Name ___________________

Solutions to Midterm Exam
October 13, 2021

This test consists of three parts. For the first and second parts, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper.

Part I: Multiple Choice Everyone: Answer all questions.
For each question, choose the best answer (2 points each)

1. The strongest evidence for dark matter in spiral galaxies like ours is that
   A) The orbital speeds of stars and gas clouds do not fall off with distance
   B) Gravitational lensing events from massive compact halo objects (MACHOs)
   C) Gravitational redshift shows that there is much more mass than can be accounted for by stars
   D) Infrared satellites can directly observe the dark matter
   E) The dimming of various distant objects because we have to see through the dark matter

2. In which part(s) of our galaxy are substantial numbers of new stars being born?
   A) The disk (only)
   B) The halo (only)
   C) The bulge (only)
   D) The halo and disk, but not the bulge
   E) The bulge and disk, but not the halo

3. Which of the following stars has the highest surface temperature?
   A) B7 B) F2 **C) B3** D) K9 E) M1

4. Galaxies with disks cannot collapse to the center because of conservation of
   A) Mass B) Energy C) Momentum D) Charge **E) Angular Momentum**

5. The best way to determine the age of a cluster of stars is by
   A) Measuring radioactive decay of elements contained in the stars
   B) Counting the ratio of red giant stars to main sequence stars
   C) Making a Hertzsprung-Russell diagram and seeing where the stars "turn off" from the main sequence
   D) Measuring how much hydrogen has been converted to helium in the stars
   E) Measuring to what extent its core has collapsed towards the center

6. The galaxy pictured at right is approximately what galaxy classification?
   A) E0 B) E7 C) SAd **D) SBc** E) Im

7. The mass of the black hole at the center of our galaxy was measured by
   A) Gravitational lensing of objects behind it
   B) Measuring the orbits of stars going around it
   C) Measuring the gravitational red shift for light waves coming from it
   D) Direct measurement of the Schwarzschild radius
   E) Studying the acceleration of exploratory spacecraft sent to study it

8. Which of the following types of active galaxies is believed to be powered by a giant black hole at the heart of the galaxy?
   A) Quasars (only)
   B) Radio galaxies (only)
   C) Blazars (only)
   D) All of the above
   E) None of the above

9. Which of the following is true about galaxy collisions?
   A) Galaxy collisions are extremely rare; most galaxies have probably never collided
   B) When galaxies collide, many of the stars are destroyed by the violent collisions
   C) Galaxy collisions have virtually no effect on the shape or structure of a galaxy
   D) The gas in galaxies is so low density it simply passes through gas in the other galaxy
   E) Galaxy collisions are common and can cause galaxies to merge

10. The name of one large galaxy that is near our galaxy is
    A) Coma **B) Andromeda** C) Fornax D) Virgo E) Laniakea

11. The most massive galaxies tend to be
    A) Spiral B) Barred Spiral C) Irregular **D) Elliptical** E) Dwarf Spheroidal

12. The way we know what stars are made of is primarily by
    A) Measuring the composition of stellar winds coming from these stars
    B) Comparing the color of stars to the color of similar materials on Earth
    C) Assuming they start as hydrogen and helium and then using computer models
    D) Studying the composition of the Solar System, made from former stars
    E) Measuring the wavelengths and strengths of the dark lines in the stellar spectra

13. Most of the mass of our galaxy is in the
    A) Nucleus B) Bulge **C) Halo** D) Disk E) None of these

14. The name of the <u>collection of galaxies</u> we live in is
    A) Milky Way **B) Local Group** C) Virgo D) Andromeda E) Coma

15. Laniakea is the name of our ______
    A) Supercluster B) Stellar cluster C) Galaxy D) Galaxy cluster E) Hypercluster

Part II: Short Answer PHY 310: Choose three of the four questions PHY 610: Answer all four questions. Write 2-4 sentences about each of the following [10 each]

PHY 310: You may skip one question

16. Explain what tidal friction is, and what effect it has on galaxies that almost collide but just miss each other.

Tidal friction is a gravitational interaction between two galaxies, where the gravitational pull of one is non-uniform, causing the stars in the other galaxy to have relative velocities change. This can distort the other galaxy, but it also robs the system of kinetic energy, causing their relative motion to slow down, and ultimately the galaxies may actually collide and then merge.

