

Name \_\_\_\_\_

**Solutions to Final Exam**  
**December 14, 2011**

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered.

**Part I: Multiple Choice (mixed new and review questions) [50 points]**

For each question, choose the best answer (2 points each)

1. Which of the following is not a prediction of general relativity?  
A) Light is deflected by the Sun or other astrophysical objects  
B) Time slows down when you are near a gravitational source  
C) If a massive object is shrunk enough, it should collapse to a black hole  
D) Rotating objects can twist space, causing a phenomenon called “frame dragging”  
**E) Actually, all of these are predictions of General Relativity**
  
2. Which of the following shows that gravitational waves are almost certainly real, based on current observations?  
A) Large scale interferometers have actually detected the fluctuations between mirrors separated by several kilometers  
B) Sets of gyroscopes in gravity probe B changed the direction they spin as they were affected by gravitational waves  
C) The orbits of satellites have indicated that they are moving randomly as pushed by gravitational waves  
**D) The orbits of pairs of neutron stars have been shown to be slowly spiraling inwards, indicating a loss of energy from gravitational waves**  
E) No measurement, direct or indirect, has yet indicated the reality of gravitational waves
  
3. An electron at rest has total energy 0.511 MeV. If we accelerate it to nearly the speed of light, what will the limit of its energy be?  
A) 0.255 MeV B) 0.511 MeV C) 0.766 MeV D) 1.022 MeV **E) Infinity**
  
4. In which type of decay is there no readily detectable particle that leaves the nucleus?  
A)  $\alpha$  decay B)  $\beta$  decay C)  $\beta^+$  decay D)  $\gamma$  decay **E) electron capture**
  
5. Suppose one observer measures the total momentum and energy of a group of particles. On which of the following quantities will all observers agree?  
A) The total momentum of the particles  
B) The total energies of the particles  
**C) The invariant mass of the set of particles**  
D) The center of mass velocity of the set of particles  
E) Observers will not agree on any of these

6. Below are listed the mass, in u, of various isotopes of phosphorous. Which one is  $^{30}\text{P}$ ?  
A) **29.9783**    B) 30.9738    C) 31.9738    D) 32.9717    E) 33.9736
7. According to quantum mechanics, suppose a particle with energy  $E$  encounters a barrier of height  $V_0$  which is smaller than  $E$ . What will happen?  
A) The wave is completely reflected  
B) The wave is completely transmitted  
C) The wave is completely absorbed  
D) **The wave is partially transmitted and partially reflected**  
E) The wave is partially transmitted and partially absorbed
8. What aspect of Mercury's orbit about the sun was explained successfully by Einstein's general theory of relativity?  
A) The fact that the orbit was more oval than elliptical  
B) **The fact that the closest point of the orbit of Mercury around the Sun (the perihelion) changed direction, very slowly (precessed)**  
C) The fact that the Sun-Mercury distance seemed to be slowly decreasing  
D) The fact that Mercury orbited slightly faster than predicted by Newton's laws  
E) The fact that clocks on spacecraft near Mercury seemed to slow down, compared to those on Earth
9. The proton contains  
A) **Three quarks**  
B) Three anti-quarks  
C) A quark and an anti-quark  
D) Two leptons  
E) None of the above; protons are believed to be elementary.
10. When an object is moving at high velocity, its length in the direction of motion is \_\_\_\_\_ while in directions perpendicular to the direction of motion it is \_\_\_\_\_  
A) decreased, decreased  
B) decreased, increased  
C) **decreased, stay the same**  
D) increased, stay the same  
E) stay the same, decreased
11. Which of the following is true about the quantum mechanical wavelength of an electron?  
A) It is proportional to the energy  
B) It is inversely proportional to the energy  
C) It is proportional to the momentum  
D) **It is inversely proportional to the momentum**  
E) None of the above is correct

