

Name \_\_\_\_\_

## Solutions to Test 1

### Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

1. For event B to be in the absolute future of event A, the time of event B must be greater than the time of event A, and
  - A) The two events must be at the same location
  - B) The two events must be at different locations
  - C) The separation must be spacelike
  - D) The separation must be lightlike
  - E) The separation must be timelike**
  
2. Inside the Sun, four hydrogen nuclei can be combined to make one helium nucleus, and a LARGE amount of energy is released; in fact, this is perhaps the most energetic nuclear reaction known. How does the mass of the hydrogen nuclei compare to the mass of the final helium nucleus?
  - A) It is somewhat higher, enough to measure**
  - B) It is a tiny bit higher, but too small to measure
  - C) It is exactly the same
  - D) It is a tiny bit lower, but too small to measure
  - E) It is somewhat lower, enough to measure
  
3. What is an inertial frame of reference?
  - A) It is a term used only in art to describe the artist's view of reality when creating a piece of artwork
  - B) Any set of coordinates that are rigidly attached to a uniformly moving body, such as the rotating Earth.
  - C) A way of thinking about the universe and objects within it in terms of how much inertia or mass is in them
  - D) A choice of coordinates in which the law of inertia holds, so that objects at rest remain at rest.**
  - E) Any choice of coordinates that accelerates with a constant acceleration.
  
4. To transmit energy instantaneously from point A to point B, take a rigid rod, and push on one end, sending a push instantaneously to the other end. This method for transmitting energy instantaneously will not work because
  - A) There is no such thing as a rigid rod in special relativity**
  - B) The motion of the rod causes it to shorten, thanks to Lorentz contraction
  - C) The motion of the rod causes time to dilate, so there is a delay
  - D) Though it works, it transports only virtual energy, which is converted from the mass of the rod
  - E) Actually, it will work

5. The formula for 4d distance is  $s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 + k(\Delta t)^2$  with  $k =$   
 A) +1    B) -1    C)  $+c^2$     **D)  $-c^2$**     E) None of the above
6. One of the formulas I gave you was:  $\frac{\vec{u}}{c} = \frac{c\vec{p}}{E}$ . When is this equation valid?  
 A) For massless particles, but not massive  
 B) For massive particles, but not massless  
 C) For particles moving slowly, but not at relativistic speeds  
 D) For particles moving at relativistic speeds, but not slowly  
**E) This formula is always valid**
7. According to the conventions I used in relativity, which of the following does *not* become large as velocity approaches the speed of light?  
 A) Energy  
 B) Momentum  
**C) Mass**  
 D) The Lorentz factor  $\gamma$   
 E) Actually, all of these *do* become large
8. Under what circumstances is the binomial expansion  $(1 + \varepsilon)^n = 1 + n\varepsilon + \dots$  most useful?  
 A) When  $\varepsilon$  is negative  
 B) When  $\varepsilon$  is positive  
**C) When  $\varepsilon$  is very small**  
 D) When  $\varepsilon$  is very large  
 E) The binomial expansion is *always* a good approximation
9. When an object passes us at high velocity in the  $x$ -direction, an observer will see it get shorter  
 A) In all three dimensions ( $x$ ,  $y$ , and  $z$ )  
**B) In the  $x$ -direction, but not in the  $y$ - or  $z$ -direction**  
 C) In the  $y$ - or  $z$ -direction, but not in the  $x$ -direction  
 D) In no dimension; only observers moving *with* the object will see it getting shorter  
 E) In no dimension; moving object look *longer*, not shorter
10. It is commonly said that nothing can go faster than light. Which of the following scenarios actually *does* allow something to go faster than light?  
 A) Make a rocket that goes at  $0.6c$ , and have it fire a bullet at  $0.6c$  (only)  
**B) Pass through a medium, like water, where the speed of light is less than  $c$  (only)**  
 C) Combine two particles of light into a single particle, which moves at  $2c$  (only)  
 D) Scenarios A and B, but not C, will work  
 E) Scenarios A, B, and C will all work

**Part II: Short answer [20 points]**

Choose **two** of the following questions and give a short answer (1-3 sentences, or a simple sketch) (10 points each).

- 11. An atom moving at half the speed of light emits a photon (particle of light) moving forward, compared to the atom. According to us, how fast will the light be moving? Explain how, if at all, your answer would change if the atom emitted the light in the backwards direction. You should NOT need equations to give the answer.**

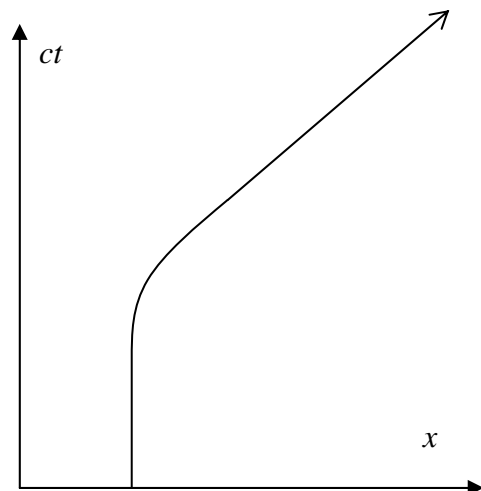
The speed of light in vacuum is  $c$ , no matter what, according to all observers. Therefore, a photon emitted from an atom will move at velocity  $c$ , whether the atom emits it forwards, backwards, or sideways. End of story.