17. Which portion of the galaxy do we live in? Besides stars, what other objects can be found in the portion we live in?

We live in the disk. The disk also contains gas, hot gas (HII regions) where the hydrogen is so hot it is ionized, medium temperature gas (HI regions) where the gas is cool enough that electrons are bound to hydrogen, but no molecules form, and cold gas (molecular clouds) where hydrogen combines with itself and other atoms to make H_2 molecules and other compounds. There is also dust, which mostly obscures our vision.

18. The Sun goes around our galaxy in an approximately circular orbit. In addition to this circular motion, what other types of motion does the Sun undergo as it orbits the center?

The Sun is also moving in and out towards and away from the center (epicycles), representing radial motion, and it moves up and down in the z-direction. The motion around the orbit is not uniform, but speeds and slows down slightly, synchronized with its motion inwards and outwards from the center.

19. Explain how the radar distancing method can be used to measure the distance to, for example, the Moon. Explain why it isn't used to measure the distance to the Andromeda galaxy.

Radar distancing involves reflection a radio beam off of a solid object and measuring the time until return. Because it moves at the speed of light, and it has to go there and back, the distance and time are related by the simple formula $2d = ct$. We don't use it beyond our Solar system because it is difficult to get a reflection from such a large distance, and even if we did, we'd have to wait a long time for it to return (millions of years for Andromeda).

Physical Constants	Units	Distance/Magnitude	Doppler Shift
$k_B = 1.381\times10^{-23}$ J/K	$\text{pc} = 3.086\times10^{16}$ m	$d = 10^{1+\frac{m-M}{5}}$ pc	$1+z = \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+v_r/c}{1-v_r/c}}$
$\hbar = 1.055\times10^{-34}$ J·s	$M_\odot = 1.989\times10^{30}$ kg	$m - M = 5\log(d) - 5$	
$h = 6.626\times10^{-34}$ J·s	$\text{rad} = 206,265''$		
$G = 6.673\times10^{-11}\,\text{m}^3/\text{kg/s}^2$			

Brightness/Magnitude	Type Ia Supernova	Black Body Radiation	Black Hole
$F = 2.518\times10^{-8}\,\text{W/m}^2\left(10^{-\frac{2}{5}m}\right)$	$M_{\max} = -19.3$	$u = \frac{\pi^2}{15}\frac{(k_BT)^4}{(\hbar c)^3}$	$R_s = \frac{2GM}{c^2}$
		$\lambda_{\max}T = 0.00290\ \text{m}\cdot\text{K}$	

Part III: Calculation:
PHY 310: Choose four of the five problems; PHY 610: Do all five problems.
For each of the following problems,
give the answer, explaining your work. [20 points each]

20. We will shortly learn that the universe is filled with black body radiation. The peak of the power is at a wavelength of $\lambda_{\max} = 1.064\times10^{-3}$ m.

(a) What is the energy of one photon with this wavelength?

The energy of a single photon is given by $E = h\nu$, where we can find the frequency from $\lambda\nu = c$. We therefore have

$$E = h\nu = h\frac{c}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{1.064\times10^{-3}\ \text{m}} = 1.867\times10^{-22}\text{J}.$$

A more convenient unit is the electron volt; this is the equivalent of $E = 0.00165$ eV.

(b) What is the black-body temperature of the Universe?

We use the formula $\lambda_{\max}T = 0.00290\ \text{m}\cdot\text{K}$ to find

$$T = \frac{0.00290\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{0.00290\ \text{m}\cdot\text{K}}{0.001064\ \text{m}} = 2.726\text{K}.$$

(c) What is the energy density of the black body radiation in J/m³?

We use the formula

$$u = \frac{\pi^2}{15}\frac{(k_BT)^4}{(\hbar c)^3} = \frac{\pi^2\left[\left(1.3806\times10^{-23}\ \text{J/K}\right)\left(2.725\ \text{K}\right)\right]^4}{15\left[\left(1.0546\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)\right]^3} = 4.172\times10^{-14}\ \text{J/m}^3.$$

(d) If we treat the energy of a photon from part (a) as typical, use the energy density from part (c) to estimate the number density of photons.

The energy density of photons will be the number density of photons times the average energy, so $u = nE$, and therefore

$$n = \frac{u}{E} = \frac{4.172\times10^{-14}\ \mathrm{J/m^3}}{1.867\times10^{-22}\ \mathrm{J}} = 2.23\times10^{8}\,\mathrm{m^{-3}}.$$

21. The star β-Cygni is actually a double star, with components β-Cygni A having an apparent magnitude of m_A = 3.10 and β-Cygni B having an apparent magnitude of m_B = 5.10.

(a) What is the flux in W/m² coming from each of these stars? By naked eye, the two stars appear to be one star. What is the total flux of this combination?