12. Quarks in a baryon are held together by the exchange of which of the following particles?  
 A) photons    **B) gluons**    C) pions    D) W-particles    E) Higgs bosons
13. Which of the following might be the size of the nucleus of an atom?  
 A)  $10^{-10}$  m    B)  $10^{-12}$  m    **C)  $3 \times 10^{-15}$  m**    D)  $3 \times 10^{-18}$  m    E) 0
14. The probability density of a particle being at the point  $x$  is given by  
 A)  $\psi^* \psi$     B)  $\psi^*$     C)  $\psi^2$     D)  $\psi^{*2}$     E)  $\psi + \psi^*$
15. What is the difference between the phase velocity and group velocity of a wave?  
 A) **Phase velocity is how fast the individual peaks move, group velocity is how fast the entire packet moves**  
 B) Phase velocity is how fast the entire packet moves, group velocity is how fast the individual peaks move  
 C) Phase velocity is how fast a specific wavelength moves; group velocity is the average over all wavelengths  
 D) Phase velocity is the average over all wavelengths; group velocity is how fast a specific wavelength moves  
 E) Phase velocity is the speed of the peaks; group velocity is the speed of the troughs
16. The percentage of protons in a stable nucleus is typically about  
 A) 50 percent for all masses of nuclei  
 B) 50 percent for light nuclei, 60 percent for heavy nuclei  
**C) 50 percent for light nuclei, 40 percent for heavy nuclei**  
 D) 60 percent for light nuclei, 50 percent for heavy nuclei  
 E) 40 percent for light nuclei, 50 percent for heavy nuclei
17. Which of the following contains a succinct summary of general relativity?  
 A) The gravitational force between two objects is proportional to the product of their masses and inversely proportional to the square of their separation  
 B) Objects follow straight lines when forces are acting on them, in which case they follow geodesics  
 C) Time slows down in the vicinity of masses  
**D) Matter tells space how to curve, and space tells matter how to move**  
 E) To err is human; to forgive, divine
18. If an electron has  $l = 2$ , then the angular momentum  $\vec{L}^2$  will be  
 A)  $\hbar^2$     B)  $2\hbar^2$     C)  $4\hbar^2$     **D)  $6\hbar^2$**     E)  $8\hbar^2$

19. In hydrogen, how do the energy of an electron in the 3d and 3p state differ?
- A) The 3p is significantly lower energy  
 B) The 3d is significantly lower energy  
**C) The energies are essentially identical**  
 D) It depends on which specific orbital the electron goes in  
 E) It depends on the spin state of the electron
20. What does the uncertainty principle say about the uncertainty in the position of a particle  $\Delta x$  and/or the uncertainty in the momentum  $\Delta p$  ?
- A) There is a fundamental limit on the position  $\Delta x$   
 B) There is a fundamental limit on the momentum  $\Delta p$   
 C) There is a fundamental limit on the ratio of these,  $\Delta p/\Delta x$   
**D) There is a fundamental limit on the product of these,  $\Delta p \cdot \Delta x$**   
 E) The uncertainty relation says nothing about these quantities
21. Suppose we have the coordinates of two events in space-time,  $A$  and  $B$ . Under what circumstances can we determine which of the two events is actually first, as viewed by any observer?
- A) Only when they are space-like separated  
**B) Only when they are time-like (or light-like) separated**  
 C) Only when they are at the same location but different times  
 D) Only when they are at the same time but different locations  
 E) Under none of these conditions will all observers agree on which came first

*In questions 22 – 25, you will be given a reaction, and you must state what type of interaction it is. A mini-table of particles with their spin, mass, strangeness, and type is listed at right. All particles have charge 0. Masses are in  $MeV/c^2$ .*

Name	Spin	Mass	St.	Type
$n^0$	$\frac{1}{2}$	940	0	baryon
$\Lambda^0$	$\frac{1}{2}$	1116	-1	baryon
$\Sigma^0$	$\frac{1}{2}$	1193	-1	baryon
$\Delta^0$	$\frac{3}{2}$	1234	0	baryon
$\pi^0$	0	139	0	meson
$\bar{K}^0$	0	495	-1	meson
$\gamma$	1	0	0	other

22.  $\Lambda^0 \rightarrow n^0 + \pi^0$   
 A) impossible    B) strong    **C) weak**    D) electromagnetic    E) gravity
23.  $\Lambda^0 \rightarrow n^0 + \bar{K}^0$   
**A) impossible**    B) strong    C) weak    D) electromagnetic    E) gravity
24.  $\Sigma^0 \rightarrow \Lambda^0 + \gamma$   
 A) impossible    B) strong    C) weak    **D) electromagnetic**    E) gravity
25.  $\Delta^0 \rightarrow n^0 + \pi^0$   
 A) impossible    **B) strong**    C) weak    D) electromagnetic    E) gravity

**Part II: Short answer (review material) [20 points]**

Choose **two** of the following three questions and give a short answer (1-3 sentences) (10 points each).

