Problems 16-20

16. Show that $\left[\nabla_a, \nabla_b\right] w_c = -w_d R^d{}_{cab}$.

17. Using the fact that $\left[\nabla_a, \nabla_b\right] g_{cd} = 0$, prove that $R_{cdab} = -R_{dcab}$

18. Show that
    (a) $\left[\left[\nabla_a, \nabla_b\right], \nabla_c\right] + \left[\left[\nabla_b, \nabla_c\right], \nabla_a\right] + \left[\left[\nabla_c, \nabla_a\right], \nabla_b\right] = 0$, no matter what it is acting on. Just write it out, and it will be trivial.
    (b) Let the operator in part (a) act on an arbitrary scalar function *f*, and show that this implies $R^d{}_{cab} + R^d{}_{abc} + R^d{}_{bca} = 0$.

19. In this problem we will consider the Riemann tensor in two dimensions
    (a) In 2D it is know that the Riemann tensor can be written $R_{abcd} = \frac{1}{2}\bar{R}\varepsilon_{ab}\varepsilon_{cd}$. Argue why this is reasonable.
    (b) Compute the Ricci tensor in terms of $\bar{R}$.
    (c) Show that $\bar{R}$ is the Ricci scalar.
    (d) Consider the surface of a sphere of radius *r*, with metric

$$ds^2 = r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

    Compute any non-vanishing component of Riemann and use it to compute the Ricci scalar.

20. Work out the components of the four acceleration A^μ in flat space relative to a Lorentz frame defined by

$$ds^2 = -dt^2 + \delta_{ij} dx^i dx^j .$$

    Show that its components are given by

$$A^t = \gamma^4 \vec{a} \cdot \vec{v} \quad \text{and} \quad \vec{A} = \gamma^2 \vec{a} + \gamma^4 \left(\vec{a} \cdot \vec{v}\right)\vec{v}$$

    where

$$\vec{v} = \frac{d\vec{x}}{dt}, \quad \vec{a} = \frac{d^2\vec{x}}{dt^2}, \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\vec{v}^2}}$$