Solutions to Test 1

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. **Which of the following is <u>not</u> true in special relativity?**
   A) The measured length of an object moving at high speeds is shorter
   B) Clocks moving at high speeds run slower, as measured by us
   C) The mass of a swiftly moving object increases
   D) The energy of a swiftly moving object can be very large
   E) The speed of a massive object is always slower than the speed of light

2. **Which of the following is true about the speed of massless objects?**
   A) Their speed is proportional to the energy, but not their momentum
   B) Their speed is proportional to their momentum, but not their energy
   C) Their speed is proportional to their momentum and their energy
   D) Their speed is always equal to the speed of light *c*
   E) Their speed is always zero

3. **Suppose that a rocket ship is traveling at ¾ of the speed of light compared to the Earth, and someone on the rocket ship fires a gun with bullets which, according to him, travel at ¾ of the speed of light. What will an observer on the Earth see?**
   A) The bullets will be moving at less than the speed of light
   B) The bullets will be moving at the speed of light
   C) The bullets will be moving faster than the speed of light
   D) It depends on which observer is actually at rest, the observer on the rocket or on the Earth
   E) It is impossible to fire bullets at this speed from a rocket moving at this speed

4. **Here is a method for communicating faster than light: I create a rod one light-year in length. I then move this end using Morse code, and you watch how the far end of the rod moves. What, if anything, is wrong with this method for instantaneous communication?**
   A) There is no such thing as a rigid rod in relativity, so the far end won't move at the same time I move this end
   B) Since everything is moving in space, you would have to perform a Lorentz transformation to figure out what really happens
   C) Since the rod is moving, it would Lorentz contract and be too short
   D) Actually, there is nothing wrong with this idea; it would work

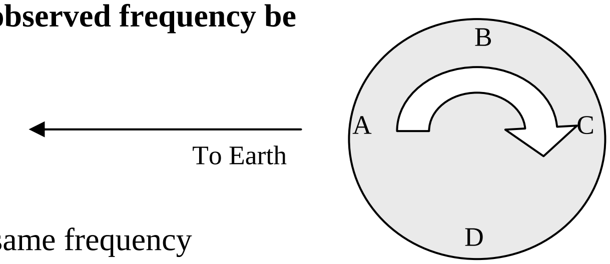
5. Which of the following statements about mass is <u>false</u>?

A) Mass can be easily computed if you know the energy and momentum of a particle
B) When an object is heated, its mass stays the same
C) There are known particles with zero mass
D) All observers agree on the mass of an object, whatever their speed
E) When you add internal energy to something, you increase its mass

6. Which of the following statements about Doppler shift is true?

A) Doppler shifts apply only to lasers; other electromagnetic radiation, like radio waves, are unaffected
B) All astronomical objects are moving at slow enough speed that relativistic effects are unimportant
C) An object moving perpendicular to the line of sight results in no Doppler shift
D) If both the observer and the source are moving at the same velocity, there is still a Doppler shift
E) Doppler shifts to a higher frequency are called blue shift; those to a lower frequency, red shift

7. A distant galaxy is rotating in the direction indicated. Neutral hydrogen atoms in four clouds, labeled A, B, C, and D each emit radiation at a frequency of 1420 MHz. From which cloud will the observed frequency be highest, as observed from Earth?



A) A
B) B
C) C
D) D
E) They will all be observed at the same frequency

8. A force of 1 Newton acts on a particle of mass 1 kg for one second. Which of the following is true?

A) The momentum will change by 1 kg·m/s (only)
B) The velocity will change by 1 m/s (only)
C) The energy will change by 1 J (only)
D) A and B are both true
E) None of the above is true

9. In ordinary physics, work is given by force times distance. In relativity, this formula must be changed by

A) Multiplying force times distance times the Lorentz factor (γ)
B) Dividing force time distance by the Lorentz factor (γ)
C) The formula does not need to be modified at all
D) There is no similar formula for work in relativity

10. Which of the following is true about time in special relativity?

A) Although time gets distorted, multiple observers will at least agree if two events are simultaneous
B) The amount of proper time between two events will depend on who is observing those events
C) Time is really just another form of energy
D) Time is money
E) "Future" and "Past" are still valid concepts, but must be modified from their conventional meanings

Part II: Short answer [20 points]

Choose **two** of the following questions and give a short answer (1-3 sentences) (10 points each).