**26. Explain qualitatively how a pole vaulter whose pole is the same length as a barn can have the far end of his pole hit the back of the barn, even though (to a stationary observer) it looks like the pole is shorter than the barn.**

In the example I gave of the pole vaulter paradox, the vaulter stops when, in his frame, the end of the pole reaches the back of the barn, and while he is still outside the barn. From the perspective of the stationary observer, however, the ends don't stop simultaneously, and the pole stretches to smash into the back of the barn. There are no rigid objects in special relativity.

**27. Why is it that we so rarely notice quantum effects when everyday events occur; for example, why don't we see diffraction when people walk through doors?**

Although the equation  $\lambda = h/p$  implies that everything, including moving people, have a wavelength, the tiny size of  $h$  causes this wavelength to be immeasurably small for macroscopic objects. Hence any diffraction effects are completely negligible.

**28. What does spin mean? What is the spin of the electron? What are the possible values of the  $z$ -component of the spin of the electron,  $S_z$ ?**

Spin is the intrinsic angular momentum of some object, in this case, the electron. The electron has spin  $\frac{1}{2}$ , which means that the  $z$ -component can take on only the two values  $S_z = \pm \frac{1}{2} \hbar$ .

**Part III: Short answer (new material) [30 points]**

Choose **three** of the following four questions and give a short answer (1-3 sentences) (10 points each).

**29. Explain qualitatively why very heavy nuclei tend to be unstable and undergo  $\alpha$ -decay.**

Nuclei are held together by a short-range force, so that for a very heavy nucleus, each nucleon gains the benefits of its nearest neighbors, but not from those far away. However, electromagnetism is a long-range force. In a very heavy nucleus, the positive charge from all the protons repel each other, and since this is cumulative, the nucleus wants to expel some charge to become more stable. A logical way to do this is  $\alpha$ -decay.

**30. In the movie “Roxanne,” Darryl Hannah, playing a scientist describing quarks, says, “There are at least six different types: Up, Down, Strange, Charmed, Bottom and Top. Top and Bottom Quarks are most common.” Explain what is right or wrong about her statements.**

There are, in fact, *exactly* six quarks, though this may not have been known at the time. And the names are correct. However, ordinary particles contain only up and down quarks, not top and bottom. There are also circumstances where strange quarks arise, but these are much rarer. Bottoms and tops are rarely seen outside of particle colliders.

**31. What are the four forces of nature? Which three are described by the standard model of particle physics? Of these three, which ones must conserve electric charge? Which ones must conserve strangeness?**

The four forces of nature are the strong, weak, electromagnetic, and gravitational forces. The standard model includes the first three, but not gravity. All forces are believed to conserve electric charge. Strangeness is conserved by the strong and electromagnetic force, but not by the weak force.

**32. Explain in your own words the distinction between curved coordinates and curved spacetime.**

General relativity can handle any coordinate system, not just Cartesian coordinates. For example, it can work in coordinates which are curved, such as spherical coordinates. Curved spacetime, however, represents an actual curving of the universe. It cannot be undone simply by changing back to Cartesian coordinates; in a sense, there is no such thing as flat coordinates in curved spacetime.

