

Physics 215– Elementary Modern Physics
Equations for Test 3

The following equations you should have memorized, and understand how to use them:

For one dimension

Momentum Operator

$$p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$$

The Hamiltonian

$$H = \frac{1}{2m} p_{op}^2 + V$$

$$\bar{E} = \langle H \rangle$$

Wave Functions

$$\Psi(x, t) = e^{-iEt/\hbar} \psi(x)$$

Expectation Value of Operator

$$\langle \mathcal{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x) \mathcal{O} \psi(x) dx$$

Schrödinger Equations

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi + V\Psi \quad \text{or} \quad i\hbar \frac{\partial}{\partial t} \Psi = H\Psi$$

$$E\psi = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi + V\psi \quad \text{or} \quad E\psi = H\psi$$

Uncertainty

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2$$

For three dimensions

Momentum Operators (3D):

$$p_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$$

$$p_y = \frac{\hbar}{i} \frac{\partial}{\partial y}$$

$$p_z = \frac{\hbar}{i} \frac{\partial}{\partial z}$$

Schrödinger Equations in 3D:

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Psi + V\Psi \quad \text{or} \quad i\hbar \frac{\partial}{\partial t} \Psi = H\Psi$$

$$E\psi = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi + V\psi \quad \text{or} \quad E\psi = H\psi$$

Wave Functions

$$\Psi(\vec{r}, t) = e^{-iEt/\hbar} \psi(\vec{r})$$

The Hamiltonian

$$H = \frac{1}{2m} \vec{p}_{op}^2 + V$$

$$\bar{E} = \langle H \rangle$$

Spin Values

$$s = \frac{1}{2}$$

$$S^2 = \hbar^2 (s^2 + s)$$

$$= \frac{3}{4} \hbar^2$$

$$S_z = \hbar m_s$$

Angular Momentum Values:

$$L^2 = \hbar^2 (l^2 + l)$$

$$L_z = \hbar m$$

Restrictions on Quantum Numbers:

$$n = 1, 2, 3, \dots$$

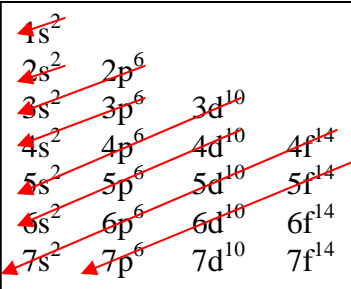
$$l = 0, 1, 2, 3, \dots, n-1$$

$$m = -l, -l+1, \dots, 0, \dots, l$$

$$m_s = \pm \frac{1}{2}$$

Filling up Atoms:

$1s^2 \ 2s^2 \ 2p^6 \ 3s^2 \ 3p^6 \ 4s^2$



The following equations you need not memorize, but you should know how to use them if given to you:

1D Square Well

$$E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi nx}{L}\right)$$

Harmonic Oscillator
Energies:

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

where $n = 0, 1, 2, \dots$

Reflection off a step:

$$R = \begin{cases} \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}}\right)^2 & \text{if } E > V_0 \\ 1 & \text{if } E < V_0 \end{cases}$$

Planck Constants:

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$$

Barrier Penetration:

$$T \approx 16 \frac{E}{V_0} \left(1 - \frac{E}{V_0}\right) \exp\left(-2L\sqrt{2m(V_0 - E)}/\hbar\right)$$

Hydrogen-like Atoms

$$E_n = -\frac{(13.6 \text{ eV})Z^2}{n^2}$$

Layout of the exam: Below is an outline of the exam

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Part I: Multiple Choice [20 points]

For each question, choose the best answer (2 points each)

[questions 1-10]

Part II: Short answer [20 points]

Choose **two** of the following three questions and give a short answer (1-3 sentences) (10 points each).

[questions 11-13]

Part III: Calculation: [60 points]

Choose **three** of the following four questions and perform the indicated calculations (20 points each)

[questions 14-17]