Physics 780 – General Relativity

Homework E

12. Consider the coordinate transformation in 2D relating Cartesian coordinates to polar,

$$\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \rho &= \sqrt{x^2+y^2} \\ \phi &= \tan^{-1}(y/x) \end{aligned}$$

(a) If we have a vector $\left(V^x, V^y\right) = (A, 0)$, what is V is $\left(V^\rho, V^\phi\right)$? Write your answers in terms of (ρ, ϕ).

(b) If we have a 1-form $\left(V_x, V_y\right) = (A, 0)$, what is V is $\left(V_\rho, V_\phi\right)$? Write your answers in terms of (ρ, ϕ).

(c) If we have a tensor of type $T^{xx} = A$, with all other components vanishing, what are the components of *T*in the (ρ, ϕ) system? Write your answers in terms of (ρ, ϕ).

13. Consider the components of a pair of vectors in 2D, $[V, W]^\mu$, where $V = V^\mu \partial_\mu$ and $W = W^\mu \partial_\mu$

(a) Find the commutator if $\left(V^x, V^y\right) = (-y, x)$ and $\left(W^x, W^y\right) = (x, y)$. Is it possible that these vectors correspond to partial derivatives of some coordinates (r,θ), *i.e.*, is it possible for $V = \partial_\theta$ and $W = \partial_r$ (hint: partial derivatives commute)?

(b) Find the commutator if $\left(V^x, V^y\right) = (y, x)$ and $\left(W^x, W^y\right) = (x, y)$. Is it possible that these vectors correspond to partial derivatives of some coordinates (r,θ)?