Physics 780 – General Relativity

Homework Set I

21. Consider a geodesic in flat 2D space, but working in polar coordinates $ds^2 = d\rho^2 + \rho^2 d\phi^2$. The non-vanishing Christoffel symbols are $\Gamma^{\phi}_{\rho\phi} = \Gamma^{\phi}_{\phi\rho} = \rho^{-1}$, $\Gamma^{\rho}_{\phi\phi} = -\rho$. Because we have space and no time, geodesics are parameterized by *s*, not *τ*.
    (a) Write both components of the geodesic equations for dU^{μ}/ds.
    (b) Show that on a geodesic, $\rho^2 U^{\phi}$ is constant, that is, $\frac{d}{ds}\left(\rho^2 U^{\phi}\right) = 0$. Hint: on the dU^{ϕ}/ds equation, replace $U^{\rho} = d\rho/ds$ and then multiply it by ρ^2.

22. Write out $\left[\nabla_{\mu}, \nabla_{\nu}\right] g_{\alpha\beta}$ in terms of the Riemann tensor, and then use the fact that the metric has vanishing covariant derivative to show that $R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\nu\mu}$

23. Show the Jacobi identity $\left[\left[\nabla_{\mu}, \nabla_{\nu}\right], \nabla_{\alpha}\right] + \left[\left[\nabla_{\nu}, \nabla_{\alpha}\right], \nabla_{\mu}\right] + \left[\left[\nabla_{\alpha}, \nabla_{\mu}\right], \nabla_{\nu}\right] = 0$

24. By letting the Jacobi identity act on a scalar ϕ, show that $R^{\beta}{}_{\alpha\mu\nu} + R^{\beta}{}_{\mu\nu\alpha} + R^{\beta}{}_{\nu\alpha\mu} = 0$.