

Solutions to Test 2

Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

- Which of the following is true about the relationship between phase velocity and group velocity?
 - Phase velocity is greater than group velocity
 - Phase velocity is less than group velocity
 - Phase velocity is equal to group velocity
 - Any of A, B, or C could be true**
 - Phase and group velocity cannot be compared because they have different units
- An X-ray photon collides with a stationary electron, and rebounds at a 90 degree angle. How does the final wavelength of the photon compare with the initial wavelength?
 - It is the same
 - It is smaller
 - It is greater**
 - It depends on the energy of the incoming photon
 - It depends on the energy of the final photon
- What does Wien's Law, $\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$, tell us?
 - It allows you to calculate the temperature of a laser at a particular wavelength
 - If you have a black body spectrum, it tells you what wavelength all the power is at
 - If you have a black body spectrum, it tells you what wavelength the most power is at**
 - It allows you to calculate the wavelength of an electron in a hot environment
 - It relates the period and the maximum wavelength inside an atom
- What, approximately, is the significance of the cross-section in a scattering experiment?
 - It is the actual area of an object that is being scattered from, like a nucleus
 - It is the size of the area where a certain action will happen, like scattering by a large angle**
 - It is the size of the region around the scattering object where the force will be felt
 - It is the combined area of the projectile and the target
 - It is the square of the distance between the target and object, times pi

5. How come we don't notice quantum mechanical effects in the everyday world? How come Newton didn't notice them?
- A) **They are too small to affect most everyday phenomena**
 - B) They affect only elementary particles, like electrons, not composite objects, like people
 - C) They affect only objects that move at the speed of light, like photons, not physical objects, like electrons
 - D) All such phenomena were attributed to "random error" up until the 20th century
 - E) Newton was an idiot and didn't do any experiments
6. Which of the following is not a consequence of the uncertainty principle?
- A) Atomic states that live a very short time do not produce perfectly monochromatic light, but a small range of different frequencies.
 - B) If you measure the momentum of a particle very well, its position is very poorly determined
 - C) The electron in an atom can't be definitely at the nucleus, since then it would have a very indefinite (and hence large) momentum
 - D) **The charge on an electron can never be precisely determined**
 - E) A particle confined to a box has "zero-point" energy, even if there is no potential energy
7. Why are the spectral lines for heavy hydrogen, ^2H , slightly different than the spectral lines for ordinary hydrogen, ^1H ?
- A) The neutron slightly neutralizes the nucleus, slightly changing the electric charge so the binding energies are a bit different
 - B) The neutron adds a bit of gravitational attraction to the atom, shifting the lines slightly
 - C) The electron spends some of its time inside the nucleus, which is now larger, and this has to be corrected for
 - D) The electron undergoes weak interactions with the neutron, shifting its energy slightly
 - E) **The binding energy has to be computed using the reduced mass, which includes the effects of the nuclear mass**
8. According to quantum mechanics, what does the wave function of an electron tell you?
- A) It tells you where a large group of electrons are, and cannot be applied to a single electron
 - B) **If you square its amplitude, it tells you the probability of finding a single electron at various places**
 - C) If you find its center of mass and its wavelength, it tells you exactly where the electron is and exactly what its momentum is
 - D) It tells you where the charge of the electron is "spread out", but the electron is in just one place
 - E) It tells you where the electron is going to be, not where it is now.

9. Which of the following experiments did not contribute to the conclusion that charge comes in multiples of a fundamental unit e ?
- A) **The measurement of electron waves bouncing off of crystals, by Davisson and Germer**
 - B) The measurement of the charge of oil drops, by R. A. Millikan
 - C) The measurement of deflection of charges in a magnetic field by J.J. Thomson
 - D) The measurement of the amount of charge needed for chemical reactions by Michael Faraday
 - E) Actually, these were all considered critical to demonstrating the existence of a fundamental unit of electric charge
10. Why was it important that when Rutherford used the highest energy α -particles and the lowest Z targets, he saw deviations from the formula he expected for a point-like nucleus?
- A) The energies were so high that relativity was playing a role, and Rutherford was able to see these effects
 - B) Electric forces become weaker at very short distances, and he was seeing that
 - C) His α -particles were breaking up the nucleus, invalidating his approach
 - D) Electrons in the innermost core of the atom were partly screening the nuclear charge
 - E) **The α -particles were beginning to penetrate inside the nucleus, and he was able to estimate the size of the nucleus**

Part II: Short answer [20 points] Choose two out of three.