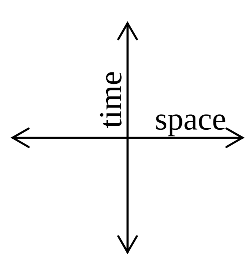
11. I stay on Earth, while my twin heads for the star Sirius at a constant velocity. Ten years later, I take off on a faster rocket, such that I overtake my twin just as he arrives at Sirius. When we get there, who is younger, and why?

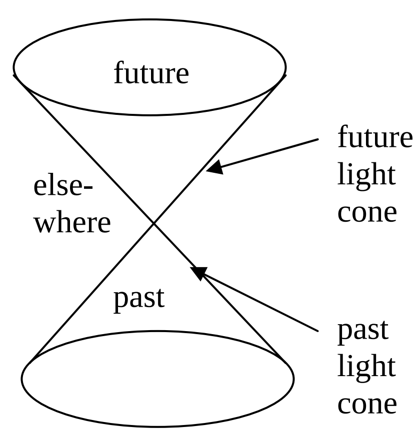
In general, the one that undergoes acceleration is younger. Since I accelerated when I changed speeds, I will end up younger. To demonstrate this explicitly, suppose my twin travels at $0.45c$ for 20 years, while I spend 10 years at zero velocity and 10 years t $0.9c$, so that we each go a total distance of 9 light-years after 20 years. You can show that during the "waiting" phase I will age 10 years, followed by an additional 4.36 years when I am rushing, for a total of 14.36 years. My twin will age 17.86 years at his constant velocity.

12. In the pole and barn paradox as described in the book, a long pole fit inside a short barn. This is demonstrated because the farmer closes both doors of the barn simultaneously as the pole is inside the barn. According to the pole vaulter, what actually happened?

The pole and barn paradox is a paradox, because the pole vaulter cannot understand how his pole, which is longer than the barn, could possibly fit inside the barn. The problem lies in the word "simultaneously." The farmer closes both doors simultaneously as measured by him, but the pole vaulter sees the front door shut and open first, and then the back door. Of course, you can close both doors of the barn if you don't do it simultaneously.

13. Draw a spacetime sketch, clearly labeling the space and time axes, illustrating what the terms "past" and "future" mean in special relativity





Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

14. Consider the three points listed below, which have coordinates (*x*,*y*,*z*,*ct*) (all in meters):

A: (0,0,0,0)   B: (-5,0,0,-3)   C: (-2,0,0,2)

For each pair of points, determine whether their separation is spacelike, timelike, or lightlike, <u>and</u> when appropriate determine either the proper distance *s* between them or the proper time $c\tau$ between them.

(a) A and B

We simply use the distance formula

$$s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2 = (0+5)^2 + (0-0)^2 + (0-0)^2 - (0+3)^2 = 25 - 9 = 16,$$
$$s = 4 \text{ m}$$

Since s^2 came out positive, the separation is spacelike, and the proper distance is s = 4 m.

(b) A and C

As before,

$$s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2 = (0+2)^2 + (0-0)^2 + (0-0)^2 - (0-2)^2 = 4 - 4 = 0$$

Since s^2 came out zero, the separation is lightlike, and there is no point in discussing the proper time or proper distance between the points.

(c) B and C

This time we have

$$s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - (c\Delta t)^2 = (-5+2)^2 + (0-0)^2 + (0-0)^2 - (-3-2)^2 = 9 - 25 = -16$$

Since the result came out negative, we rewrite this as $-(c\tau)^2$, so we have $(c\tau)^2 = 16 \text{ m}^2$ or $c\tau = 4 \text{ m}$.

15. The Relativity Express, traveling at 2.40 × 10^8 m/s, passes me. As it passes, I discover that the train, as it passes, is 3 m tall and takes 10^{-6} s to pass me.

(a) According to me, how long is the train?

This is straightforward, it is just velocity times time, or

$$L = vt = \left(2.4\times10^8\ \text{m/s}\right)\left(10^{-6}\ \text{s}\right) = 240\ \text{m}$$

(b) How long is the train really, as measured by someone traveling with the train?

