Physics 712 – Electricity and Magnetism

Equations for Final Exam

The following equations you should have memorized

Maxwell's equations

$$\nabla\cdot\mathbf{D}=\rho$$
$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$$
$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$$
$$\nabla\cdot\mathbf{B}=0$$

Linear materials

$$\mathbf{D}=\varepsilon\mathbf{E}$$
$$\mathbf{B}=\mu\mathbf{H}$$
$$\mathbf{J}=\sigma\mathbf{E}$$

Boundary conditions

$\mathbf{D}_\perp,\mathbf{E}_\parallel$ continuous

$\mathbf{B}_\perp,\mathbf{H}_\parallel$ continuous

On conductors:

$$\mathbf{E}_\parallel=0=\dot{\mathbf{B}}_\perp$$

Poynting Vector

$$\mathbf{S}=\mathbf{E}\times\mathbf{H}^*$$
$$\mathbf{g}=\mathbf{S}/c^2$$

Vector and Scalar Potentials

$$\mathbf{B}=\nabla\times\mathbf{A}$$
$$\mathbf{E}=-\nabla\Phi-\partial\mathbf{A}/\partial t$$

Gauge Choice

Gauge change: $\begin{cases}\mathbf{A}\to\mathbf{A}+\nabla\Lambda\\ \Phi\to\Phi-\partial\Lambda/\partial t\end{cases}$

Coulomb guage: $\nabla\cdot\mathbf{A}=0$

Lorentz: $\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}=0$

Vacuum

$$\mu=\mu_0$$
$$\varepsilon=\varepsilon_0$$
$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

Energy Density

$$u=\tfrac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H})$$

Lorentz Force

$$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

Periodic Fields

$$\mathbf{E}(t)=\mathrm{Re}\left(\mathbf{E}e^{-i\omega t}\right)$$

And so on

$$\langle\mathbf{S}\rangle=\tfrac{1}{2}\mathrm{Re}\left(\mathbf{E}\times\mathbf{H}^*\right)$$

Index of refraction

$$n=\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$
$$\frac{\omega}{k}=v_p=\frac{c}{n}$$
$$v_g=d\omega/dk$$

EM waves

$$\mathbf{E}(\mathbf{x})=\mathbf{E}_0e^{i\mathbf{k}\cdot\mathbf{x}}$$
$$\mathbf{E}_0\perp\mathbf{k}$$
$$\mathbf{B}=\frac{n}{c}\hat{\mathbf{k}}\times\mathbf{E}$$
$$I=\tfrac{1}{2}\varepsilon\mathbf{E}\cdot\mathbf{E}^*$$

Rotations of vectors, tensors

$$x_i\to x_i'=\sum_j R_{ij}x_j$$
$$E_i(x)\to E_i'(x')=\sum_j R_{ij}E_j(x)$$
$$T_{ij}\to T_{ij}'=\sum_{kl}R_{ik}R_{jl}T_{kl}$$

Snell's Law

$$n\sin\theta=n'\sin\theta'$$
$$k_x=k_x'=k_x''$$

Relativity

$$x=(ct,\mathbf{x})$$
$$g_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$$
$$c^2\tau^2=(x-y)\cdot(x-y)$$
$$\gamma=1\Big/\sqrt{1-v^2/c^2}$$
$$U^\mu=\frac{dx^\mu}{d\tau}=\gamma(c,\mathbf{v})$$
$$p^\mu=mU^\mu=(E/c,\mathbf{p})$$
$$E^2=c^2\mathbf{p}^2+m^2c^4$$

Cavities

TE: $B_z=\psi\sin(kz)$

TM: $E_z=\psi\cos(kz)$

$$k=\frac{\pi p}{d},\quad k=0,1,2,\ldots$$

Wave Guides

$$\nabla_t^2\psi+\gamma^2\psi=0$$

TE: $B_z=\psi,E_z=0,\left.\frac{\partial\psi}{\partial n}\right|_s=0$

TM: $E_z=\psi,B_z=0,\psi|=0$

TEM: $E_z=B_z=0,\gamma=0$

$$\mu\varepsilon\omega^2=k^2+\gamma^2$$

all waves $\sim e^{ikz-i\omega t}$

Lorentz Boost

$$x'=\gamma(x-vt)$$
$$t'=\gamma\left(t-vx/c^2\right)$$
$$y'=y,\quad z'=z$$

Capacitors

$$Q=C\Delta V$$
$$U=\tfrac{1}{2}Q\Delta V$$

Surface Charge $\mathbf{D}=\sigma\hat{\mathbf{n}}$

Phase Shift

$$\Delta\phi=\arg\left(\frac{E''}{E}\right)$$

Effective Permittivity

$$\varepsilon_{\mathrm{eff}}=\varepsilon+i\sigma/\omega$$

Some other things to know:

