Name ___________________

Solutions to Test 2
October 15, 2008

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. The equations below may be helpful with some problems.

Constants

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$
$$k_B = 1.3807\times10^{-23}\ \text{J/K} = 8.6173\times10^{-5}\ \text{eV/K}$$
$$k = 8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$$
$$e = 1.602\times10^{-19}\ \text{C}$$
$$\alpha = \frac{ke^2}{\hbar c} = 7.29735\times10^{-3} \simeq \frac{1}{137}$$

Black Bodies

$$U = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3}$$
$$\lambda_{\max} T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$$

Rutherford Scattering

$$b = \frac{kqQ}{m_\alpha v^2}\cot\left(\frac{\theta}{2}\right)$$
$$R = \frac{2Ze^2k}{E}$$

Compton Effect

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$
$$\frac{h}{mc} = 2.426\times10^{-12}\ \text{m}$$

Wave Relationships

$$\lambda = \frac{2\pi}{k}$$
$$\frac{\omega}{2\pi} = f = \frac{1}{T}$$

Hydrogen-Like Atoms

$$E = -\frac{k^2e^4\mu Z^2}{2\hbar^2 n^2} = -\frac{(\mu c^2)\alpha^2 Z^2}{2n^2}$$
$$E = \frac{-(13.6\ \text{eV})Z^2}{n^2}$$

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. What is one difference between the wave amplitude Ψ of quantum mechanics and other types of waves, such as the electric field **E**?
   A) It is not a function of time, while the electric field is
   B) It is a function of position, while the electric field is not
   C) It will be a complex function, while the electric field is real
   D) It can look like cosine or sine, while the electric field can only be a cosine
   E) It can be used to calculate forces, while the electric field cannot

2. Which of the following is not an assumption of the Bohr model of the atom?
   A) Electrons orbit the nucleus in circular orbits
   B) The angular momentum comes in integer multiples of $\hbar$
   C) The electron does not have a definite position or momentum
   D) When an electron goes from one level to another in an atom, the energy comes out as one photon, or particle of light
   E) Actually, all of these are assumptions of the Bohr model

3. When you excite atoms from a gas such as hydrogen, light is emitted, which only contains certain wavelengths of light. What is causing these particular wavelengths?
   A) Electrons moving from one level to another within the atom
   B) Light that has scattered off of the electrons in the atom
   C) Electrons being absorbed by the nucleus
   D) Electrons being emitted by the nucleus
   E) A little guy who lives in the atom and is watching a TV

4. What is the difference between group velocity and phase velocity?
   A) Group velocity is the overall speed of a packet of waves; phase velocity is the speed of the peaks and valleys inside the wave
   B) Phase velocity is the overall speed of a packet of waves; group velocity is the speed of the peaks and valleys inside the wave
   C) Phase velocity takes into account that the waves are complex, group velocity does not
   D) Group velocity takes into account that the waves are complex, phase velocity does not
   E) There is no difference; they are two names for the exact same thing.

5. What was deBroglie's explanation for why electrons only go in certain orbits in the Bohr model?
   A) They have to be an integer number of wavelengths away from the nucleus
   B) They have to have a wavelength that matches the size of the nucleus
   C) They have to have a wave such that diffraction around the nucleus causes it to change direction and go the other way, like an orbit
   D) The orbit actually vibrates, producing a wave, and the electron is caught in the wave
   E) They have to have an integer number of wavelengths that fit around the orbit

6. What did Millikan discover by studying the motion of tiny drops of oil in an electric field in 1909?
   A) He measured the ratio of the electron's mass to charge
   B) He was able to measure the fundamental charge, the charge on a single electron or proton
   C) He was able to tell that electrons were negatively charged
   D) He discovered the charge of a mole of electrons, but he was unable to discover the charge of a single electron
   E) He was able to show that light waves could only have certain discrete amounts of energy

7. When we send something through a small slit, when are we likely to get significant diffraction effects?

A) You get significant effects for photons, but not electrons
B) You get significant effects for electrons, but not photons
C) You get significant effects if the wavelength is much shorter than the size of the slit, but not if it is comparable
D) You get significant effects if the wavelength is comparable to the size of the slit, but not if it is much shorter
E) Diffraction effects are always significant

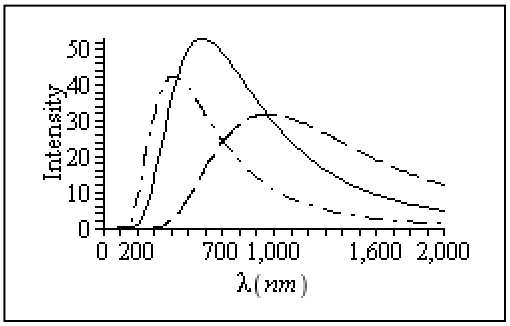
8. One version of the uncertainty principle relates the uncertainty in the position and momentum of a particle. What two quantities are related in the other version we have discussed?
A) Energy and angular momentum
B) Energy and time
C) Time and angular momentum
D) Wavelength and Energy
E) Wavelength and time