Because the train is traveling at such an enormous velocity, it will be Lorentz contracted and will be perceived (by me) as shorter than its true length. The apparent length and the proper length are related by $L = L_p/\gamma$, so we have

$$L_p = \gamma L = \frac{L}{\sqrt{1-\left(u/c\right)^2}} = \frac{240\ \text{m}}{\sqrt{1-\left(\dfrac{2.4\times10^8\ \text{m/s}}{3.0\times10^8\ \text{m/s}}\right)^2}} = \frac{240\ \text{m}}{\sqrt{1\text{-}0.64}} = 400\ \text{m}$$

So the actual, or proper length of the train is 400 m.

(c) How tall is the train really, as measured by someone traveling with the train?

This is a trick question. There is no Lorentz contraction in the vertical direction, only in the direction of motion, so the actual height of the train is the same as the observed height, or 3 m.

16. A timer on a rocket is set to exactly 10^7 s, and then the rocket is brought to a speed of exactly 10 km/s. It is aimed to go into orbit around a planet 10^8 km

away. The idea is that when the timer goes off, the rocket will be at the distant planet. The rocket scientists forgot about relativity.

(a) Because the rocket is traveling so fast, will the timer come on too early or too late?

Relativistic clocks move more slowly than stationary ones, so the timer will go off late.

(b) Calculate the difference in time between when the rocket is supposed to turn on, and when it actually turns on. You may find it helpful to know the binomial expansion

$$(1+\varepsilon)^n = 1 + n\varepsilon + \tfrac{1}{2}n(n-1)\varepsilon^2 + \cdots$$

The time at which the timer goes off is given by the formula $\Delta t = \gamma\tau$. The amount by which it is late is $\Delta t - \tau = (\gamma - 1)\tau$. Since we are working with a (relatively) small velocity, the difference between γ and 1 will be tiny, so it is helpful to use the binomial expansion. For small velocities, this implies that

$$\gamma = \left(1 - u^2/c^2\right)^{-1/2} \approx 1 - \left(-\tfrac{1}{2}\right)\frac{u^2}{c^2} = 1 + \frac{u^2}{2c^2}$$

The amount by which the timer comes on late is therefore

$$\Delta t - \tau = (\gamma - 1)\tau \approx \frac{v^2}{2c^2}\tau = \frac{1}{2}\left(\frac{10^4 \text{ m/s}}{3\times 10^8 \text{ m/s}}\right)^2 \left(10^7 \text{ s}\right) = 0.0056 \text{ s}$$

(c) Calculate how far the rocket ship overshoots/undershoots its destination due to this effect.

The rocket is traveling at a respectable 10^4 m/s, and if we multiply this by the time, we find that the rocket overshoots its destination by 55.6 m.

17. **A negatively charged pion (mass $m_\pi c^2$ =139 MeV) moving at a velocity of v = 0.908c collides with a stationary proton (mass $m_p c^2$ =938 MeV) to produce a neutral Δ. The reaction is described by**

$$p^+ + \pi^- \rightarrow \Delta^0$$

(a) What is the momentum and energy of the pion?

We simply use the formulae for energy and momentum, namely

$$p = \gamma m v = \frac{\left(139 \text{ MeV}/c^2\right)\left(0.908c\right)}{\sqrt{1-\left(0.908c/c\right)^2}} = 301 \text{ MeV}/c$$

$$E = \gamma m c^2 = \frac{\left(139 \text{ MeV}/c^2\right)c^2}{\sqrt{1-\left(0.908c/c\right)^2}} = 332 \text{ MeV}$$

(b) What is the momentum and energy of the Δ^0?

We will use conservation of momentum and energy, but to do so, we must include the momentum and energy of the proton. The proton is stationary, so it has no momentum, $p_p = 0$, and its energy is $E = \gamma m_p c^2 = m_p c^2 = 938 \text{ MeV}$. The total energy and momentum, therefore, is

$$p_\Delta = p_\pi + p_p = 301 \text{ MeV}/c + 0 = 301 \text{ MeV}/c,$$
$$E_\Delta = E_\pi + E_p = 332 \text{ MeV} + 938 \text{ MeV} = 1270 \text{ MeV}.$$

(c) What is the mass of the Δ^0?

We simply use the formula for invariant mass, namely

$$\left(m_\Delta c^2\right)^2 = E_\Delta^2 - \left(p_\Delta c\right)^2 = \left(1270 \text{ MeV}\right)^2 - \left(301 \text{ MeV}\right)^2 = 1.522\times10^6 \text{ MeV}^2,$$
$$m_\Delta c^2 = 1234 \text{ MeV}$$

So the mass is 1234 MeV/c^2. This is, in fact, the mass of the Δ^0.