Solutions to Test 2

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered. The equations below may be helpful with some problems.

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s} = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$$

$$k = 8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$$
$$e = 1.602\times10^{-19}\ \text{C}$$

$$U = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3}$$
$$\lambda_{\max} T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$$

$$k_B = 1.2306\times10^{-23}\ \text{J/K} = 7.6807\times10^{-5}\ \text{eV/K}$$

$$b = \frac{kqQ}{m_\alpha v^2}\cot\left(\frac{\theta}{2}\right)$$
$$R = \frac{2Ze^2k}{E}$$

$$E = -\frac{k^2e^4\mu Z^2}{2\hbar^2 n^2} = -\frac{(\mu c^2)\alpha^2 Z^2}{2n^2}$$
$$\alpha = \frac{ke^2}{\hbar c} \simeq \frac{1}{137}$$

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$
$$\frac{h}{mc} = 2.426\times10^{-12}\ \text{m}$$

Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

1. If the complex number z is given by $z = 5 + 2i$, what is $|z|^2$?

   A) $21\text{-}20i$ B) 21 **C) 29** D) $\sqrt{21}$ E) $\sqrt{29}$

2. Which of the following is ***not*** an assumption of the Bohr model of the atom?
   A) The difference of energies between the levels determines the energy/frequency of the emitted photons
   B) The electron orbits the nucleus in a circle
   C) The electron is not at a specific point, but rather is a wave that is spread out
   D) The angular momentum of the electron is an integer multiple of $\hbar$
   E) The force that holds the electron to the nucleus is the electric attraction between them

3. What did Davisson and Germer demonstrate by studying the way electrons scattered from crystals?
   A) They showed that electrons satisfy Schrödinger's equation
   B) They showed that electrons are waves, as predicted by deBroglie
   C) They showed that the Bohr model is correct
   D) They showed that photons satisfy the relationship $E = hf$.
   E) They showed that atoms have only specific discrete energy levels.

4. According to deBroglie, the wavelength of a particle is:
   A) Proportional to its momentum
   B) Inversely proportional to its momentum
   C) Proportional to its energy
   D) Inversely proportional to its energy
   E) None of these are the deBroglie relation

5. What does the uncertainty principle tell us about the position and the momentum of a physical particle?
   A) The uncertainty in the position cannot be small
   B) The uncertainty in the momentum cannot be small
   C) The uncertainty in the position <u>or</u> the momentum can be small, but not both
   D) The uncertainty principle states nothing about these two uncertainties
   E) I am uncertain of the answer to this question; please mark it wrong

6. How does the binding energy of a single electron around a fluorine nucleus ($Z = 9$) compare to the binding energy of a single electron around a hydrogen nucleus?
   A) It is 81 times smaller
   B) It is 9 times smaller
   C) It is 3 times smaller
   D) It is 9 times bigger
   E) It is 81 times bigger

7. A typical size for a nucleus is around __________ and for an atom around __________.
   A) 10^{-8} m, 10^{-15} m
   B) 10^{-10} m, 10^{-15} m
   C) 10^{-10} m, 10^{-10} m
   D) 10^{-15} m, 10^{-10} m
   E) 10^{-15} m, 10^{-8} m

8. Suppose you know the speed of an electron. What can you learn by studying the curve it makes in a magnetic field?
   A) Its mass *m*
   B) Its charge *e*
   C) The product of its mass and its charge, *me*
   D) The ratio of its mass and its charge, *e*/*m*
   E) None of these can be learned from this experiment

9. Why do spectral lines in atoms come only at certain very special frequencies or wavelengths?
   A) The atom can have only certain energy levels, and the photons must have the right energies to equal the difference between these energy levels
   B) The atoms have a certain physical size, and only certain wavelengths can "fit into" the atoms.
   C) Atoms collide with each other at certain frequencies, and only these frequencies are emitted
   D) Diffraction effects cause all but certain wavelengths to be diffracted back inside the atoms.
   E) It is an experimental artifact, and it is now known that these spectral lines were the result of the poor instruments available in the 19^{th} century.

