Problems 6-10

6. If we assume $\Gamma^i_{[jk]} = 0$ (the torsion-free condition, show that

$$\Gamma^i_{jk} = \tfrac{1}{2} g^{i\ell} \left(\partial_j g_{\ell k} + \partial_k g_{j\ell} - \partial_\ell g_{jk} \right)$$

hint: start with $\nabla_k g_{ij} + \nabla_j g_{ki} - \nabla_i g_{jk} = 0$

7. For spherical coordinates,
   (a) Determine all eighteen components of Γ^k_{ij}.
   (b) Determine the standard formula for the Laplacian ∇^2 acting on a scalar

8. Show that $\Gamma^i_{ki} = \partial_k \ln \sqrt{g}$, where g is the determinant of g_{ij}.

9. For cylindrical coordinates with metric $ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$
   (a) Find an orthonormal basis of vectors and forms. To save your instructor from insanity, use the most obvious choice.
   (b) Find ∇f, $\nabla \cdot \mathbf{v}$ and $\nabla^2 f$ in the coordinate basis
   (c) Find ∇f, $\nabla \cdot \mathbf{v}$ and $\nabla^2 f$ in the orthonormal basis.

10. A rocket follows a path with coordinates given by

$$x = \sqrt{t^2 + a^{-2}} - a^{-1},$$
$$y = z = 0.$$

   (a) Write all four coordinates in terms of the proper time τ.
   (b) Show that the proper acceleration is given by $\sqrt{\mathbf{A}^2} = a$.
   (c) Calculate the proper time required to reach α Centauri (4.3 ly), the center of the Milky Way galaxy (25 kly) and the edge of the universe (40 Gly), if accelerating at a constant proper acceleration a = 9.80 m/s^2. A light-year (ly) is the speed of light c times one year.