**Solution Set 7**

We first need to figure out if the rotation given is a "proper" one, which means it can actually be performed or not. This can be done by checking if the determinant is +1 (proper rotations) or -1 (improper rotations). This has determinant one, so we don't have to worry about improper rotations.

We start with Natalie's equations (20) through (23), which help us get the various Euler angles.

$$\cos\beta = \mathcal{R}_{zz} = 0$$
$$\sin\beta = 1$$
$$e^{-i\alpha} = \frac{\mathcal{R}_{zx} - i\mathcal{R}_{zy}}{\sin\beta} = \frac{1}{1} = 1$$
$$e^{-i\gamma} = \frac{\mathcal{R}_{xz} + i\mathcal{R}_{yz}}{-\sin\beta} = \frac{i}{-1} = -i$$

We now plug these into Tinkham (5-36):

$$D^{(1)}(\alpha,\beta,\gamma) = \begin{pmatrix} \frac{1}{2}(-i) & -\frac{1}{\sqrt{2}} & \frac{1}{2}i \\ \frac{1}{\sqrt{2}}(-i) & 0 & -\frac{1}{\sqrt{2}}i \\ \frac{1}{2}(-i) & \frac{1}{\sqrt{2}} & \frac{1}{2}i \end{pmatrix}$$

Note that the columns denote the values of *m* and the rows the values of *m'*, starting at the most positive value of *m*, so to figure out $Y_1^1(\mathcal{R}\hat{\mathbf{r}})$, we need to multiply the first column by the corresponding spherical harmonics. We now just need to check these. For example,

$$Y_1^1(\mathcal{R}\hat{\mathbf{r}}) = -\tfrac{1}{2}iY_1^1(\hat{\mathbf{r}}) - \tfrac{1}{\sqrt{2}}iY_1^0(\hat{\mathbf{r}}) - \tfrac{1}{2}iY_1^{-1}(\hat{\mathbf{r}}) = \sqrt{\frac{3}{8\pi}}\frac{-y-iz}{r}$$
$$Y_1^0(\mathcal{R}\hat{\mathbf{r}}) = \tfrac{1}{\sqrt{2}}\left[-Y_1^1(\hat{\mathbf{r}}) + Y_1^{-1}(\hat{\mathbf{r}})\right] = \sqrt{\frac{3}{4\pi}}\frac{x}{r}$$
$$Y_1^{-1}(\mathcal{R}\hat{\mathbf{r}}) = \tfrac{1}{2}iY_1^1(\hat{\mathbf{r}}) - \tfrac{1}{\sqrt{2}}iY_1^0(\hat{\mathbf{r}}) + \tfrac{1}{2}iY_1^{-1}(\hat{\mathbf{r}}) = \sqrt{\frac{3}{8\pi}}\frac{y-iz}{r}$$

Since $\mathcal{R}\mathbf{r} = (y,z,x)$, we can see that this is exactly what we want.