Physics 712
Chapter X Problems

1. **A 6 volt battery is connected between two parallel metal plates of area 100 cm² and separation 1 mm, initially both neutral.**
   (a) How much charge flows when the battery is connected?

This is trivial if we have the capacitance of the capacitor. Since the plate area is large compared to the separation, this can be treated as a standard parallel plate capacitor. If the voltage difference is ΔV and the separation is d, then the electric field will be $E = \Delta V/d$. If there is a dielectric with permittivity ε, then we would have a displacement field $D = \varepsilon E = \varepsilon \Delta V/d$, which means this will be the surface charge $\sigma = \mathbf{D} \cdot \hat{\mathbf{n}}$. Multiplying by the area, we find the charge on each plate will be

$$Q = \frac{\varepsilon A \Delta V}{d}$$

The capacitance can be deduced from $Q = C\Delta V$, so $C = A\varepsilon/d$. With the numbers provided, the charge in the capacitor will be

$$Q = \frac{\varepsilon_0 A \Delta V}{d} = \frac{\left(8.85 \times 10^{-12}\ \text{m}^{-1}\text{J}^{-1}\text{C}^2\right)\left(10^{-2}\ \text{m}^2\right)\left(6.00\,\text{J/C}\right)}{10^{-3}\ \text{m}} = 5.31 \times 10^{-10}\ \text{C} = 0.531\,\text{nC}.$$

   (b) With the battery still connected, the plates are moved to 0.5 mm separation. What is the change in electrostatic energy stored in the field?

The energy in a capacitor is $U = \frac{1}{2}Q\Delta V$. The energy *before* you bring the plates together is therefore

$$U_i = \tfrac{1}{2}Q\Delta V = \tfrac{1}{2}\left(5.31 \times 10^{-10}\ \text{C}\right)\left(6.00\ \text{J/C}\right) = 1.59 \times 10^{-9}\ \text{J} = 1.59\ \text{nJ}.$$

When you cut the distance in half, the capacitance doubles, so the charge doubles and the energy doubles (since the voltage stays the same), so we have $U_f = 2U_i = 3.2 \times 10^{-10}$ J. The change in energy is, therefore,

$$\Delta U = U_f - U_i = 2U_i - U_i = U_i = 1.59\ \text{nJ}.$$

   (c) How much energy was supplied by the battery as the plates were moved?

As a small charge δQ is moved from one plate to the other, the work required is $\delta W = (\delta Q)(\Delta V)$. Since the potential difference remains constant, the work can be integrated to get

$$W = (\Delta Q)(\Delta V) = (Q_f - Q_i)(\Delta V) = (2Q_i - Q_i)(\Delta V) = \left(5.31 \times 10^{-10}\ \text{C}\right)\left(6.0\ \text{J/C}\right) = 3.18\ \text{nJ}$$

This exceeds the energy increase in the capacitor. The rest of the energy went into mechanical work, since there is an attractive force trying to pull the capacitor plates together.

    (d) Starting over with the 1 mm gap and the plates at 6 volt potential difference, disconnect the battery. Find the work required by an external force (e.g. you) to insert a slab of dielectric slab of permittivity $\varepsilon = 2\varepsilon_0$ between the plates. Is the slab pulled in by the capacitor, or must it be pushed in?

    Doubling the permittivity doubles the capacitance, but since the charge must remain the same, this implies that the voltage difference decreases by a factor of 2. The energy of the capacitor is still given be $U = \frac{1}{2}Q\Delta V$, so it must also be cut in half. Hence the change in energy is

$$\Delta U = U_f - U_i = \tfrac{1}{2}U_i - U_i = -\tfrac{1}{2}U_i = -0.797 \text{ nJ}.$$

This is the work you do on the dielectric slab. Since it is negative, the slab is being pulled into the capacitor.