Name ________________

Solutions to Final Exam
December 13 or 15, 2007

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered. Some helpful equations and a short table of isotopes can be found on the last page

Part I: Multiple Choice (mixed new and review questions) [50 points]
For each question, choose the best answer (2 points each)

1. Which type of decay can happen *only* when the nucleus is in an excited state, having more energy than the ground state?
   A) β^+ decay B) β^- decay **C) γ decay** D) α decay E) electron capture

2. According to the deBroglie formula (and experiments), which types of objects should have quantum-mechanical wave-like properties?
   A) Electrons (only)
   B) Photons (only)
   C) Atoms (only)
   D) Electrons and photons, but not atoms
   E) Electrons, atoms, and photons

3. It is now believed that the total number of different types of quarks (up, down, strange, etc.) is
   A) 3 B) 4 C) 5 **D) 6** E) more than 6

4. Why is it so difficult to get really heavy stable nuclei, say, with $A > 210$ or so?
   A) The strong force starts to become repulsive at large A values
   B) The electrostatic repulsion of the protons becomes excessively large
   C) Electrons start getting sucked into the nucleus, changing Z
   D) The gravitational force becomes strong, and the nucleus sucks up neutrinos causing inverse neutron decay
   E) The quarks inside the proton and neutron become unstable at high densities

5. In a complex atom (NOT hydrogen), which of the following electron states would generally have the highest energy?
   A) 2p B) 2d C) 3p **D) 3d** E) 3p and 3d would be tied

6. When does light move fastest in vacuum?
    - A) When it is emitted by a source moving the same direction as the light wave
    - B) When it is emitted by a source moving the opposite direction as the light wave
    - C) When it is emitted by a source moving perpendicular to the direction of the light wave
    - D) When it is emitted by a source that is not moving
    - **E) The speed of light in vacuum is exactly the same in all four cases**

7. Which of the following is <u>not</u> a prediction of special relativity?
    - A) The size of a fast-moving object appears to change
    - B) There are actually no rigid objects
    - C) A high-velocity clock moves slower compared to one at rest
    - **D) The position of an object is not definite, but must have some uncertainty**
    - E) Actually, all of these <u>are</u> predictions of special relativity

8. Element 118 is the heaviest one ever made, and the only isotope ever seen is ^{293}Uuo. How many neutrons are there in this isotope?
    A) 118 B) 293 C) 411 **D) 175** E) 57

9. Which of the following is true about momentum in quantum mechanics?
    - A) It must always be an integer times Planck's reduced constant $\hbar$.
    - **B) It becomes an operator which acts like a derivative of the wave function**
    - C) If you calculate its expectation value (average value) for a real wave function, you always get a non-zero value
    - D) It is identical to the Hamiltonian, which measures the energy of the wave function
    - E) Momentum itself becomes meaningless, and must be written as mass times velocity

10. What is the value of the spin quantum number *s* of an electron?
    A) -1 B) 0 C) +1 D) +2 **E) + ½**

11. How come we don't worry about the Schwarzschild radius for objects like the Sun or the Earth?
    - A) It is microscopically small, and therefore its effects can never be seen
    - B) It is so huge it effectively can never be reached
    - **C) It would be inside the Sun or the Earth, and since the metric is only valid <u>outside</u> the object, it is irrelevant**
    - D) Because the Earth and the Sun are rotating, there is no such Schwarzschild radius
    - E) I have no idea, please mark this one wrong

12. Simplify the expression: $e^{i\pi/2}$
    A) *i* B) 1 C) -*i* D) -1 E) None of the above

13. What’s the fundamental difference between using curved coordinates (like spherical coordinates) and actual gravity (like the Schwarzschild metric)
    A) Only with real gravity will your motion in coordinates look non-linear
    B) Only with real gravity will spacetime actually have curvature, not just the coordinates
    C) Only with real gravity will there be any advantage to using spherical coordinates
    D) Only with real gravity will the formula for distance actually look different
    E) Without gravity, you follow geodesics, but with gravity you follow different curves