**Part IV: Calculation (review material) [40 points]**

Choose **two** of the following three questions and perform the indicated calculations (20 points each)

**33. According to rumors, there is some indication that the Higgs particle may have been discovered at the LHC by its decay to pairs of (massless) photons,  $H \rightarrow \gamma\gamma$ . Suppose the photons come out back to back, as illustrated below, and have energies of  $E_1 = 42.0$  GeV and  $E_2 = 93.0$  GeV.**



**(a) What is the momentum of each of the photons, in GeV/c? Remember that momentum is a vector!**

Since the photon is massless,  $E^2 - \vec{p}^2 c^2 = 0$ , so  $pc = E$ . The problem is one-dimensional, but keeping in mind that momentum is still a vector, it is clear that one particle has positive momentum and the other negative. So the two momenta are

$$p_1 = 42.0 \text{ GeV}/c, \quad \text{or} \quad \vec{p}_1 = 42.0 \hat{i} \text{ GeV}/c, \\ p_2 = -93.0 \text{ GeV}/c, \quad \vec{p}_2 = -93.0 \hat{i} \text{ GeV}/c.$$

**(b) What is the total energy and momentum of the initial Higgs?**

By conservation of energy and momentum, these are given by

$$E = E_1 + E_2 = 135.0 \text{ GeV}, \\ p = p_1 + p_2 = -51.0 \text{ GeV}/c.$$

If we prefer we can write this as  $\vec{p} = -51.0 \hat{i} \text{ GeV}/c$ .

**(c) What is the mass of the Higgs, in GeV/c<sup>2</sup>?**

We use the invariant mass formula again, and find

$$m^2 c^4 = E^2 - \vec{p}^2 c^2 = (135.0 \text{ GeV})^2 - (51.0 \text{ GeV})^2 = 15624 \text{ GeV}^2, \\ mc^2 = \sqrt{15624 \text{ GeV}^2} = 125.0 \text{ GeV}, \\ m = 125.0 \text{ GeV} / c^2.$$

**(d) What was the initial velocity of the Higgs, as a fraction of the speed of light?**

We use the velocity formula,

$$\frac{u}{c} = \frac{pc}{E} = \frac{-51.0 \text{ GeV}}{135.0 \text{ GeV}} = -0.378.$$

**34. A laser with wavelength 237.0 nm is shone on a sample of cesium, which has a work function of  $\phi = 2.14$  eV .**

**(a) What is the frequency of the incident photon?**

We use the formula  $\lambda f = c$  to find

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{237.0 \times 10^{-9} \text{ m}} = 1.265 \times 10^{15} \text{ s}^{-1} .$$

**(b) What is the kinetic energy of those electrons that are liberated by these photons?**

The energy of these initial photons is

$$E = hf = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(1.265 \times 10^{15} \text{ s}^{-1}) = 5.23 \text{ eV} .$$

From this we must subtract the work function,  $\phi = 2.14$  eV , which is expended to liberate the electron from the metal, and the remaining energy is the kinetic energy.

$$T = E - \phi = (5.23 \text{ eV}) - (2.14 \text{ eV}) = 3.09 \text{ eV} .$$

**(c) Another laser of unknown frequency is then brought to bear, and it is found that the electrons escaping have a kinetic energy of 1.05 eV. What is the frequency of this laser?**

The electrons ended with an energy of 1.05 eV after having overcome the work function of 2.14 eV. Therefore the photons must have had energy

$$E = \phi + T = (2.14 \text{ eV}) + (1.05 \text{ eV}) = 3.19 \text{ eV} .$$

The corresponding frequency can then be calculated from  $E = hf$  to yield

$$f = \frac{E}{h} = \frac{3.19 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 7.71 \times 10^{14} \text{ s}^{-1}$$

**(d) What is the minimum frequency and corresponding wavelength that can free an electron?**

The minimum frequency is when the energy of the photon equals the work function, so we have

$$f_{\min} = \frac{\phi}{h} = \frac{2.14 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 5.17 \times 10^{14} \text{ s}^{-1}$$

The corresponding wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{5.17 \times 10^{14} \text{ s}^{-1}} = 5.799 \times 10^{-7} \text{ m} = 580. \text{ nm} .$$

**35. A particle has wave function given by**

$$\psi(x) = \begin{cases} x\sqrt{12(a-x)}/a^2 & \text{for } 0 \leq x \leq a, \\ 0 & \text{otherwise.} \end{cases}$$

