Physics 780 – General Relativity

Homework Set Q

41. The Reissner-Nordström solution has many properties similar to the Schwarzschild solution, and therefore many of the same techniques can be used.
    (a) Based on the Killing Vectors ∂_t and ∂_ϕ, argue that two components of the four-velocity must be conserved. For consistency with Schwarzschild, call these $-E$ and *J*.
    (b) Argue in a manner similar to Schwarzschild, that if the instantaneous velocity satisfied $U^\theta = U^\phi = 0$ at any time, that these will continue to be true.
    (c) Argue in a manner similar to Schwarzschild that if at any time $\theta = \frac{1}{2}\pi$ and $U^\theta = 0$, these will continue to be true.
    (d) For Schwarzschild, we showed that for a massive particle moving in the $\theta = \frac{1}{2}\pi$ plane,

$$\tfrac{1}{2}\left(E^2 - 1\right) = \tfrac{1}{2}\left(U^r\right)^2 + V_{eff}\left(r\right) \quad \text{where} \quad V_{eff}\left(r\right) = -\frac{GM}{r} + \frac{J^2}{2r^2} - \frac{GMJ^2}{r^3}.$$

    Is there a comparable equation for Reissner-Nordström, and if so, what changes must be made to these equations?

42. Consider an object falling into a charged black hole $Q \neq 0$ in the $\theta = \frac{1}{2}\pi$ plane
    (a) Show that no matter the value of the energy *E*, it will *never* reach the origin; that is, it will stop moving inwards ($U^r = 0$) before it reaches the center. I had to prove it using two cases, $J = 0$ and $J \neq 0$.
    (b) Consider now the case of dropping something in from infinity, initially at rest, so $E = 1$ and $J = 0$. Find the turning radius $r_{\min}$ where it stops falling and makes a U-turn. Where does this point compare to the two event horizons, $r_\pm$ (assume we don't have a naked singularity, so $Q > M\sqrt{G}$)?
    (c) Find a formula for the proper time to fall from distance *r* to $r_{\min}$.

43. This problem has a lot to do with units. The goal is to keep careful track of them.
    (a) Working in SI units, if you have a charge *q*, what is the electric field at a distance *r*? Compare with our formula for the electric field for the Reissner-Nordström. At least at large *r*, they should be the same. Based on this, find a formula relating *Q* to *q*.
    (b) You're not done with units! Because we are working in general relativity, there can easily be some factors of *c*, the speed of light hidden in your formula for part (a). Given that GQ^2 has units of m^2, revise your formula from part (a) by adding an appropriate power of *c* to the relationship you found there.
    (d) At large distances, we can use classical formulas to calculate forces. Supposed a black hole of mass *M* and charge *q* is so charged up that a proton with mass *m* and charge *e* far from the black hole feels exactly balancing forces from gravity and electromagnetism. What is the ratio *q*/*M* for this black hole in C/kg? You can use classical formulas, since we are far from the black hole.

(e) For the black hole in part (c), find the value of $Q/\left(M\sqrt{G}\right)$. You may have to include factors of c to make this expression dimensionless.