14. If I throw a spherical ball at very high velocities, then according to special relativity, as viewed by a stationary observer
    A) The ball would get <u>smaller</u> in the direction of motion and <u>larger</u> in the other directions
    B) The ball would get <u>smaller</u> in the direction of motion and <u>unchanged</u> in the other directions
    C) The ball would get <u>larger</u> in the direction of motion and <u>smaller</u> in the other directions
    D) The ball would get <u>larger</u> in the direction of motion and <u>unchanged</u> in the other directions
    E) The ball would be <u>unchanged</u> in all directions.

15. Which of the following is produced during nuclear β^- decay?
    A) Anti-Neutrino B) Anti-electron C) Neutron D) Photon E) Anti-proton

Because of an error in the questions, all answers were treated as correct for problem 15.

16. Which of the following is a pretty good description of the principle of equivalence?
    A) Gravity is equivalent to any other force, such as electromagnetism
    B) The force of gravity is equivalent to the force from a spring with fixed spring constant k
    C) The effects of gravity are equivalent to the effects of being in an accelerated reference frame
    D) Because the force of gravity is so small, having gravitational forces and not having gravitational forces are equivalent
    E) Curved and flat spacetimes are equivalent

17. What effect, if any, does the <u>rotation</u> of the Earth have on objects near it, according to General Relativity?
    A) It causes objects to be twisted around, as if spacetime itself were rotating
    B) It causes a significant additional <u>slowing down</u> of time
    C) It causes a significant additional <u>speeding up</u> of time
    D) It causes the Earth’s effective gravity to be significantly <u>increased</u>
    E) It causes the Earth’s effective gravity to be significantly <u>decreased</u>

18. If an electron moves from one state with energy E_1 to another state with energy E_2 by emitting a photon of frequency f, what is the relationship between these three quantities?
A) $hf = E_1 - E_2$
B) $hf = E_2 - E_1$
C) $f = h(E_1 - E_2)$
D) $f = h(E_2 - E_1)$
E) There is insufficient information to answer this question

***In questions 19-21, you will answer questions about a particle made of an up quark, down quark, and strange quark,* [uds]**

19. What type of particle is made of [uds]?
A) lepton B) anti-lepton C) meson **D) baryon** E) anti-baryon

20. The charge of the particle made from [uds] is
A) -1 B) -1/3 **C) 0** D) +1/3 E) +1

21. The strangeness of the particle [uds] is
A) -2 **B) -1** C) 0 D) +1 E) +2

***In questions 22 – 25, you will be given a reaction, and you must state what type of interaction it is. A mimi-table of particles is listed at right.* All of them have strangeness 0. The mesons are all spin 0 and the leptons are spin ½.**

Mesons

Name	Mass
π^+	139
π^0	135
π^+	139

Leptons

Name	Mass
e^-	0.5
ν_e	0

22. $\pi^+ \rightarrow e^+ + \nu_e$ (the e^+ is the anti-particle of the e^-)
A) impossible B) strong **C) weak** D) electromagnetic E) gravity

23. $\pi^0 \rightarrow \pi^+ + \pi^-$
A) impossible B) strong C) weak D) electromagnetic E) gravity

24. $\pi^0 + \pi^0 \rightarrow \pi^+ + \pi^-$
A) impossible **B) strong** C) weak D) electromagnetic E) gravity

25. $\pi^0 \rightarrow e^+ + e^-$ (the e^+ is the anti-particle of the e^-)
A) impossible B) strong C) weak **D) electromagnetic** E) gravity

Part II: Short answer (review material) [20 points]

Choose two of the following three questions and give a short answer (1-3 sentences) (10 points each).

26. According to special relativity, you can't go faster than light. But according to electromagnetism, if you push on, say, a proton with charge *e* using an electric field *E*, it will feel a force $F = eE$ and therefore will accelerate at a rate $a = eE/m$, so that after a time $t = mc/eE$, it will exceed the speed of light. Explain what, if anything, is wrong with this argument.

