Physics 712
Chapter 1 Problems

1. Consider a long (infinite) cylinder of charge with radius R, centered along the z-axis, with charge density uniformly spread over its volume, with linear charge λ C/m. What is the electric field everywhere?

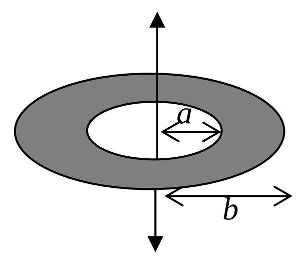
2. Consider the electric potential from a neutral hydrogen atom, given in spherical coordinates by

$$\Phi = \frac{q}{4\pi\varepsilon_0} e^{-2r/a}\left(\frac{1}{r}+\frac{1}{a}\right)$$

   where q is the fundamental charge, a is the Bohr radius. Find the electric field everywhere. Then find the charge density everywhere. Be careful when finding the charge at the origin; you may have to apply Gauss's Law to a small sphere around the origin.

3. Is it possible to design a trap for a particle? Suppose we wish to trap a particle with positive charge q at the origin, but such that there is no other charge very close, so that $\rho = 0$ in a small neighborhood of the origin.
   (a) What condition(s) can you place on the potential $\Phi(\mathbf{x})$ or its derivative so that it is at a local minimum of energy in the x-direction. Repeat for the y and z-direction.
   (b) Given that $\rho = 0$ at the origin, prove or disprove that you can have a local minimum of the potential there.

4. Consider an annulus (hollow circle) of surface charge density σ with inner radius a and outer radius b centered on the origin in the xy-plane. Find the potential and electric field everywhere on the z-axis. If a charge q, initially at rest, were released from the origin, what would be its speed v when it reaches infinity?



5. Consider the region $z > 0$, with boundary condition $\partial\Phi/\partial n = 0$ on the boundary.
   (a) Show that the Green function for this region and boundary condition is given by

$$G(\mathbf{x},\mathbf{x}') = \frac{1}{|\mathbf{x}-\mathbf{x}'|} + \frac{1}{|\mathbf{x}_R-\mathbf{x}'|}, \quad \text{where} \quad \mathbf{x}_R = (x, y, -z)$$

   (b) Find the electric potential everywhere if there is a line of charge along part of the z-axis, so that

$$\rho(\mathbf{x}) = \begin{cases} \lambda\delta(x)\delta(y) & \text{for } z < a, \\ 0 & \text{for } z > a. \end{cases}$$

6. Estimate the capacitance of a sphere of radius R using the trial potential function $\Psi(\mathbf{x}) = e^{-\lambda(r-R)/2}$, and compare to the exact value from lecture.

7. Consider a square of side *a* with *V* = 0 on three sides and *V* = 1 on the surface *y* = *a* in two dimensions. Our goal is to compute the potential $\Phi\left(\frac{1}{4}a,\frac{1}{2}a\right)$ using the relaxation method. To do so, you can download some helpful spreadsheets at

http://users.ecarlson.wfu/eandm/relax.xlsx

(a) We will first work in a low-resolution matrix with grid spacing $\frac{1}{4}a$. Check that the formula in square B2 is correct for computing using Φ_{ij}^{+}, then copy it and paste the formula into the rest of the interior of the spreadsheet (by selecting Paste, f_x from the pulldown menu). Then press F9 repeatedly to force recalculation until it converges.

(b) Now switch to the medium-res tab at the bottom with grid spacing $\frac{1}{8}a$. Redo the calculation. Then switch to the high-res tab and redo it with spacing $\frac{1}{16}a$.

(c) Now, redo the formula so we are instead using $\Phi_{ij} = \frac{4}{5}\Phi_{ij}^{+} + \frac{1}{5}\Phi_{ij}^{\times}$. Redo the calculation in each case. Record all the values for $\Phi\left(\frac{1}{4}a,\frac{1}{2}a\right)$ in a table (it should have six values in it).

(d) Comment on which technique you think is most accurate. Based on all your computations, how accurate do you think your final answer is (about how many digits?)