Problems 32-36

32. Rindler spacetime is a two-dimensional spacetime of considerable importance in the understanding of quantum field theory in curved spacetime. The metric is given by

$$ds^2 = -x^2 dt^2 + dx^2$$

   (a) Find all non-zero components of the connection $\Gamma^{\alpha}_{\mu\nu}$. Find the four-acceleration (actually, two-acceleration) A^{μ} and its magnitude $a^2 = A^{\mu} A_{\mu}$ experienced by an observer staying at fixed position *x*. Show that it diverges at the apparent coordinate singularity $x = 0$.

   (b) Calculate a relationship between *x* and *t* for a light beam moving leftwards/rightwards. From this relationship, define two null coordinates *v* and *w*. Write the metric in terms of these coordinates. The metric should look something like $ds^2 = f(v, w)\, dv\, dw$.

   (c) Define new coordinates $v' = v'(v)$ and $w' = w'(w)$ that makes the metric look as simple as possible. Show that in these new coordinates, the former coordinate singularity at $x = 0$ has disappeared.

33. Consider a particle moving radially in the FRW (Friedmann-Robertson-Walker) metric, so that $u^{\phi} = u^{\theta} = 0$. I recommend using the (t, ψ, θ, ϕ) coordinates

   (a) Find an equation for how the radial velocity changes $du^{\psi}/d\tau$ for a particle following a geodesic.

   (b) Define $p = mu^{\hat{\psi}}$, where $u^{\hat{\psi}}$ is the radial velocity in orthonormal coordinates, *m* is the (constant) mass of the particle and *p* is the momentum. Show that ap is constant; that is, that $d(ap)/d\tau = 0$.

34. Find the volume of the universe, defined as $V = \int \sqrt{\gamma}\, d^3\mathbf{r}$, where γ is the determinant of the space part of the metric, for a closed universe ($k = 1$). I recommend using the (t, ψ, θ, ϕ) coordinates.

   (a) In terms of the scale factor *a* at any time.

   (b) At present, in terms of H_0 and Ω (assuming $\Omega > 1$).

35. Assume a flat universe $k = 0$ that is completely dominated by either radiation or matter. A photon leaves the origin at $t = 0$ and travels radially outwards. Find the distance the photon travels by time *t* for each of the two cases.

36. Suppose a universe contains only radiation, but does not necessarily have $\Omega = 1$. Find a relationship between the Hubble constant H_0 and the age of the universe in terms of Ω.