The formula for electric force, $F = eE$ is indeed correct; however, the formula for acceleration $F = ma$ that is implied by the next formula is incorrect. Instead, we use $F = dp/dt$, which will demonstrate that the momentum p rises indefinitely, but since this rises towards infinity as the velocity approaches c, the particle will simply slowly approach the limiting velocity c.

27. Why is it necessary to use a light source with a minimum *frequency* to get electrons to pop loose from a piece of metal, in accordance with the photoelectric effect?

According to Planck, the energy of a photon of frequency f is given by $E = hf$, where f is the frequency and h is Planck's constant. Since it takes a certain minimum amount of work ϕ to remove an electron from a piece of metal, the frequency f must be high enough that $hf > \phi$.

28. Explain, qualitatively, the meaning of the quantum numbers *l* and *m* (the latter is sometimes called m_l) when describing an electron in hydrogen. Formulas may be appropriate, but are not necessary.

l and m describe the total orbital angular momentum and the angular momentum around the z-axis of an electron in a spherically symmetric potential, such as hydrogen. The formulas for these two quantities are

$$\vec{L}^2 = \hbar^2\left(l^2 + l\right) \qquad \text{and} \qquad L_z = \hbar m$$

Part III: Short answer (new material) [30 points]

Choose three of the following four questions and give a short answer (1-3 sentences) (10 points each).

29. What are the three forces discussed in the standard model? For each of them, give the name of at least one particle that "mediates" that force.

The three forces in the standard model are the strong force (mediated by the eight gluons), the weak force (mediated by the $W^{\pm}$ and the Z^0), and electromagnetism (mediated by the photon).

30. What is another name for a β-particle? How does emission of it change the values of Z and A for a nucleus? What is the formula for the left-over energy (Q) when a nucleus emits a β-particle?

A particle is the same thing as an electron. When you have β-decay, the Z-value of the nucleus increases by one, but the mass number A is unchanged. The energy left over afterwards is

$$Q = \left(M_P - M_D\right)c^2$$

Where M_P and M_D are the masses of the parent and daughter atoms respectively.

31. There is some controversial experimental evidence for a particle named θ which is believed to have quark content [$\bar{u}udds$]. Explain why this is controversial.

Strongly interacting particles are supposed to be baryons (made of three quarks), anti-baryons (made of three anti-quarks), and mesons (made of a quark and an anti-quark). Since this fits none of the categories, it is controversial. Some people might consider it a combination of a baryon and a meson.

32. According to Newton's law of gravitation, two objects orbiting each other should orbit at a constant distance forever, but studies of some binary pulsars indicate that they are slowly moving closer to each other. Explain how this is possible.

Newton's laws and Einstein's *almost* agree, but not exactly. In particular, in Einstein's equations, orbiting neutron stars are accelerating masses, and these will produce gravitational radiation. This loss of energy causes the orbits of the particles to decay, resulting in them gradually spiraling together.

Part IV: Calculation (review material) [40 points]

Choose two of the following three questions and perform the indicated calculations (20 points each)

33. Two observers are observing the same pair of events *A* and *B* in spacetime. The coordinates of these two events, according to the first observer are

A: (*x*, *y*, *z*, *t*) = (0 m, 0 m, 0 m, 0 ns)

B: (*x*, *y*, *z*, *t*) = (9.00 m, 0 m, 0 m, 10.0 ns)

A second observer is moving at speed $v = 1.80\times10^8$ m/s (= 0.600*c*) in the +*x* direction. Some possibly useful formulas can be found at the end of the exam.

(a) What is the value of γ for this velocity?

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.600^2}} = \frac{1}{\sqrt{0.640}} = \frac{1}{0.800} = 1.25$$

(b) What are the coordinates of *A* and *B* as viewed by the moving observer?