**This wave function has already been properly normalized.**

**(a) What is the probability that this particle lies in the region  $0 < x < \frac{1}{2}a$  ?**

We work this out by simply integrating the probability density:

$$\begin{aligned} P(x < \frac{1}{2}a) &= \int_0^{\frac{1}{2}a} \psi^*(x)\psi(x) dx = \int_0^{\frac{1}{2}a} \psi^2(x) dx = \frac{1}{a^4} \int_0^{\frac{1}{2}a} 12x^2(a-x) dx \\ &= \frac{1}{a^4} (4ax^3 - 3x^4) \Big|_0^{\frac{1}{2}a} = \frac{1}{a^4} \left( 4a \frac{a^3}{8} - 3 \frac{a^4}{16} \right) = \frac{1}{2} - \frac{3}{16} = \frac{5}{16} = 0.3125 \\ &= 31.25\% . \end{aligned}$$

**(b) Find the expectation values  $\langle x \rangle$  and  $\langle x^2 \rangle$  for this wave function.**

We straightforwardly use the formula for expectation values:

$$\begin{aligned} \langle x \rangle &= \int \psi^*(x)x\psi(x) dx = \frac{1}{a^4} \int_0^a 12x^3(a-x) dx = \frac{1}{a^4} (3ax^4 - \frac{12}{5}x^5) \Big|_0^a \\ &= a(3 - \frac{12}{5}) = \frac{3}{5}a, \\ \langle x^2 \rangle &= \int \psi^*(x)x^2\psi(x) dx = \frac{1}{a^4} \int_0^a 12x^5(a-x) dx = \frac{1}{a^4} (\frac{12}{5}ax^5 - 2x^6) \Big|_0^a \\ &= a^2(\frac{12}{5} - 2) = \frac{2}{5}a^2. \end{aligned}$$

**(c) What is the uncertainty  $\Delta x$  in the position of this particle?**

Again, we simply use the formula, which says that

$$\begin{aligned} (\Delta x)^2 &= \langle x^2 \rangle - \langle x \rangle^2 = \frac{2}{5}a^2 - \left(\frac{3}{5}a\right)^2 = a^2\left(\frac{2}{5} - \frac{9}{25}\right) = \frac{1}{25}a^2, \\ \Delta x &= \frac{1}{5}a. \end{aligned}$$

**Part V: Calculation (new material): [60 points]**

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

**36. Plutonium was once thought to be produced only artificially, but it is now known that a very small amount of  $^{244}\text{Pu}$ , with a half-life of 80.0 million years, is probably primordial.**

**(a) Suppose you had 1.000 g of  $^{244}\text{Pu}$  in a sample of rock when the Earth first formed. How many atoms of  $^{244}\text{Pu}$  is this?**

To a good approximation,  $^{244}\text{Pu}$  will have a mass of 244 u. Hence the number of atoms is

$$N_0 = \frac{M}{m} = \frac{1.000 \text{ g}}{244. \text{ u}} = \frac{N_A}{244.} = \frac{6.022 \times 10^{23}}{244.} = 2.468 \times 10^{21} .$$

**(b) What is the decay rate  $\lambda$  in  $\text{s}^{-1}$  for this isotope?**

The decay rate is given by

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{8.00 \times 10^7 \text{ y}} = 8.664 \times 10^{-9} \text{ y}^{-1} \frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}} = 2.746 \times 10^{-16} \text{ s}^{-1}$$

**(c) When the sample first existed, how many decays per second would there have been?**

We simply use the equation

$$R_0 = \lambda N_0 = (2.746 \times 10^{-16} \text{ s}^{-1})(2.468 \times 10^{21}) = 6.78 \times 10^5 \text{ s}^{-1} .$$

**(d) The Earth is approximately 4.55 billion years old. How many atoms of  $^{244}\text{Pu}$  would remain from the original 1.00 g?**

This is most conveniently done with  $\lambda$  written in units of years, so we have

$$\begin{aligned} N &= N_0 e^{-\lambda t} = (2.468 \times 10^{21}) \exp\left[-(8.664 \times 10^{-9} \text{ y}^{-1})(4.55 \times 10^9 \text{ y})\right] \\ &= (2.468 \times 10^{21}) e^{-39.4} = 1.87 \times 10^4 . \end{aligned}$$

So there would be less than 20,000 atoms left from the original gram of plutonium.