- How various fields transform under proper and improper rotations
- How various fields transform under time reversal
- What Brewster's angle means

The following equation you need not memorize, but you should be able to use them if given to you:

Green's Function for Lorentz Gauge

$$\Phi(\mathbf{x},t) = \int \frac{\rho\left(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|/c\right)}{4\pi\varepsilon_0 |\mathbf{x}-\mathbf{x}'|} d^3\mathbf{x}'$$

$$\mathbf{A}(\mathbf{x},t) = \mu_0 \int \frac{\mathbf{J}\left(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|/c\right)}{4\pi |\mathbf{x}-\mathbf{x}'|} d^3\mathbf{x}'$$

Maxwell Stress Tensor

$$T_{ij} = \varepsilon_0 E_i E_j + \mu_0 H_i H_j - \tfrac{1}{2}\delta_{ij}\left(\varepsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{H}^2\right)$$

Wave Guides

TE: $\mathbf{B}_t = ik\gamma^{-2}\nabla_t \psi$, $\mathbf{E}_t = -i\omega\gamma^{-2}\hat{\mathbf{z}}\times\nabla_t\psi$

TM: $\mathbf{E}_t = ik\gamma^{-2}\nabla_t E_z$, $\mathbf{B}_t = i\varepsilon\mu\omega\gamma^{-2}\hat{\mathbf{z}}\times\nabla_t E_z$

Relativity and EM

$$J = (c\rho, \mathbf{J})$$

$$\partial_\mu J^\mu = 0$$

$$A^\mu = (\Phi/c, \mathbf{A})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$$

$$\varepsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\alpha\beta} = 0$$

$$\frac{d}{d\tau} p^\nu = qF^\nu{}_\mu U^\mu$$

Reflection Amplitudes

$$\frac{E'_\perp}{E_\perp} = \frac{2n\cos\theta}{n\cos\theta + \sqrt{n'^2 - n^2\sin^2\theta}}, \quad \frac{E''_\perp}{E_\perp} = \frac{n\cos\theta - \sqrt{n'^2 - n^2\sin^2\theta}}{n\cos\theta + \sqrt{n'^2 - n^2\sin^2\theta}}$$

$$\frac{E'_\parallel}{E_\parallel} = \frac{2nn'\cos\theta}{n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}, \quad \frac{E''_\parallel}{E_\parallel} = \frac{n'^2\cos\theta - n\sqrt{n'^2 - n^2\sin^2\theta}}{n'^2\cos\theta + n\sqrt{n'^2 - n^2\sin^2\theta}}$$

Boosts and Fields

$$\mathbf{E}'_\parallel = \mathbf{E}_\parallel, \quad \mathbf{B}'_\parallel = \mathbf{B}_\parallel$$

$$\mathbf{E}'_\perp = \gamma\left(\mathbf{E}_\perp + \mathbf{v}\times\mathbf{B}\right)$$

$$\mathbf{B}'_\perp = \gamma\left(\mathbf{B}_\perp - \mathbf{v}\times\mathbf{E}/c^2\right)$$

Kramers-Kronig Relations

$$\mathrm{Re}\left[\varepsilon(\omega)\right] = \varepsilon_0 + \frac{2}{\pi} P\int_0^\infty \frac{\omega' \,\mathrm{Im}\left[\varepsilon(\omega')\right]}{\omega'^2 - \omega^2} d\omega'$$

$$\mathrm{Im}\left[\varepsilon(\omega)\right] = -\frac{2\omega}{\pi} P\int_0^\infty \frac{\mathrm{Re}\left[\varepsilon(\omega')\right] - \varepsilon_0}{\omega'^2 - \omega^2} d\omega'$$

EM Fields

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$