We simply substitute into the formula $F = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-\frac{2}{5}m}\right)$, which gives

$$F_A = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-0.4\times3.10}\right) = 1.45\times10^{-9}\ \mathrm{W/m^2},$$

$$F_B = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-0.4\times5.10}\right) = 2.30\times10^{-10}\ \mathrm{W/m^2},$$

$$F_{tot} = F_A + F_B = \left(1.45 + 0.23\right)\times10^{-9}\ \mathrm{W/m^2} = 1.68\times10^{-9}\ \mathrm{W/m^2}.$$

(b) What is the apparent magnitude *m* of the stars together?

We simply use the same equation in reverse:

$$F = 2.518\times10^{-8}\,\mathrm{W/m^2}\left(10^{-\frac{2}{5}m}\right) = 1.68\times10^{-9}\ \mathrm{W/m^2},$$

$$10^{-\frac{2}{5}m} = \frac{1.68\times10^{-9}\ \mathrm{W/m^2}}{2.518\times10^{-8}\,\mathrm{W/m^2}} = 0.0668,$$

$$-\tfrac{2}{5}m = \log\left(0.0667\right) = -1.175,$$

$$m = \tfrac{5}{2}\left(1.175\right) = 2.94.$$

(c) The system has a parallax of 0.00816″. What is the distance to β-Cygni?

We use the formula relating parallax (in arc-seconds) to distance (in parsecs), so

$$d = \frac{1}{p} = \frac{1}{0.00816} = 123\ \mathrm{pc}.$$

(d) What is the absolute magnitude *M* for the combined star system?

We use the formula $m - M = 5\log\left(d\right) - 5$, which we rearrange and solve for M to yield

$$M = m - 5\log\left(d\right) + 5 = 2.94 - 5\log\left(123\right) + 5 = -2.51.$$

22. The calcium K-line has a normal wavelength of $\lambda_0 = 393.4\text{ nm}$, but is detected in a distant galaxy to occur at a wavelength of $\lambda = 457.3\text{ nm}$.

(a) Find the value of $1+z$, and the velocity of this galaxy in km/s. The value of *z* is too large to use the non-relativistic approximation.

The value of $1+z$ is just given by the red-shift formula:

$$1+z = \frac{\lambda}{\lambda_0} = \sqrt{\frac{1+v_r/c}{1-v_r/c}} = \frac{457.3\text{ nm}}{393.4\text{ nm}} = 1.162.$$

We now do some algebra to solve for the velocity.

$$\frac{1+v_r/c}{1-v_r/c} = 1.162^2 = 1.351\,,$$

$$1+\frac{v_r}{c} = 1.351\left(1-\frac{v_r}{c}\right) = 1.351 - 1.351\frac{v_r}{c},$$

$$\frac{v_r}{c}(1+1.351) = 1.351-1,$$

$$\frac{v_r}{c} = \frac{0.351}{2.351} = 0.149,$$

$$v_r = 0.149\left(2.998\times10^8\text{ m/s}\right) = 4.478\times10^7\text{ m/s} = 44780\text{ km/s}.$$

If we had used the nonrelativistic approximation, we would have gotten an error of about 15%.

(b) A type Ia supernova goes off in this galaxy! It peaks at an apparent magnitude $m_{\max} = 19.7$. Find the distance to this galaxy in Mpc.

We simply use the distance formula together with the maximum absolute magnitude $M_{\max} = -19.3$ to yield

$$d = 10^{1+\frac{m-M}{5}}\text{ pc} = 10^{1+\frac{1}{5}(19.7+19.3)}\text{ pc} = 10^{8.80}\text{ pc} = 6.310\times10^8\text{ pc} = 631\text{ Mpc}.$$

(c) Using this single data point, estimate the value of Hubble's constant in km/s/Mpc.

Hubble's Law says that $v = H_0 d$, so solving for H_0, we have

$$H_0 = \frac{v}{d} = \frac{44780\text{ km/s}}{631\text{ Mpc}} = 71.0\text{ km/s/Mpc}.$$

23. Perhaps the dark matter is black holes of mass $M = 500 M_\odot$. If they are this large, could we notice them if they happen to be in front of a nebula that glows, as a dark spot?

(a) Find the Schwarzschild radius for a black hole of this mass.