9. Why do X-rays reflect so much better off of crystals when they have a particular wavelength?
A) They can only bounce off if they have the right energy, and this implies they must have the right frequency and hence wavelength
B) They must have exactly the right wavelength so they can fit between the atoms in the crystal structure
C) They actually work for any wavelength, but can only be detected if they have the right wavelength
D) They must exactly match the wavelength of the electrons orbiting the atoms
E) They are actually bouncing off different layers of atoms, and the spacing must be right so the reflected waves add together, rather than canceling out

10. When Rutherford scattered alpha particles off of nuclei, why did the results change substantially when he used very high energy alpha particles?
A) The alphas had enough energy to knock electrons loose from the atom
B) The alphas had enough energy to break the nucleus apart
C) The alphas reached a wavelength short enough to fit inside the atom
D) The alphas had enough energy to reach the repulsive positively charged nucleus
E) The alphas had enough energy to knock the atoms they were hitting out of the lattice of other atoms

Part II: Short answer [20 points]

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).



11. At right are the spectra from three different stars. Which one is hottest, and how can you tell? Give any relevant formula.

The wavelength where the intensity is highest is related to the temperature by Wien's Law, given above, or $\lambda_{\max} T = 2.898\ \text{mm} \cdot \text{K}$. The hottest one is the one with the peak at the shortest wavelength, or the dot-dash curve.

12. Why is there a minimum frequency for which electrons can be ejected from a metal? Give a formula for the work function in terms of this minimum energy

A certain amount of work ϕ is required to extract an electron from metal. Since photons come with energy $E = hf$, the minimum frequency that can extract an electron is just $hf_{\min} = \phi$.

13. When you take an electron and accelerate it through a thin gas, the probability the electron makes it through the gas (the current) increases as you increase the voltage, but then, as you increase it more, it suddenly decreases (the Franck-Hertz experiment). What causes these sudden decreases in the current?

Atoms have a minimum amount of energy, called the ground state, and many levels above the ground state. When an electron collides with an atom, it cannot transfer any energy to the atom unless it has a certain minimum, the minimum energy separating the first level about the ground state. It is when you reach this threshold that there is a sudden drop in the current.

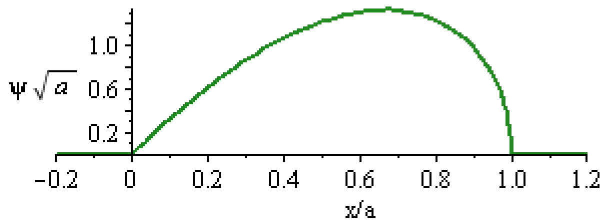
Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each).

14. At a particular time, the wave function of a particle is given by

$$\psi(x) = \begin{cases} x\sqrt{12(a-x)}\big/a^2 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

This wave function is sketched at right.



(a) What is the most likely place(s) that particle might be?

The most likely place is wherever the wave function is most positive or most negative. We can work either with the wave function (when it's real) or with the square of the magnitude, and we'll use the latter, since it is easy to square this. To find a maximum, we simply set the derivative of the square of the wave function to zero.

$$0 = \frac{d}{dx}\left|\psi(x)\right|^2 = \frac{d}{dx}\frac{12x^2(a-x)}{a^4} = \frac{12}{a^4}\left(2ax - 3x^2\right) = \frac{12x}{a^4}(2a - 3x)$$

There are two solutions to this, $x = 0$ and $x = \frac{2}{3}a$, but the former is clearly a minimum, not a maximum, so the answer is $x = \frac{2}{3}a$.

(b) What is the probability that the particle is in the region $0 < x < \frac{1}{2}a$?

We simply integrate the amplitude squared over the appropriate region to find

$$P\left(0 < x < \tfrac{1}{2}a\right) = \int_0^{\frac{1}{2}a}\left|\psi(x)\right|^2 dx = a^{-4}\int_0^{\frac{1}{2}a} 12x^2(a-x)\,dx = a^{-4}\int_0^{\frac{1}{2}a}\left(12ax^2 - 12x^3\right)dx$$

$$= a^{-4}\left(4ax^3 - 3x^4\right)\Big|_{x=0}^{\frac{1}{2}a} = a^{-4}\left(\tfrac{1}{2}a^4 - \tfrac{3}{16}a^4\right) = \tfrac{5}{16}$$

15. A lithium ion has nuclear charge *Z* = 3 and a single electron in the *n* = 10 level. The electron moves suddenly from this level to the *n* = 7 level.

(a) What is the energy of each of these levels?

The energy can be found from the formula

$$E = \frac{-(13.6\text{ eV})Z^2}{n^2} = \begin{cases} -(13.6\text{ eV})3^2/10^2 = -1.224\text{ eV} \\ -(13.6\text{ eV})3^2/7^2 = -2.498\text{ eV} \end{cases}$$

These are the energies of the two levels.

(b) Would this process require the emission or absorption of a photon? What would be the energy of that photon?