We simply use the Lorentz transformation. For point *A*, we have:

$$\begin{aligned} x' &= \gamma(x-vt) = \gamma(0-0) = 0 \\ t' &= \gamma\left(t - vx/c^2\right) = \gamma(0-0) = 0 \end{aligned} \qquad \text{and} \qquad \begin{aligned} y' &= y = 0 \\ z' &= z = 0 \end{aligned}$$

For point *B*, the computation is a bit more complicated.

$$x' = \gamma(x-vt) = \gamma\left[9.00\text{ m} - \left(1.80\times10^8\text{ m/s}\right)\left(1.00\times10^{-8}\text{ s}\right)\right] = 1.25(7.20\text{ m}) = 9.00\text{ m}$$

$$t' = \gamma\left(t - vx/c^2\right) = \gamma\left[1.00\times10^{-8}\text{ s} - \left(1.80\times10^8\text{ m/s}\right)(9.00\text{ m})\Big/\left(3.00\times10^8\text{ m/s}\right)^2\right]$$

$$= 1.25\left(1.00\times10^{-8}\text{ s} - 1.80\times10^{-8}\text{ s}\right) = 1.25(-8.0\text{ ns}) = -10.0\text{ ns}$$

and we again have *y'* = *z'* = 0. In summary, our two points in the primed frame are

A': (*x*, *y*, *z*, *t*) = (0 m, 0 m, 0 m, 0 ns)

B': (*x*, *y*, *z*, *t*) = (9.00 m, 0 m, 0 m, – 10.0 ns)

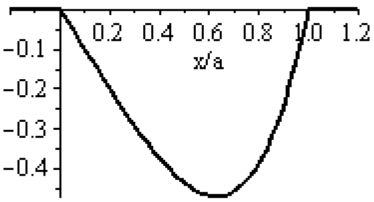
Note that while *B* is in the future of *A* as measured by the unmoving observer, it is in the past as measured by the moving observer.

(c) What is the proper distance *s* between *A* and *B* as measured by the initial observer?

$$s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 = (9.00\text{ m})^2 - \left[\left(3.00\times10^8\text{ m/s}\right)\left(1.00\times10^{-8}\text{ s}\right)\right]^2$$

$$= 72.0\text{ m}^2.$$

Taking the square root, we find *s* = 8.49 m.



34. A particle of mass *m* has wave function given by

$$\psi(x) = \begin{cases} N\left(x^4 - a^3 x\right) & \text{if } 0 < x < a, \\ 0 & \text{otherwise.} \end{cases}$$

where *a* is known and *N* is an unknown normalization constant. This function is graphed *qualitatively* at right.

(a) In the region 0 < *x* < *a*, is there any point where the particle definitely is not present?

The only places where the particle is not present is when the function vanishes, which implies

$$0 = N\left(x^4 - a^3 x\right) = Nx\left(x^3 - a^3\right) .$$

This can only happen at $x = 0$ or $x = a$, which is not in the region we are discussing.

(b) What is (are) the most likely point(s) to find the particle?

The most likely place is where the function is the largest in magnitude. This will be a minimum, which is found by setting the derivative equal to zero.

$$\begin{aligned} 0 = \frac{d\psi}{dx} &= N\left(4x^3 - a^3\right), \\ 4x^3 &= a^3, \\ x &= a\big/\sqrt[3]{4}. \end{aligned}$$

(c) What is the correct value of the normalization constant *N*?

The function is correctly normalized when its integral equals 1, so we have

$$1 = \int_{-\infty}^{\infty} \left|\psi(x)\right|^2 dx = \int_0^a N^2\left(x^4 - a^3 x\right)^2 dx = N^2 \int_0^a \left(x^8 - 2a^3 x^5 + a^6 x^2\right) dx$$

$$= N^2\left(\tfrac{1}{9}x^9 - \tfrac{2}{6}a^3 x^6 + \tfrac{1}{3}a^6 x^3\right)\Big|_0^a = N^2\left(\tfrac{1}{9}a^9 - \tfrac{1}{3}a^9 + \tfrac{1}{3}a^9 - 0\right) = \tfrac{1}{9}N^2 a^9$$

Solving for *N*, we have

$$N^2 = \frac{9}{a^9}, \qquad \text{or} \qquad N = 3a^{-9/2}$$

35. Joe Atom is driving his molecular car, which has a mass of 1.00×10^{-19} kg. He wants to park it in a space that is 6.40×10^{-8} m long.