- 12. Explain why BOTH of the formulas below cannot be correct in special relativity. Which one is correct?**

$$\vec{F} = m\vec{a} \quad \text{and} \quad \vec{F} = \frac{d\vec{p}}{dt}$$

It is easy to match the expression on the right with the expression on the left if you assume  $\vec{p} = m\vec{v}$ . But this is not true in special relativity. The equality to the right is the correct one, because it is the one that lets us preserve conservation of momentum.

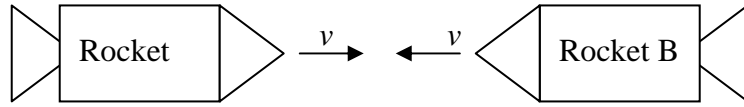
- 13. Make a crude sketch of the world line of a particle initially at rest which then accelerates in the  $+x$  direction for a short time, then travels at constant velocity.**



**Part III: Calculation: [60 points]**

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

**13. Two rockets are traveling towards each other at equal speeds. It is observed that a meter stick on one of the rockets appears to be only 80 cm long, according to an unmoving observer.**



**(a) What is the speed of the rockets? I recommend writing your answer as a fraction of  $c$ .**

The formula for Lorentz contraction tells us that

$$L = L_p / \gamma = L_p \sqrt{1 - v^2/c^2}$$

$$\sqrt{1 - v^2/c^2} = \frac{80 \text{ cm}}{100 \text{ cm}} = \frac{4}{5}$$

$$1 - v^2/c^2 = \frac{16}{25}$$

$$v^2/c^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$v = \frac{3}{5}c$$

**(b) What is the speed of rocket B, according to rocket A? You may want to write your answer as a fraction of  $c$ .**

We treat rocket A with velocity  $v = \frac{3}{5}c$  as the observer, and rocket B as moving at  $u = -\frac{3}{5}c$ . We then use the subtraction of velocities formula:

$$u'_x = \frac{u_x - v}{1 + u_x v / c^2} = \frac{-\frac{3}{5}c - \frac{3}{5}c}{1 + \frac{3}{5} \cdot \frac{3}{5}} = -\frac{\frac{6}{5}c}{1 + \frac{9}{25}} = -\frac{6}{5} \cdot \frac{25}{34}c = -\frac{15}{17}c = -0.882c$$

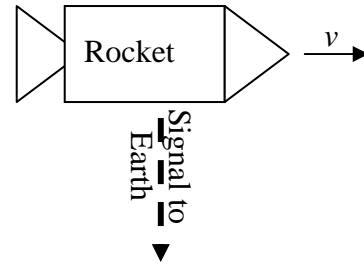
Since we were asked for speed, not velocity, we should write this as  $0.882c$ .

**(c) According to an observer on the ground, how long does it take for a clock on either rocket to advance one hour?**

According to an observer on the ground, the clock advances by an amount  $\Delta t = \gamma \tau$ , where  $\tau = 1$  hour. So we have

$$\Delta t = \gamma \tau = \frac{L_p \tau}{L} = \frac{(100 \text{ cm})(1 \text{ h})}{80 \text{ cm}} = 1.25 \text{ h} = 75 \text{ minutes}$$

14. A rocket is traveling at high speed perpendicular to Earth. The rocket is supposed to signal Earth at a frequency of 130 MHz, but the signal reaches Earth with a frequency of 120 MHz



(a) What is the speed of the rocket? You may give your answer as a fraction of the speed of light.

The frequency is shifted due to Doppler shift. In the Doppler shift formula, the angle is the angle compared to straight towards us, so we have  $\theta = 90^\circ$  and therefore

$$f = \frac{f_0}{\gamma(1 - v \cos \theta / c)} = f_0 \sqrt{1 - v^2 / c^2}$$

$$120 = 130 \sqrt{1 - v^2 / c^2}$$

$$1 - v^2 / c^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$$

$$v^2 / c^2 = 1 - \frac{144}{169} = \frac{25}{169} = \left(\frac{5}{13}\right)^2$$

$$v = \frac{5}{13} c = 0.3846c$$

(b) If the rocket were coming straight at us, what frequency would we detect?

In this case  $\theta = 0^\circ$ , so

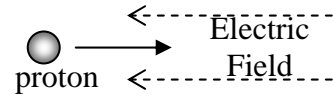
$$f = \frac{f_0}{\gamma(1 - v \cos \theta / c)} = \frac{f_0 \sqrt{1 - v^2 / c^2}}{1 - v / c} = \frac{\frac{12}{13} f_0}{1 - \frac{5}{13}} = \frac{12}{8} f_0 = 195 \text{ MHz}$$

(c) If the rocket were going straight away from us, what frequency would we detect?