10. Which of the following is a good description of the meaning of the group velocity of a wave?
    A) It describes the rate at which a group of waves is produced by some source
    B) It is the velocity of on observer with respect to a wave
    C) It is the velocity of the individual "peaks" or highest points within a wave packet
    D) It is the velocity of the individual "troughs" or lowest points within a wave packet
    E) It is the velocity at which a wave packet moves; the velocity of the "envelope" of the wave packet

Part II: Short answer [20 points]

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).

11. In 1905, Einstein explained the photoelectric effect. Explain this effect, giving any relevant equations.

Einstein explained that light comes in little packets of energy called photons, whose energy (following Planck) was given by $E = hf$. If it takes a certain amont of energy ϕ to remove an electron from metal, then the photon can only remove an electron if $E = hf > \phi$. The remaining energy $hf - \phi$ will take the form of the kinetic energy of the electron which can then move against an electric field until the potential difference matches the kinetic energy, or $eV = hf - \phi$.

12. Compton discovered that when you scatter X-ray photons off of electrons, the scattered X-rays have a longer wavelength than the originals. Why?

Photons have momentum and energy. When they scatter off of electrons, they are transferring some of that momentum and energy to the electron, and the photon then has less energy and momentum. This corresponds to a longer wavelength, according to the deBroglie relationship.

13. Explain how Rutherford's scattering of α-particles off of various atoms changed our picture of the atom, and what he was able to learn from these experiments.

The fact that atoms contain negatively charged electrons was already known, but the distribution of the positive charge was a bit of a mystery. By studying the way the α-particles scattered, he was able to deduce that the positive charge was concentrated in a small, massive portion called the nucleus. By using the highest energy α-particles and the lowest charge targets, he was able to get the α-particles to actually reach the nucleus, and was able to get an approximate estimate of its size.

Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each). You may find the following formulas helpful:

14. The wave sketched below represents a sound wave, with the "peaks" representing the high pressure places in space, and the "valleys" representing the low pressure places in space, and the size of the wave representing its physical size.

(a) What is the wavelength λ for this wave?

I've drawn in one wavelength, which points from the peak of one wave to the peak of the next. Measuring with my ruler, I found that λ = 4.60 cm = 0.046 m.

(b) What is the wave number *k* for this wave?

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.0460\text{ m}} = 137\text{ m}^{-1}$$

(c) The speed of sound (phase velocity) is *v* = 300 m/s. What is the frequency *f* and angular frequency ω of this sound?

We can calculate this in a variety of ways, but I'd like to use

$$f = \frac{v}{\lambda} = \frac{300\text{ m/s}}{0.0460\text{ m}} = 6520\text{ Hz}, \qquad \text{and} \qquad \omega = 2\pi f = 4.10\times10^{4}\text{ s}^{-1},$$

(d) Sound waves, like light waves, can be quantized (the particles are called *phonons*). What would be the momentum of one phonon with this wavelength?

This is most readily calculated from the deBrolie relationship, namely,

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ kg}\cdot\text{m}^2\text{/s}}{0.0460\text{ m}} = 1.44\times10^{-32}\text{ kg}\cdot\text{m/s}.$$

15. The universe is filled with a nearly perfect thermal background at a temperature of T = 2.73 K.

(a) At what wavelength λ is this radiation the strongest?

We simply use Wien's Law to find this:

$$\lambda_{\max} T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.062\times10^{-3}\text{ m}.$$

(b) For that same wavelength, what is the corresponding frequency (in Hz) and energy (in eV) of the photons?

These are light waves, so they obey $c = f\lambda$, or

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{1.062\times10^{-3}\text{ m}} = 2.824\times10^{11}\text{ Hz}.$$

For the energy, we then just use

$$E = hf = \left(4.136\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(2.824\times10^{11}\text{ Hz}\right) = 1.168\times10^{-3}\text{ eV}.$$

(c) What is the total energy density in photons from this thermal background?

This is nasty but straightforward. It is given by the formula

$$U = \frac{\pi^2 (k_B T)^4}{15(\hbar c)^3} = \frac{\pi^2\left[\left(7.6807\times10^{-5}\text{ eV/K}\right)\left(2.73\text{ K}\right)\right]^4}{15\left[\left(6.582\times10^{-16}\text{ eV}\cdot\text{s}\right)\left(2.998\times10^{8}\text{ m/s}\right)\right]^3}$$

$$= 1.65\times10^{5}\text{ eV/m}^3 = 2.65\times10^{-14}\text{ J/m}^3$$

16. An electron is accelerated from rest using a potential of 12 V. It then collides with some neutral hydrogen atoms which have their electrons in the ground state *n* = 1 of Hydrogen.