(a) Estimate the minimum uncertainty in the car's position, if the owner wants to make sure it is within the parking space.

In general, if we want to assure that something is within a region of size *l*, it must have an uncertainty in its position of not more than about ¼*l*. Hence the uncertainty in its position is about $\Delta x = 1.60\times 10^{-8}\ \text{m}$.

(b) Estimate the corresponding uncertainty in its momentum. Assuming the owner TRIED to put it in park (*v* = 0), estimate the ACTUAL velocity caused by this uncertainty in momentum.

By the uncertainty principle, there is a minimum uncertainty in the momentum, because $\Delta x \Delta p \geq \tfrac{1}{2}\hbar$, so we have

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.055\times 10^{-34}\ \text{J}\cdot\text{s}}{2\left(1.60\times 10^{-8}\ \text{m}\right)} = 3.30\times 10^{-27}\ \text{kg}\cdot\text{m/s}$$

This represents a sort of random motion of his car due to molecular effects. If we divide by the mass of the car, we get an estimate of the velocity, namely

$$\Delta v = \frac{\Delta p}{m} = \frac{3.30\times 10^{-27}\ \text{kg}\cdot\text{m/s}}{1.00\times 10^{-19}\ \text{kg}} = 3.30\times 10^{-8}\ \text{m/s}$$

(c) Estimate how long Joe can safely leave his car parked before the quantum velocity found in (b) will cause his car to drift out of its parking space.

If Joe parks in the middle of the parking space, then he will be 3.20×10^{-8} m from either end. If his car drifts at the rate given above, the amount of time before he leaves his space will be about

$$t = \frac{d}{v} = \frac{3.20\times 10^{-8}\ \text{m}}{3.30\times 10^{-8}\ \text{m/s}} = 0.97\ \text{s}$$

So it will remain in the parking space for only about one second.

Part V: Calculation (new material): [60 points]

Choose three of the following four questions and perform the indicated calculations (20 points each)

36. One of the unfortunate byproducts of nuclear weapons is radioactive Iodine, ^{131}I with a half-life of 8.021 days and an atomic mass of 130.906 u. Iodine is absorbed by the thyroid gland, where it can cause cancer.

(a) What is the decay constant λ for this isotope?

The half-life is related to the decay constant λ by

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.6931}{8.021\text{ d}} = \left(0.08642\text{ d}^{-1}\right)\frac{1\text{ d}}{86,400\text{ s}} = 1.000\times10^{-6}\text{ s}^{-1}$$

(b) A person's thyroid is discovered to be producing 953 decays/s from radioactive iodine. How many atoms of 131I are in their thyroid?

The decay rate is given by $R = \lambda N$, so we have

$$N = \frac{R}{\lambda} = \frac{953\text{ s}^{-1}}{1.000\times10^{-6}\text{ s}^{-1}} = 9.53\times10^{8}$$

(c) How many grams of ^{131}I are in their thyroid?

Each atom weighs 130.906 u, so this works out to

$$M = mN = \left(9.53\times10^{8}\right)\left(130.906\text{ u}\right)\left(1.661\times10^{-27}\text{ kg}\right)\left(10^{3}\text{ g/kg}\right) = 2.07\times10^{-13}\text{ g},$$

or about 0.207 pg, a very small amount.

(d) How long will they have to wait until the decay rate drops to 1.00 decays/s from radioactive ^{131}I?