In this case  $\theta = 180^\circ$ , so

$$f = \frac{f_0}{\gamma(1 - v \cos \theta / c)} = \frac{f_0 \sqrt{1 - v^2 / c^2}}{1 + v / c} = \frac{\frac{12}{13} f_0}{1 + \frac{5}{13}} = \frac{12}{18} f_0 = 86.7 \text{ MHz}$$

15. A proton with mass  $m = 938 \text{ MeV}/c^2$  and charge  $+e$  is initially moving to the right at  $v = 2.1 \times 10^8 \text{ m/s}$ . A constant electric field  $E = 1.00 \text{ MV/m}$  is pushing on the proton, trying to make it stop.



(a) What is the initial energy and momentum of the proton, in MeV and MeV/c?

We simply use the formulas for momentum and energy

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}} = \frac{938 \text{ MeV}}{\sqrt{1-(2.1/3)^2}} = \frac{938 \text{ MeV}}{\sqrt{1-(0.700)^2}} = 1313 \text{ MeV}$$

$$p = \gamma mv = \frac{mc(v/c)}{\sqrt{1-v^2/c^2}} = \frac{0.700(938 \text{ MeV}/c)}{\sqrt{1-(0.700)^2}} = 919 \text{ MeV}/c$$

(b) What is the final energy and momentum of the proton? This is easy.

The proton is at rest, so the final energy is  $E = \gamma mc^2 = mc^2 = 938 \text{ MeV}$ , and the final momentum is  $p = \gamma mv = 0$ .

(c) How much work has been done on the proton?

Work is the change in energy, so

$$W = \Delta E = E_f - E_i = (938 \text{ MeV}) - (1313 \text{ MeV}) = -375 \text{ MeV}$$

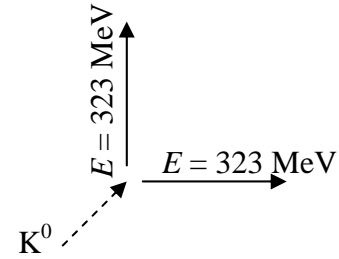
The work is technically negative, since it lost energy.

(d) What distance does it go before it stops?

Work is force times distance. The force is charge times the electric field, so we have

$$d = \frac{W}{F} = \frac{W}{eE} = \frac{-375 \text{ MeV}}{e(-1 \text{ MV/m})} = \frac{375 \text{ MeV}}{1 \text{ MeV/m}} = 375 \text{ m}$$

16. A  $K^0$  particle decays into two  $\pi^0$  particles, each having a mass of  $m_\pi = 139 \text{ MeV}/c^2$  and an energy of  $E = 323 \text{ MeV}$ . The two  $\pi^0$  particles after the decay are each moving perpendicular to each other, as illustrated at right.



(a) What is the momentum of each of the  $\pi^0$ s?

We use the formula

$$E^2 - p^2c^2 = m^2c^4$$

$$p^2c^2 = E^2 - m^2c^4 = (323 \text{ MeV})^2 - (139 \text{ MeV})^2 = 85,000 \text{ MeV}^2$$

$$pc = \sqrt{85,000 \text{ MeV}^2} = 292 \text{ MeV}$$

Since momentum is a vector, we should probably write this as  $\mathbf{p}_1 = 292\hat{\mathbf{i}} \text{ MeV}/c$  and  $\mathbf{p}_2 = 292\hat{\mathbf{j}} \text{ MeV}/c$ .

(b) What is the initial energy of the  $K^0$  in MeV? What is the initial momentum of the  $K^0$  in MeV/c? You may give the latter either as a vector with components or as a magnitude and a direction.

By conservation of energy, the initial energy of the  $K^0$  was

$$E = E_1 + E_2 = 2(323 \text{ MeV}) = 646 \text{ MeV}$$

Similarly, the initial momentum of the  $K^0$  was

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 = (292\hat{\mathbf{i}} + 292\hat{\mathbf{j}}) \text{ MeV}/c$$

This can also be described as a vector of length  $412 \text{ MeV}/c$  and direction  $45$  degrees between the  $x$ - and  $y$ -directions.

(c) What is the mass of the  $K^0$  in  $\text{MeV}/c^2$ ?

We again use the formula

$$m^2c^4 = E^2 - p^2c^2 = (646 \text{ MeV})^2 - (292 \text{ MeV})^2 - (292 \text{ MeV})^2 = 247300 \text{ MeV}^2$$

$$mc^2 = \sqrt{247300 \text{ MeV}^2} = 497 \text{ MeV}$$

So we have  $m = 497 \text{ MeV}/c^2$ . The actual experimental value is  $498 \text{ MeV}/c^2$ .