The energy is decreasing, because it is falling to a lower level. This can also be seen by looking directly at the number. In either case, the conclusion is that energy is emitted, so the photon is emitted. The energy of the resulting photon is simply the difference in energies

$$E_\gamma = \Delta E = (-1.224\text{ eV}) - (-2.498\text{ eV}) = 1.274\text{ eV}$$

(c) Find the frequency, angular frequency, (circular) wave number *k*, and the wavelength for this photon.

To find the frequency of the photon, it is easiest to use $E = hf$, and solve for f, which gives us

$$f = \frac{E}{h} = \frac{1.274\text{ eV}}{4.136\times10^{-15}\text{ eV}\cdot\text{s}} = 3.08\times10^{14}\text{ Hz}$$

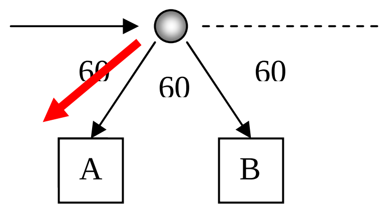
From here we just use various wave relations to work out the other quantities desired:

$$\omega = 2\pi f = 2\pi(3.08\times10^{14}\text{ Hz}) = 1.94\times10^{15}\text{ s}^{-1}$$

$$\lambda = \frac{c}{f} = \frac{2.998\times10^8\text{ m/s}}{3.080\times10^{14}\text{ /s}} = 9.73\times10^{-7}\text{ m} = 973\text{ nm}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{9.73\times10^{-7}\text{ m}} = 6.455\times10^6\text{ m}^{-1}$$

16. A beam of X-rays of unknown wavelength comes from the left and hits a target. They then scatter into two detectors A and B, as illustrated



(a) The waves entering detector A, which is 60 degrees away from back-scattering, are discovered to have wavelength 12.40 pm (pm = 10^{-12} m). What is the wavelength of the incoming X-rays?

This is Compton scattering at an angle of 120 degrees compared to forward, and the resulting waves have wavelength $\lambda' = 12.40\text{ pm}$ We use the formula for Compton scattering, namely

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta),$$

$$\lambda = \lambda' - \frac{h}{mc}(1-\cos\theta) = (12.40\text{ pm}) - (2.426\text{ pm})(1-\cos 120°)$$
$$= (12.40\text{ pm}) - (3.64\text{ pm}) = 8.76\text{ pm}$$

(b) What will be the wavelength detected at detector B, 60 degrees away from straight forward scattering?

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta) = (8.76\text{ pm}) + (2.426\text{ pm})(1-\cos 60°) = 9.97\text{ pm}$$

(c) A third detector C is supposed to be placed at a location such that the scattered X-rays have a wavelength of 13.00 pm. Where should this detector be placed?

In this case, the angle is unknown. We start working to solve for it

$$\frac{h}{mc}(1-\cos\theta) = \lambda' - \lambda = (13.00\text{ pm}) - (8.76\text{ pm}) = 4.24\text{ pm}$$

$$1-\cos(\theta) = \frac{4.24\text{ pm}}{h/mc} = \frac{4.24\text{ pm}}{2.426\text{ pm}} = 1.747$$

$$\theta = \cos^{-1}(1-1.747) = 138°$$

The direction is about 42 degrees away from straight backwards, as sketched into the diagram above.

17. **A physicist wishes to trap some electrons with mass m = 9.11 × 10^{-31} kg in a trap that only keeps them in the trap if their velocity is in the range** $-0.3 \text{ m/s} < v < 0.3 \text{ m/s}$.

(a) **What is the most positive/most negative momentum they can have? Using Carlson's rule, estimate the uncertainty Δp in the momentum of the electrons.**

The momentum is just the mass times velocity, so

$$p = mv = \pm(0.3 \text{ m/s})\left(9.11\times10^{-31} \text{ kg}\right) = \pm 2.73\times10^{-31} \text{ kg}\cdot\text{m/s}$$

Now, this implies that momentum lies in a "box" whose width is twice this, or $5.46\times10^{-31} \text{ kg}\cdot\text{m/s}$. According to Carlson's rule, we divide this by four to give us the uncertainty in the momentum, or $\Delta p = \frac{1}{4}\left(5.46\times10^{-31} \text{ kg}\cdot\text{m/s}\right) = 1.367\times10^{-31} \text{ kg}\cdot\text{m/s}$.

(b) **What is the corresponding minimum of the uncertainty in the position Δx of the electron?**

According to the uncertainty principle, $\Delta x \Delta p \geq \frac{1}{2}\hbar$, so

$$\Delta x \geq \frac{\hbar}{2\Delta p} = \frac{1.055\times10^{-34} \text{ J}\cdot\text{s}}{2\left(1.367\times10^{-31} \text{ kg}\cdot\text{m/s}\right)} = 3.86\times10^{-4} \text{ m}$$

(c) **Using Carlson's rule, estimate how large this trap must be.**

The uncertainty is suppose to be one-fourth of the size of the "container" the particle is in, so the container must be about four times larger, or about 1.54 mm in size.