The rate decays according to the formula

$$R = R_0 e^{-\lambda t}$$

Rearranging and taking the natural logarithm, we have

$$e^{\lambda t} = R_0/R, \quad \text{or} \quad \lambda t = \ln\left(R_0/R\right)$$

Dividing by the decay constant gives us the time

$$t = \frac{\ln\left(R_0/R\right)}{\lambda} = \frac{\ln\left(953/1.00\right)}{0.0862\text{ d}^{-1}} = 79.4\text{ d}$$

37. Photocopied with the equation on the next page is a portion of Appendix A from the text. ^{7}Be is an unstable atom which decays with a half-life of 53.3 days.

(a) What would be the resulting isotope if this isotope underwent α decay? What if it underwent electron capture? What if it underwent β^+ decay?

Under α decay, the value of *Z* decreases by two, and *A* decreases by 4, so the isotope that would be left over would be ^{3}He (though in this case, it is unclear which is emitting which!). Under both electron capture and β^+ decay, *A* is unchanged, while *Z* decreases by one, which leaves us with ^{7}Li.

(b) What is the *Q*-value for each of these processes?

For α decay, the *Q*-value is given by

$$Q = \left(M_P - M_D - M_{4He}\right)c^2 = \left(7.016928 - 3.016029 - 4.002602\right)\text{u}c^2$$
$$= \left(-0.001703\right)\left(931.494 \text{ MeV}\right) = -1.586 \text{ MeV}$$

For electron capture, the formula is simply

$$Q = \left(M_P - M_D\right)c^2 = \left(7.016928 - 7.016003\right)\text{u}c^2$$
$$= \left(0.000925\right)\left(931.494 \text{ MeV}\right) = 0.862 \text{ MeV}$$

For β^+ decay, the formula is the same except we subtract twice the rest energy of an electron

$$Q = \left(M_P - M_D\right)c^2 - 2m_ec^2 = 0.862 \text{ MeV} - 1.022 \text{ MeV} = -0.160 \text{ MeV}$$

(c) Cosmic rays from deep space, many light years away, sometimes contain ^{7}Be^{+4} nuclei, which have no electrons. With such a short half-life, how is this possible?37. Photocopied with the equation on the next page is a portion of Appendix A from the text. ^{7}Be is an unstable nucleus which decays with a half-life of 53.3 days.

Looking at the three decays, we see that only electron capture is possible. However, ^{7}Be^{+4} is a bare nucleus, and therefore has no electrons to capture. So this *nucleus* is in fact stable; it is only the atom (which includes electrons) which can decay by electron capture.

38. There is a particle Ξ^{*-} which decays by strong interactions as follows:

$$\Xi^{*-} \rightarrow \Sigma^- + X$$

The Ξ^{*-} and Σ^- are both baryons, and the other particles in the table at right are all mesons. The spin and strangeness of the other particles are listed at right. The charges are implied by their names.

All masses in MeV/c^2

Name	Mass	Spin	Strange
Ξ^{*-}	1820	3/2	-2
Σ^-	1197	1/2	-1
π^0	135	0	0
K^-	494	0	-1
K^0	498	0	+1
$\bar{K}^{*0}$	892	1	-1

(a) What is the charge and strangeness of the *X* particle?

The charge of the Ξ^{*-} and the Σ^- are both -1, so the *X* must have no charge to make it balance out. As for strangeness, we start with -2, which is broken into -1 and *X*, so the strangeness of *X* must be -1 as well.

(b) Is it a baryon, anti-baryon, or a meson?

We have one baryon on each side, so we'd better not have another baryon. This implies that *X* must be a meson, since it's strongly interacting.

(c) What, if anything, can you conclude about the mass of the *X*?

Since this is a decay, the mass of the products must be less than the mass of the decaying particle. This tells us

$$M_X < M_{\Xi^*} - M_\Sigma = (1820 - 1197)\ \text{MeV}/c^2 = 623\ \text{MeV}/c^2$$

(d) Could the *X* be any of the particles in the table given? Could it be any of the anti-particles?