37. Photocopied with the equation on the next page is a portion of Appendix A from the text.  $^{50}\text{V}$  is an unstable nucleus which may have multiple decay modes.

(a) What would be the resulting isotope if this isotope underwent  $\beta^-$  decay? What if it underwent  $\beta^+$  decay? What if it underwent  $\alpha$  decay?

For  $\beta^-$  decay, the number of nucleons remains unchanged, but  $Z$  increases by one, so it would become  $^{50}\text{Cr}$ . For  $\beta^+$  decay, the number again remains unchanged, but the charge decreases by one to become  $^{50}\text{Ti}$ . For  $\alpha$  decay, the number of nucleons decreases by 4, and  $Z$  decreases by two to become  $^{46}\text{Sc}$ . All of these values, as well as the resulting  $Q$ -values, are listed in the table at right.

Mode	Daughter	$Q$ (MeV)	?
$\beta^-$	$^{50}\text{Cr}$	1.038	Yes
$\beta^+$	$^{50}\text{Ti}$	1.184	Yes
	$^{46}\text{Sc}$	-9.884	No

(b) What is the  $Q$ -value for each of these processes? Which of these modes is allowed or excluded?

For  $\beta^-$  decay, the formula for the energy produced is

$$Q = (M_P - M_D)c^2 = (49.947161 \text{ u} - 49.946047 \text{ u})c^2 = 0.001114 \text{ uc}^2 \\ = 0.001114(931.494 \text{ MeV}) = 1.038 \text{ MeV} .$$

For  $\beta^+$  decay, the formula is a little different, as are the numbers:

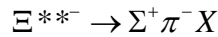
$$Q = (M_P - M_D)c^2 - 2m_e c^2 = (49.947161 \text{ u} - 49.944792 \text{ u})c^2 \\ = 0.002369 \text{ uc}^2 - 1.022 \text{ MeV} = 0.002369(931.494 \text{ MeV}) - 1.022 \text{ MeV} = 1.184 \text{ MeV} .$$

Finally, for  $\alpha$  decay, we have

$$Q = (M_P - M_D - M_{4\text{He}})c^2 = (49.947161 \text{ u} - 45.955170 \text{ u} - 4.002602 \text{ u})c^2 \\ = -0.010611 \text{ uc}^2 - 1.022 \text{ MeV} = -0.010611(931.494 \text{ MeV}) = -9.884 \text{ MeV} .$$

These results are all summarized in the table above. Clearly, the two types of  $\beta$  decay are allowed, while  $\alpha$  decay is not.

38. There is a particle  $\Xi^{*-}$  which decays by strong interactions as follows:



The  $\Xi^{*-}$  and  $\Sigma^+$  are baryons and the  $\pi^-$  is a meson. The mass, spin, strangeness and charge of the other particles are listed above.

All masses in $\text{MeV}/c^2$				
Name	Mass	Spin	Strange	Charge
$\Xi^{*-}$	2025	5/2	-2	-1
$\Sigma^+$	1179	1/2	-1	+1
$\pi^-$	135	0	0	-1

(a) What is the charge and strangeness of the  $X$  particle?

Since the decay is strong, both charge and strangeness are conserved. For charge, let  $q$  be the charge of  $X$ , then

$$\begin{aligned} -1 &= +1 - 1 + q, \\ q &= -1. \end{aligned}$$

If we let  $s$  stand for the strangeness of  $X$ , then

$$\begin{aligned} -2 &= -1 + 0 + s, \\ s &= -1. \end{aligned}$$

So the charge and strangeness are both -1.

(b) Is it a baryon, anti-baryon, or a meson?

There is one baryon on each side of the equation (the pion, since it is a meson, we ignore). Hence baryon number is conserved, and  $X$  must carry no baryon number, and therefore it is not a baryon. Since it is strongly interacting, it must therefore be a meson.

(c) Is it a fermion or a boson?