We simply use the formula to give

$$R_s = \frac{2GM}{c^2} = \frac{2\left(6.674\times10^{-11}\ \text{m}^3/\text{kg/s}^2\right)(500)\left(1.989\times10^{30}\ \text{kg}\right)}{\left(2.998\times10^8\ \text{m/s}\right)^2} = 1.477\times10^6\ \text{m} = 1477\ \text{km}.$$

(b) The local mass density (mass/volume) of dark matter is estimated to be $0.0914 M_\odot/\text{pc}^3$. Find the size of the average volume containing one black hole of this mass. Take the cube root to get an approximate distance to the nearest such black hole in pc.

The local mass density is just the mass divided by the volume, $\rho = M/V$, so

$$V = \frac{M}{\rho} = \frac{500\ M_\odot}{0.0914 M_\odot / \text{pc}^3} = 5470\,\text{pc}^3.$$

If we divide the local universe into cubes of size $V = L^3$, then the size of the cubes will be

$$L = V^{1/3} = \left(5470\,\text{pc}^3\right)^{1/3} = 17.6\ \text{pc}.$$

If each such cube contains one of these black holes, then there is probably black hole like this within about this distance.

(c) What would be the angular radius of such a black hole at the distance found in part (b) in arc-seconds? We probably can't see anything smaller than 0.001″. Are we able to detect these black holes by this method?

The angular size in radians is just the physical size divided by the distance, so we have

$$\alpha = \frac{R_s}{L} = \frac{1.477\times10^6\ \text{m}}{17.6\,\text{pc}}\cdot\frac{\text{pc}}{3.086\times10^{16}\ \text{m}} = \left(2.719\times10^{-12}\ \text{rad}\right)\left(206205''/\text{rad}\right)$$

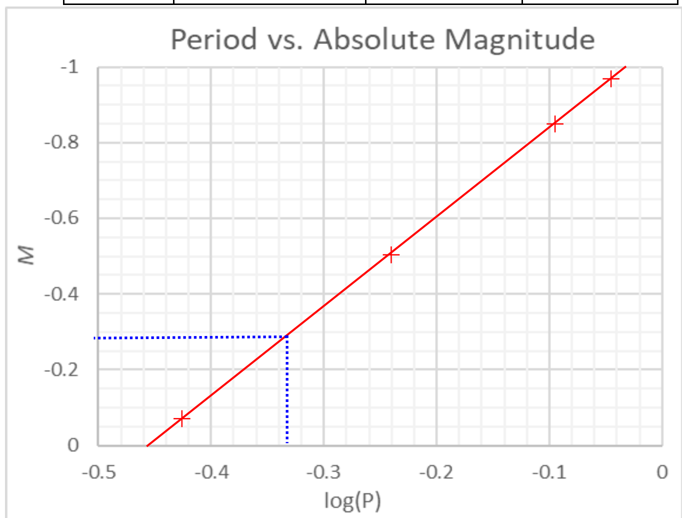
$$= \left(5.61\times10^{-7}\right)'' = 0.561\ \mu\text{as}.$$

You get to multiply this by two to get the diameter, and because black holes suck up nearby light, it can be shown that you get to multiply by another number (I think it's 3) to get the total size of the black spot, but something 3 micro arc-seconds across is really hard to see.

24. A distance method we didn't discuss uses a period-luminosity relation for RR-Lyrae stars, another type of variable star. Listed at right is a table of five RR-Lyrae stars and their period of pulsation in days.

Star	P (days)	log(P)	M
A	0.375	-0.426	-0.070
B	0.903	-0.044	-0.968
C	0.804	-0.095	-0.849
D	0.573	-0.242	-0.503

(a) Calculate log(P) for each of the stars listed. Then use the provided chart to plot log(P) vs. absolute magnitude M. You may put your answers directly on the provided table and chart.



I simply used my calculator to get the logarithms, then I plotted the points on the graph as red + signs.

(b) Explain roughly why RR-Lyrae stars can be used as a distance indicator. A trend line on the graph might help.

I have included a trend line. There is a pretty clear linear relationship between log(P) and M, so that we can determine their luminosity from their period, and then compare the apparent and absolute magnitudes to get the distance.

(c) Star X is an RR-Lyrae star with a period of 0.462 days and an apparent magnitude m = 18.03. Estimate the distance to star X.

We have log(0.462) = –0.335. Reading off the graph (dotted blue lines), this implies an absolute magnitude of about M = –0.29. We then substitute this into the distance formula to give

$$d = 10^{1+\frac{m-M}{5}}\ \text{pc} = 10^{1+\frac{1}{5}(18.03+0.29)}\ \text{pc} = 10^{4.664}\ \text{pc} = 46100\,\text{pc} = 46.1\ \text{kpc}.$$