(a) For which final levels *n* does the incoming electron have enough energy to bump the bound electron up into the new level?

Because the electron has charge –*e*, and it was accelerated using 12 V, the electron gains 12 eV of kinetic energy as it accelerates. It can only bump up the hydrogen atom, therefore, if the difference in energy is less than 12 V.

The energy of an electron bound in hydrogen is

$$E = -\frac{13.6 \text{ eV}}{n^2}.$$

The *difference* between the ground state energy (n= 1) and some other level, is then

$$\Delta E_{1n} = (13.6 \text{ eV})\left(1 - \frac{1}{n^2}\right)$$

This works out to:

$$\Delta E_{12} = 10.2 \text{ eV}, \qquad \Delta E_{13} = 12.1 \text{ eV}, \qquad \Delta E_{14} = 12.75 \text{ eV}, \qquad \text{etc.}$$

Hence we see that only the shift to n = 2 is possible. There isn't enough energy to get from n= 1 to n = 3 or above.

(b) Suppose the bound electron is bumped up to the *n* = 2 level. How much energy will the initial electron have left over?

The initial electron had 12 eV of energy going in, but it spent 10.2 eV bumping up the bound electron, so it has only 1.8 eV left over.

(c) When the bound electron falls back down from *n* = 2 to *n* = 1, what will be the energy and frequency of the resulting photon that comes out?

The energy of the photon coming out is just the difference in the energy levels of the atom, or

$$E = \Delta E_{12} = 10.2 \text{ eV}.$$

The frequency can be found from $E = hf$, so we have

$$f = \frac{E}{h} = \frac{10.2 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 2.47 \times 10^{15} \text{ Hz}.$$

17. Neutron stars are made of pure neutrons (m_n = 1.675 × 10^{-27} kg) with the incredible density of about 10^{17} kg/m^3.

(a) What is the average volume *V* occupied by each neutron? Assuming each neutron is in a cubical box of size *l*, so that $V = l^3$, what is the size of the box?

If each neutron is in a volume *V*, the density would be $\rho = m_n/V$, so we have

$$V = \frac{m_n}{\rho} = \frac{1.675\times10^{-27}\ \text{kg}}{10^{17}\ \text{kg/m}^3} = 1.675\times10^{-44}\ \text{m}^3.$$

If we have $V = l^3$, then *l* is the cube root of *V*,

$$l = V^{1/3} = \left(1.675\times10^{-44}\ \text{m}^3\right)^{1/3} = 2.55\times10^{-15}\ \text{m}$$

(b) What is the approximate uncertainty Δ*x* of the position of each neutron, based on the box? What is the corresponding minimum uncertainty Δ*p* of the momentum of each neutron?

This is hard to do exactly, but using Carlson's rule, if you force something into a box of size *l*, the uncertainty in its position is around one-fourth of that value, so

$$\Delta x = \tfrac{1}{4} l = \tfrac{1}{4}\left(2.56\times10^{-15}\ \text{m}\right) = 6.40\times10^{-16}\ \text{m}$$

By the Heisenberg uncertainty principle, $\Delta x \Delta p \geq \tfrac{1}{2}\hbar$, so

$$\Delta p \approx \frac{\hbar}{2\Delta x} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2\left(6.40\times10^{-16}\ \text{m}\right)} = 8.25\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

(c) Calculate the typical velocity *v* corresponding to this confinement of the neutron. Ignore the effects of relativity, if there are any.

If the momentum is typically around the value of Δp , then the velocity is around

$$v = \frac{p}{m} \approx \frac{\Delta p}{m} = \frac{8.25\times10^{-20}\ \text{kg}\cdot\text{m/s}}{1.675\times10^{-27}\ \text{kg}} = 4.92\times10^{7}\ \text{m/s}$$

This is fast enough that, in fact, you probably should include some relativistic effects, but we are only doing an estimate anyway, so this is good enough.