Physics 310/610 – Cosmology

Solution Set S

1. **[15] For this problem we are going to predict properties of an undiscovered particle(s) and predict its properties. We are going to make a table of the nine lightest spin 3/2 baryons**
   (a) Looking at all the spin 3/2 baryons, divide them into categories based on their strangeness. Invent a generic name for each category. Note that there is a relatively small mass difference *among* the particles in each category.
   (b) Now make a table, with one row for each category. The columns should contain the following information (i) your generic name, (ii) strangeness, (iii) number of particles in that category (iv) average electric charge (v) average mass.
   (c) There is an additional row which corresponds to an undiscovered category, called Ω. By extrapolation, predict all of the entries of the additional row.

Cat.	S	#	Q_{ave}	m_{ave} (MeV)
Δ	0	4	+½	1233
Σ^*	–1	3	0	1384.7
Ξ^*	–2	2	–½	1533.5
Ω	–3	1	–1	1684

It is pretty obvious what the strangeness, number of particles, and the average charge is: one particle with strangeness –3 and charge –1. The mass increases almost exactly 150 MeV at each step, so the next row should be right around 1684 MeV.

(d) Look up the properties of the "Omega baryon," and see how well you did on your predictions. For any that are wrong, calculate the percent error.

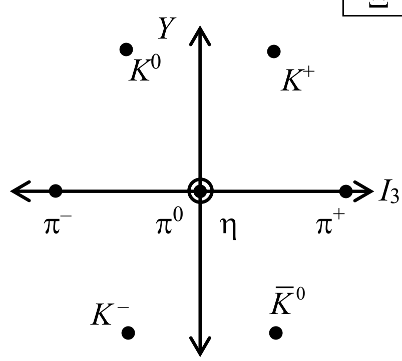
The charge and strangeness of the lightest omega baryon are right, and there is only one such particle. The mass is 1672.4, so we are about 0.7% off. Not too bad.

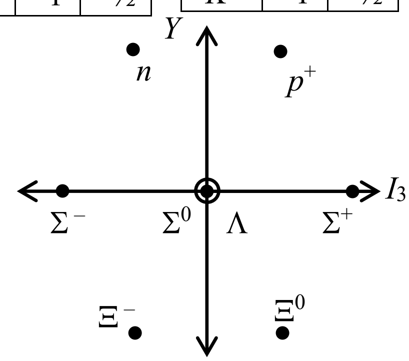
2. **[15] For this problem we are going to need to calculate the *hypercharge* of various particles, which is defined as $Y = S + B$, the sum of strangeness and baryon number, and the *third component of isospin*, $I_3 = Q - \frac{1}{2}Y$, the sum of charge and half of hypercharge.**
   (a) Find these quantities for the eight lightest spin-0 mesons. Plot the result as points on a 2D graph, with I_3 on the horizontal axis, and Y on the vertical axis. If two particles have the same value, designate it by drawing a circle around a dot.
   (b) Repeat for the eight lightest spin-½ baryons.

Baryons		
Part	Y	I_3
p^+	1	+½
n^0	1	–½
Λ	0	0
Σ^+	0	+1
Σ^0	0	0
Σ^-	0	–1
Ξ^0	–1	+½
Ξ^-	–1	–½

Mesons		
Part	Y	I_3
K^+	1	+½
K^0	1	–½
η	0	0
π^+	0	+1
π^0	0	0
π^-	0	–1
$\bar{K}^0$	–1	+½
K^-	–1	–½

The calculations for both are trivial and straightforward.





Tables are provided above. The resulting plots are given at right.

3. **[10] For each of the following particles (which are made only of up, down, and strange quarks), predict the quark content, based on their baryon number, charge, and strangeness. Denote your answers using *u*, *d*, or *s* for quarks, and $\bar{u}$, $\bar{d}$ and $\bar{s}$ for anti-quarks. It is possible for two particles to have the same quark content**
   (a) Λ^0 $\{\mathrm{uds}\}$ A baryon with strangeness -1 (one strange quark), and total charge zero
   (b) K^+ $\{\mathrm{u\bar{s}}\}$ Meson with strangeness +1 (strange anti-quark) and total charge +1
   (c) $\bar{\Sigma}^+$ $\{\mathrm{\bar{d}\bar{d}\bar{s}}\}$ Anti-baryon with strangeness +1 (strange anti-quark) and total charge +1
   (d) Σ^0 $\{uds\}$ A baryon with strangeness -1 (one strange quark), and total charge zero
   (e) π^- $\{d\bar{u}\}$ Meson with no strangeness, with total charge -1.

Graduate Problem: Do the following problem only if you are in PHY 610.

4. **[10] There is a symmetry of strong interactions called *isospin*. We will explore its properties in the nucleons, a two state system consisting of protons $|p\rangle$ and neutrons $|n\rangle$**
   (a) Define the *I*3 operator as was done in problem 2. Write *I*3 as a 2×2 matrix in $\{|p\rangle,|n\rangle\}$ space. Since *I*3 is conserved in all interactions, it commutes with the Hamiltonian.

A proton or neutron have definite values of I_3 which tells us that they are eigenstates, with eigenvalues as found above. So we have $I_3|p\rangle=+\frac{1}{2}|p\rangle$ and $I_3|n\rangle=-\frac{1}{2}|n\rangle$, or as a matrix,

$$I_3=\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

(b) Consider the operator $2I_1$, which converts protons and neutrons and vice-versa. Write I_1 as a 2×2 matrix. Since protons and neutrons interact the same in strong interactions, I_1 commutes with the strong part of the Hamiltonian.

We want $2I_1|p\rangle = |n\rangle$ and $2I_1|n\rangle = |p\rangle$. Writing this as a matrix (and dividing by two) we have

$$I_2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(c) Define I_2 by the commutator $[I_3, I_1] = iI_2$. Write I_2 as a 2×2 matrix.

Ain't nothin' to it but to do it:

$$I_2 = \frac{1}{i}[I_3, I_1] = \frac{1}{i}(I_3I_1 - I_1I_3) = \frac{1}{4i}\left\{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$$
$$= \frac{1}{4i}\left\{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right\} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

(d) You already have $[I_3, I_1]$. Work out $[I_1, I_2]$ and $[I_2, I_3]$.

We have:

$$[I_1, I_2] = \frac{1}{4}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \frac{1}{4}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = iI_3\,,$$

$$[I_2, I_3] = \frac{1}{4}\left[\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} - \frac{1}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = iI_1\,.$$

These are, in fact, the same commutation relationships that spin satisfy (though those relationships have an extra factor of $\hbar$ included), which explains the "spin" part of "isospin."