The total number of fermions (particles with half-integer spin) on the two sides of the equation must always be even. We have two such particles so far, since the pion doesn't count, so the  $X$  must be a boson, not a fermion.

(d) What, if anything, can you conclude about the mass of the  $X$ ?

Since the process is a decay, the mass of the starting particle must exceed the sum of the masses of the final particles. Hence

$$\begin{aligned} 2025 \text{ MeV}/c^2 &> (1179 \text{ MeV}/c^2) + (135 \text{ MeV}/c^2) + M_X, \\ M_X &< 711 \text{ MeV}/c^2. \end{aligned}$$

**39. A neutron star is discovered to have a radius of  $R = 15.24$  km. It is found that the hydrogen- $\alpha$  line, with a normal wavelength of  $\lambda_0 = 656.28$  nm, is observed instead with a wavelength of  $\lambda = 700.35$  nm.**

**(a) If we spent 60.0 minutes on the surface of the neutron star, as viewed by a distant observer, how much would we age (assuming we can survive the gravity)?**

Comparing the wavelength and time dilation formulas, we see that they can be written in a common form as

$$\frac{\tau}{t} = \sqrt{1 - \frac{2GM}{c^2 r}} = \frac{\lambda_0}{\lambda}$$

Substituting in our specific values, we have

$$\begin{aligned} \frac{\tau}{t} &= \sqrt{1 - \frac{2GM}{c^2 r}} = \frac{\lambda_0}{\lambda} = \frac{656.28 \text{ nm}}{700.35 \text{ nm}} = 0.9371, \\ \tau &= 0.9371t = (0.9371)(60.0 \text{ min}) = 56.22 \text{ min}. \end{aligned}$$

**(b) If this neutron star's mass were compressed to make a black hole, what would be the Schwarzschild radius  $R$ ?**

Comparing with the formula for the Schwarzschild radius, we see that we have

$$\begin{aligned} \sqrt{1 - \frac{R_s}{R}} &= 0.9371, \\ 1 - \frac{R_s}{R} &= 0.9371^2, \\ R_s &= R(1 - 0.9371^2) = (15.24 \text{ km})(0.1219) = 1.858 \text{ km}. \end{aligned}$$

**(c) What is the mass of the neutron star, in solar masses ( $M_\odot = 1.989 \times 10^{30}$  kg).**

We rearrange our standard formula for the Schwarzschild radius to yield

$$\begin{aligned} M &= \frac{c^2 R_s}{2G} = \frac{(2.998 \times 10^8 \text{ m/s})^2 (1858 \text{ m})}{2(6.673 \times 10^{-11} \text{ m}^3 / \text{kg} / \text{s}^2)} = 1.250 \times 10^{30} \text{ kg} \frac{1M_\odot}{1.989 \times 10^{30} \text{ kg}} \\ &= 0.629 M_\odot. \end{aligned}$$

### Equations

Constants:  $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$        $u = 931.494 \text{ MeV} / c^2$   
 $\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$        $u = 1.661 \times 10^{-27} \text{ kg}$   
 $G = 6.673 \times 10^{-11} \text{ m}^3 / \text{kg} / \text{s}^2$        $2m_e c^2 = 1.022 \text{ MeV}$   
 $N_A = 6.022 \times 10^{23}$        $M_{\text{He}} = 4.002602 u$   
 $1 \text{ y} = 3.156 \times 10^7 \text{ s}$

Gravitational time dilation:  $\tau = t \sqrt{1 - \frac{2GM}{c^2 r}}$       Red Shift:  $\lambda = \lambda_0 \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}$

Photoelectric Effect:  $eV_{\text{max}} = hf - \phi$

Mass, energy, momentum:  $\vec{p} = \gamma m \vec{u}$ ,       $E = \gamma mc^2$ ,       $E^2 - \vec{p}^2 c^2 = m^2 c^4$ ,       $\frac{\vec{p}c}{E} = \frac{\vec{u}}{c}$

Schwarzschild radius:  $R_s = 2GM / c^2$

Expectation values:  $\langle \mathcal{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x) \mathcal{O} \psi(x) dx$       Uncertainty:  $(\Delta \mathcal{O})^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$

### Isotope Masses