

Physics 780 – General Relativity
Homework Set T

47. The universe will be finite in size if $\Omega > 1$.
- (a) The value given in class for the density parameter is $\Omega_0 = 0.9993 \pm 0.0037$. Taking this literally, for a closed universe, what is the *smallest* possible value for $H_0 a_0$ assuming $1 < \Omega \leq 0.9993 + 0.0037$?
 - (b) What is the spatial volume for a closed universe with scale factor a ? You will probably have to use the version of the metric in terms of ψ to get the full range $\psi \in [0, \pi]$ of the whole universe.
 - (c) Write the scale factor a from part (a) in Gpc if $H_0 = 67.7$ km/s/Mpc. Don't forget to add factors of c to get the units right! Then find the minimum volume of the visible universe in Gpc^3 using the result of part (b).
48. We found an integral formula for the current age of the universe times the current Hubble constant $t_0 H_0$ and the current density parameter if there is *only* matter with density Ω_m .
- (a) Repeat this exercise and find $t_0 H_0$ if there is *only* radiation with density Ω_r . Perform the integral.
 - (b) Repeat this exercise and find $t_0 H_0$ if there is radiation with density Ω_r and matter with density Ω_m , but the universe is flat, so $\Omega_r + \Omega_m = 1$. Perform the integral.