- 11. What was the ultraviolet catastrophe, and what assumption did Planck have to make to fix the problem?**

According to classical (non-quantum) ideas, a thermal or black-body distribution at fixed temperature T should have an infinite amount of energy in it. The energy was infinite because of the contribution from the short wavelength, or ultraviolet, portion of the spectrum. Planck was able to fix the problem by assuming that electromagnetic energy came only in chunks of hf , where f was the frequency and h was a new fundamental constant now called Planck's constant.

- 12. Give a short explanation of the Franck-Hertz experiment and what it demonstrated.**

In the Franck-Hertz experiment, electrons were electrically accelerated to a fixed energy and then collided with a low density of atoms. It was found that the electrons scattered elastically, without losing any energy, until they reached a certain threshold corresponding to the energy difference between the atoms and the first excited state of those atoms. Furthermore, it was found that the low density gas glowed with the wavelength corresponding to the energy loss of the electrons. This made clear that atoms did, indeed, only have certain distinct energy levels, and the photons that were coming out were merely carrying the energy difference between those levels.

13. The discovery that particles act like waves meant that several classical ideas about particles had to be overturned. List at least three.

- Particles do not generally have a definite position
- Particles do not generally have a definite momentum
- Particles can have diffraction effects when they pass through small apertures
- Particles can scatter off of layers in a crystal in a way that only waves should
- Particles are described by a wave function $\psi(x, t)$.

Part III: Calculation: [60 points] Choose three out of four

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$$

14. A photon of wavelength 437 nm collides with a metal with an unknown work function. An electron is ejected with energy 0.513 eV

(a) What is the frequency of the photons of this light?

The frequency can be calculated using $\lambda f = v_p = c$, so that

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{4.37 \times 10^{-7} \text{ m}} = 6.86 \times 10^{14} \text{ s}^{-1}$$

(b) What is the energy of the photons of this light?

Anticipating future developments, it is easiest to work in electron volts, so we have

$$E = hf = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(6.86 \times 10^{14} \text{ s}^{-1}) = 2.837 \text{ eV}$$

(c) What is the work function for the metal?

The work function is the amount of energy it takes to pry an electron out of the metal. Since a photon with an energy of 2.837 eV can pry an electron out and the electron still has 0.513 eV left over, it follows that the amount of energy used to remove the electron must be

$$\phi = hf - E = 2.837 \text{ eV} - 0.513 \text{ eV} = 2.324 \text{ eV}$$

(d) What will be the lowest frequency that is capable of ejecting electrons from this metal?

The lowest frequency that can eject photons will correspond to photons whose energy is equal to the work function, $hf = \phi$. Therefore, they will have a frequency of

$$f = \frac{\phi}{h} = \frac{2.324 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 5.619 \times 10^{14} \text{ s}^{-1}$$

15. Consider the Bohr model for a constant radial force inward F . The energy of the system is given by

$$E = \frac{1}{2}mv^2 + Fr$$

(a) Equate the force to the centripetal force to get an equation for velocity as a function of r .

Centripetal force is given by $F = mv^2/r$, so solving for the velocity, we have

$$v = \sqrt{\frac{Fr}{m}}$$

(b) Apply the Bohr quantization condition to the angular momentum L to find a second equation for the velocity, this time in terms of r and n .

Bohr says that the angular momentum is always an integer multiple of Planck's reduced constant, $L = n\hbar$, where n is an integer. Since the angular momentum is $L = pr = mvr$, we can solve this for the velocity to give

$$v = \frac{L}{mr} = \frac{n\hbar}{mr}$$

(c) Combine the results of parts (a) and (b) to find an equation for r as a function of n .

The two equations for the velocity must match, so we have

$$v = \frac{n\hbar}{mr} = \sqrt{\frac{Fr}{m}}$$

Squaring both sides, and then solving for the radius r , we have

$$\frac{n^2\hbar^2}{m^2r^2} = \frac{Fr}{m} \quad \text{so that} \quad r^3 = \frac{n^2\hbar^2}{Fm} \quad \text{or} \quad r = \left(\frac{n^2\hbar^2}{Fm}\right)^{1/3}$$

(d) Substitute your results to find the energy E as a function of n . You do not have to simplify it all the way if you find it is getting messy.

