

Physics 215 – Elementary Modern Physics
Hydrogen-Like Wave Functions

Angular Momentum

$$L^2 = \hbar^2 (l^2 + l)$$

$$L_z = \hbar m$$

$$S^2 = \frac{3}{4} \hbar^2$$

$$S_z = \hbar m_s$$

Quantum Numbers

$$(n, l, m, m_s)$$

$$n = 1, 2, 3, \dots$$

$$l = 0, 1, 2, 3, \dots, n-1$$

$$m = -l, -l+1, \dots, 0, \dots, l$$

$$m_s = \pm \frac{1}{2}$$

Energy

$$E_n = -\frac{k^2 Z^2 e^4 \mu}{2 \hbar^2 n^2} = -\frac{(13.6 \text{ eV}) Z^2}{n^2}$$

Characteristic radius:

$$a = \frac{\hbar^2}{kZe^2 \mu} = \frac{a_0}{Z} = \frac{0.0529 \text{ nm}}{Z}$$

$$\langle r \rangle \sim an^2$$

Wave function (ignoring spin):

$$\psi_{n,l,m}(r, \theta, \phi) = R_{n,l}(r) Y_{l,m}(\theta, \phi)$$

Spherical Harmonics

$$Y_{0,0} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{1,\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi}$$

$$Y_{1,0} = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$Y_{2,\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{\pm 2i\phi}$$

$$Y_{2,\pm 1} = \mp \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$$

$$Y_{3,\pm 3} = \mp \sqrt{\frac{35}{64\pi}} \sin^3 \theta e^{\pm 3i\phi}$$

$$Y_{3,\pm 2} = \sqrt{\frac{105}{32\pi}} \sin^2 \theta \cos \theta e^{\pm 2i\phi}$$

$$Y_{3,\pm 1} = \mp \sqrt{\frac{21}{64\pi}} \sin \theta (5 \cos^2 \theta - 1) e^{\pm i\phi}$$

$$Y_{3,0} = \sqrt{\frac{7}{16\pi}} (5 \cos^3 \theta - 3 \cos \theta)$$

Radial Wave Functions

$$R_{1,0} = \frac{2}{\sqrt{a^3}} e^{-r/a}$$

$$R_{2,0} = \frac{1}{\sqrt{2a^3}} \left(1 - \frac{r}{2a} \right) e^{-r/2a}$$

$$R_{2,1} = \frac{r}{2\sqrt{6a^5}} e^{-r/2a}$$

$$R_{3,0} = \frac{2}{3\sqrt{3a^3}} \left(1 - \frac{2r}{3a} + \frac{2r^2}{27a^2} \right) e^{-r/3a}$$

$$R_{3,1} = \frac{4\sqrt{2}r}{27\sqrt{3a^5}} \left(1 - \frac{r}{6a} \right) e^{-r/3a}$$

$$R_{3,2} = \frac{2\sqrt{2}r^2}{81\sqrt{15a^7}} e^{-r/3a}$$

$$R_{4,0} = \frac{1}{4\sqrt{a^3}} \left(1 - \frac{3r}{4a} + \frac{r^2}{8a^2} - \frac{r^3}{192a^3} \right) e^{-r/4a}$$

$$R_{4,1} = \frac{\sqrt{5}r}{16\sqrt{3a^5}} \left(1 - \frac{r}{4a} + \frac{r^2}{80a^2} \right) e^{-r/4a}$$

$$R_{4,2} = \frac{r^2}{320\sqrt{a^7}} \left(1 - \frac{r}{12a} \right) e^{-r/4a}$$

$$R_{4,3} = \frac{r^3}{768\sqrt{35a^9}} e^{-r/4a}$$