The *X* must be a neutral meson with strangeness -1 and mass less than 623 MeV/c^2. There is no such particle in the table. However, the K^0 has all the right properties except the strangeness has the wrong sign. Hence its anti-particle, which is also neutral, has all the requisite properties.

39. The center of the galaxy M87 is believed to have black hole with mass of approximately 3.0 × 10^9 times the Sun's mass. The Sun's mass is 1.989 × 10^{30} kg.

(a) What is the Schwarzschild radius of this black hole? Give the answer in AU, where 1 AU = 1.496 × 10^{11} m.

The Schwarzschild radius is given by

$$R_S = \frac{2GM}{c^2} = \frac{2\left(6.673\times10^{-11}\ \mathrm{m^3/kg/s^2}\right)\left(3.0\times10^{9}\right)\left(1.989\times10^{30}\ \mathrm{kg}\right)}{\left(2.998\times10^{8}\ \mathrm{m/s}\right)^2}$$

$$= \frac{8.86\times10^{12}\ \mathrm{m}}{1.496\times10^{11}\ \mathrm{m/AU}} = 59\ \mathrm{AU}$$

(b) Suppose you spent one day at a distance of 1 AU from the Schwarzschild radius of the galactic black hole (*i.e. r* = *R*$_S$ + 1 AU). How much time would pass, according to an observer far from the black hole?

The relationship between proper time, as measured by you near the black hole, and external time, as measured by someone far away, is given by

$$\tau = t\sqrt{1-\frac{2GM}{c^2}} = t\sqrt{1-\frac{R_S}{r}} = t\sqrt{1-\frac{59\ \mathrm{AU}}{(59+1)\ \mathrm{AU}}} = 0.13t,$$

$$t = \tau/0.13 = 7.8\ \mathrm{d}$$

(c) If someone far away shone a laser on you with a wavelength of 730 nm, what wavelength would you detect? Warning: note that this is the reverse of the usual situation, in that we have light going *into* the gravitational potential.

This problem is a little confusing, because the wavelength λ_0 is the wavelength inside the gravitational potential, while λ is the wavelength outside of it. Hence these wavelengths are related by

$$\lambda_0 = \lambda\left(1-\frac{2GM}{c^2r}\right)^{1/2} = 0.13\lambda = 0.13(730\ \mathrm{nm}) = 95\ \mathrm{nm}$$

This light is such a short wavelength that it is well into the ultraviolet.

Another way to make sure you get the correct effect is to notice that since you are near the black hole, time slows down for you, and hence any source outside will seem to vibrate much faster. This means the frequency of the light increases, which in turn implies that the light you see will be at a shorter wavelength.

Equations

Special Relativity Formulas

$$s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 = -(c\tau)^2$$

$$x' = \gamma(x - vt) \qquad y' = y$$

$$t' = \gamma(t - vx/c^2) \quad \text{and} \quad z' = z$$

Constants

$$h = 6.626\times10^{-34}\ \mathrm{J\cdot s} = 4.136\times10^{-15}\ \mathrm{eV\cdot s}$$

$$\hbar = 1.055\times10^{-34}\ \mathrm{J\cdot s} = 6.582\times10^{-16}\ \mathrm{eV\cdot s}$$

$$G = 6.673\times10^{-11}\mathrm{m^3/kg/s^2}$$

Nuclear Decay

$$\mathrm{u} = 931.494\ \mathrm{MeV}/c^2$$

$$= 1.661\times10^{-27}\,\mathrm{kg}$$

$$N_A = 6.022\times10^{23}$$

$$2m_ec^2 = 1.02200\ \mathrm{MeV}.$$

General Relativity

$$\tau = t\sqrt{1 - \frac{2GM}{c^2 r}}$$

$$\lambda = \lambda_0\left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}$$

$$R_S = \frac{2GM}{c^2}$$

Isotope Masses