First, we can substitute it in to our equation for v to give

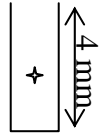
$$v = \frac{n\hbar}{mr} = \frac{n\hbar}{m} \left(\frac{Fm}{n^2\hbar^2}\right)^{1/3} = \left(\frac{n\hbar F}{m^2}\right)^{1/3}$$

Now we simply substitute in everything for the energy E and find

$$E = \frac{1}{2}mv^2 + Fr = \frac{1}{2}m\left(\frac{n\hbar F}{m^2}\right)^{2/3} + F\left(\frac{n^2\hbar^2}{Fm}\right)^{1/3} = \frac{3}{2}\sqrt[3]{\frac{F^2\hbar^2 n^2}{m}}$$

16. An electron, having mass $m = 9.109 \times 10^{-31}$ kg is placed in little test tube 4 mm high.

(a) What is the approximate uncertainty in the vertical position of the electron Δx ?



The electron is somewhere within the test tube, and therefore is no more than 2 mm from the half way up point. More typically, it will be about half this distance away, 1 mm, we approximate

$$\Delta x = 0.001 \text{ m}$$

(b) What is the corresponding uncertainty in the momentum of the electron Δp ?

By the Heisenberg uncertainty principle, the electron has a minimum momentum uncertainty of

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}{2(0.001 \text{ m})} = 5.275 \times 10^{-32} \text{ kg}\cdot\text{m/s}$$

(c) What is the approximate kinetic energy of the electron due to the momentum found in part (b)?

If this is the uncertainty in the momentum, then a typical momentum equal to about this amount, and therefore the kinetic energy will typically be

$$E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} = \frac{(5.275 \times 10^{-32} \text{ kg}\cdot\text{m/s})^2}{2(9.109 \times 10^{-31} \text{ kg})} = 1.527 \times 10^{-33} \text{ J}$$

(d) Compare the kinetic energy in part (c) with the amount of potential energy $E = mgh$ required to climb out of the test tube, where $g = 9.8 \text{ m/s}^2$

Climbing out of the test tube under one gravity requires an energy

$$E = mgh = (9.109 \times 10^{-31} \text{ kg})(9.8 \text{ m/s}^2)(0.004 \text{ m}) = 3.571 \times 10^{-32} \text{ J}$$

This is considerably more than the kinetic energy we found in part (c), so the electron doesn't quite have enough energy to climb out of the test tube.

17. Because of its extremely high temperature, many atoms in the Sun are multiply ionized. Consider a carbon atom ($Z = 6$) in the Sun that has been ionized so that it contains only a single electron

(a) What are the energy levels of this Carbon atom, in eV, as a function of n ?

For an arbitrary single-electron atom, the energy levels are given by

$$E = -\frac{(13.6 \text{ eV})Z^2}{n^2} = -\frac{490. \text{ eV}}{n^2}$$

(b) How much energy would be released by an electron moving from the $n = 7$ level of Carbon to the $n = 5$ level.

To find this energy, we simply subtract the relevant energy between the two different levels

$$\Delta E = (490. \text{ eV})\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (490. \text{ eV})\left(\frac{1}{25} - \frac{1}{49}\right) = 9.59 \text{ eV}$$

(c) What is the frequency, angular frequency, period, wavelength, and wave number (f , ω , T , λ , and k) of the corresponding light waves that come out?

The frequency is most easily calculated from $E = hf$, so we have

$$f = \frac{E}{h} = \frac{9.59 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 2.32 \times 10^{15} \text{ s}^{-1}$$

We then can get the angular frequency and the period very quickly

$$\omega = 2\pi f = 1.46 \times 10^{16} \text{ s}^{-1} \quad \text{and} \quad T = 1/f = 4.31 \times 10^{-16} \text{ s}$$

The wavelength can be obtained from $f\lambda = v_p = c$, so

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{2.32 \times 10^{15} \text{ s}^{-1}} = 1.29 \times 10^{-7} \text{ m} = 129 \text{ nm}$$

The wave number can now be obtained either from $k\lambda = 2\pi$ or $\omega = kv_p = ck$.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.29 \times 10^{-7} \text{ m}} = 4.87 \times 10^7 \text{ m}^{-1}$$