

Problems 41-43

41. Write the formula for the size of gravitational waves from a circular Newtonian system in the form

$$h \sim h_0 \left(\frac{M}{M_0} \right)^a \left(\frac{T_0}{T} \right)^b \left(\frac{r_0}{r} \right)$$

where M , T , and r are the total mass, period, and distance to the orbiting bodies, a and b are simple powers. Choose the distance r_0 such that h_0 is of order 10^{-21} to 10^{-22} . Choose M_0 and T_0 as appropriate, for:

- (a) Two white dwarfs of total mass $2 \times 1 M_\odot$, orbiting with a period appropriate for a separation around $R = 4 \times 10^5$ km .
 (b) Two neutron stars of total mass $2 \times 1.4 M_\odot$, orbiting with a period appropriate for a separation around $R = 100$ km

42. Consider Lie derivatives.

- (a) Show that the Lie derivative of a 1-form is given by $\mathcal{L}_\xi \alpha_i = \xi^j \partial_j \alpha_i + \alpha_j \partial_i \xi^j$. You may assume the Leibnitz rule applies; that is, $\mathcal{L}_\xi (AB) = (\mathcal{L}_\xi A)B + A\mathcal{L}_\xi B$.
 (b) Show that $\mathcal{L}_\xi v^i$ is a tensor. Hint: show that $\mathcal{L}_\xi v^i = \xi^j \nabla_j v^i - v^j \nabla_j \xi^i$

43. Show that the rate of loss of angular momentum from a Newtonian binary is given by

$$\frac{dJ}{dt} = \frac{32}{5} \frac{\mu^2}{M} (GM\Omega)^{7/3}$$

Hint: You can write the angular momentum J and energy E in terms of the radius or angular velocity. Then you can use either of

$$\frac{dJ}{dt} = \left(\frac{dJ}{d\Omega} \right) \left(\frac{d\Omega}{dt} \right) = \left(\frac{dJ}{dR} \right) \left(\frac{dR}{dt} \right)$$