Physics 712
Chapter 6 Problems

1. A wire along the z-axis has a current that turns on suddenly at $t = 0$, so the current density is $\mathbf{J}(\mathbf{x},t) = I\hat{\mathbf{z}}\delta(x)\delta(y)\theta(t)$, where $\theta(t)$ is the Heaviside function, with $\theta(t<0)=0$ and $\theta(t>0)=1$. There is no charge density, $\rho(\mathbf{x},t)=0$.
   (a) Working in Lorentz gauge, find $\mathbf{A}(\mathbf{x},t)$ in cylindrical coordinates.
   (b) Find the electric and magnetic fields $\mathbf{E}(\mathbf{x},t)$ and $\mathbf{B}(\mathbf{x},t)$.

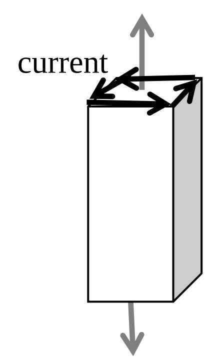
2. For question 1, find the total energy flux per unit length flowing out of a cylinder of radius a centered on the z-axis as a function of time.

3. An oscillating point dipole with dipole moment $\mathbf{p} = p\hat{\mathbf{z}}\sin(\omega t)$ at the origin results in scalar and vector potentials (in Lorentz gauge) at large r of approximately

$$\Phi = \frac{p\omega\cos\theta}{4\pi\varepsilon_0 rc}\cos(\omega t - \omega r/c), \quad \mathbf{A} = \frac{\mu_0 p\omega}{4\pi r}\cos(\omega t - \omega r/c)\left(\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta\right).$$

   (a) Find the leading order terms at large r for the electric and magnetic fields (these will be terms of order r^{-1}). As a check, $\mathbf{E}$ should be entirely in the $\hat{\boldsymbol{\theta}}$ direction and $\mathbf{B}$ in the $\hat{\boldsymbol{\varphi}}$ direction.
   (b) Find the total power flowing out of a sphere of radius r centered on the origin at large r.

4. Consider a cylinder of arbitrary cross-sectional shape, such as a square, circle, or other similar shape. This cylinder will be infinitely long in the z-direction. It will have a surface current K, with units A/m, running around it in a counter-clockwise direction as viewed from above.
   (a) In which direction(s) can you translate this cylinder and leave it unchanged? What can you conclude about the resulting magnetic field?
   (b) Across which plane can you reflect this current and leave it unchanged? Based on this, which components of the magnetic field must vanish?



5. Consider an infinite plane of surface current in the plane $z = 0$ flowing in the direction $\mathbf{K} = K\hat{\mathbf{x}}$, where K has units of A/m.
   (a) Which direction(s) can you translate this current and leave it unchanged? What conclusions can you draw about the $\mathbf{B}$-field?
   (b) By reflecting this problem across the $y = 0$ plane, which of the components of $\mathbf{B}$ can you conclude must vanish?
   (c) By reflecting this problem across the $z = 0$ plane, show that you can relate the field above the plane to the field below the plane.
   (d) Using an appropriate Ampere loop, find $\mathbf{B}$ everywhere.