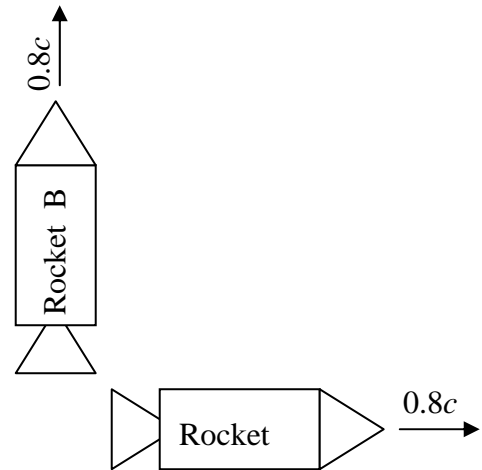
**Part III: Calculation: [60 points]**

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

14. An observer is currently at the point in space-time  $(x_1, y_1, z_1, t_1) = (2 \text{ m}, 3 \text{ m}, 4 \text{ m}, 0 \text{ s})$ . A second observer is stationary at  $(x_2, y_2, z_2, t_2) = (0 \text{ m}, 2 \text{ m}, 6 \text{ m}, t)$  where  $t$  is unknown.
  - (a) For what values of  $t$  will the separation between these two points be timelike, spacelike, or lightlike? You may give your answer either in terms of  $t$  or  $ct$ , whichever you prefer.
  - (b) For which values of  $t$  will the second point be in the absolute past, absolute future, past light cone, future light cone, or elsewhere, compared to the first point?

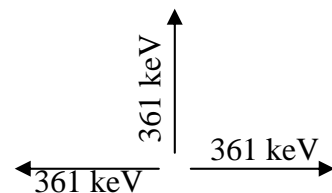
15. A 1.50 m tall man plus his rocket ship (total mass 100 kg) are to be accelerated from rest to  $2.13 \times 10^8$  m/s. This is to be achieved by a constant force of 980 N (about the weight on Earth).
- What is the initial energy and momentum of the system, in J and kg·m/s? What is the final momentum and final energy of the system, in the same units?
  - How long does it take to reach this speed, as measured by us? Give the answer in years ( $1 \text{ yr} = 3.156 \times 10^7$  s).
  - How far has the system traveled by this time? Give the answer in light-years ( $1 \text{ ly} = 9.461 \times 10^{15}$  m).
  - Assuming the rocket is moving in the direction the man is standing, how tall does he seem to us?

16. Rocket A and Rocket B start at the same place at the same time, but simultaneously accelerate to  $0.8c$ , but with rocket A moving in the  $+x$  direction and rocket B moving in the  $+y$  direction.



- Write all three components  $\vec{u}$  of the velocity of rocket B in the original system (this is trivial)? Write all three components  $\vec{u}'$  of the velocity of rocket B as viewed by Rocket A. I recommend writing all velocities as fractions of  $c$ .
- Find the magnitude of the velocity  $\vec{u}'^2$  of rocket B and check that it is less than  $c^2$ .
- Unfortunately for the person on Rocket B, after 2.73 hours of travel (as measured by B), rocket B develops engine trouble. According to the observer on rocket A, how much time has passed when this occurred?

17. An Ortho-Positronium particle that is not at rest decays to three photons. Suppose that the three photons all have mass 0, energy  $E_{1,2,3} = 361.3$  keV, and are moving in the  $+x$ ,  $-x$ , and  $+y$  direction respectively, as illustrated at right.



- What is the momentum of each of the three photons?
- What is the initial energy in keV and momentum in keV/ $c$  of the ortho-positronium?
- What is the mass of ortho-positronium in keV/ $c^2$ ?