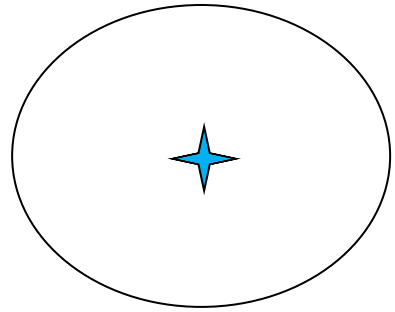
Homework Set F



1. **[15] A supernova in the galaxy M33 produces a point of bright light. Over time, a ring of light forms around the star, as illustrated at right. This is presumably a circular ring, but because the ring is tilted, it looks like an ellipse, with 1 cm representing 10 mas. The top edge of the ellipse begins to glow 50 days after the supernova appears. The bottom edge of the ellipse begins to glow 199 days after the supernova.**
   (a) [4] In class we found a formula for the time delay for the leading edge (top) as a function of the angle β compared to the vertical the ring is tilted at (so $\beta = 0$ would mean we see a circle) and the actual radius *R* of the ring. Derive a similar formula for the trailing edge (bottom) of the ring.

If you look at the ellipse from the side, it is clear that the leading edge of the ellipse is at a distance $R\sin\beta$ closer than the star itself, so that if the star is at a distance of *d*, then the leading edge is at a distance $d - R\sin\beta$. The total distance an echo from the leading edge must travel is then $d - R\sin\beta + R$, or an extra distance of $R(1-\sin\beta)$. This causes a time delay of $\Delta t = R(1-\sin\beta)/c$. In a similar manner, the trailing edge is farther than the star by an amount $R\sin\beta$, and hence the trailing edge is at a distance $d + R\sin\beta$. Hence the light from the trailing edge travels a total distance of $d + R\sin\beta + R$, resulting in a time delay

$$\Delta t = R(1+\sin\beta)/c.$$

(b) [3] Based on the shape of the ellipse, estimate β.

The long axis of the ellipse has an actual size of 2*R*, and the short axis of $2R\sin\beta$. Measuring with a ruler, I found the ellipse to have a major axis of 5.0 cm and minor axis of 4.0 cm, so we have

$$\cos\beta = \frac{2R\cos\beta}{2R} = \frac{4.0}{5.0} = 0.80, \quad \text{or} \quad \beta = 36.9°.$$

(c) [4] Using the time until illumination of the top edge, estimate the true radius of the ring (in AU, or any convenient unit). Repeat for the bottom edge. Your numbers should be fairly close.

We solve each of the equations for the radius *R*, and find

$$R_1 = \frac{c\Delta t_1}{1-\sin\beta} = \frac{(2.998\times10^8\text{ m/s})(50\text{ d})}{1-\sin(36.9°)}\cdot\frac{86{,}400\text{ s/d}}{1.496\times10^{11}\text{ m/AU}} = 21{,}700\text{ AU},$$

$$R_2 = \frac{c\Delta t_2}{1+\sin\beta} = \frac{(2.998\times10^8\text{ m/s})(199\text{ d})}{1+\sin(36.9°)}\cdot\frac{86{,}400\text{ s/d}}{1.496\times10^{11}\text{ m/AU}} = 21{,}500\text{ AU}.$$

Since the numbers came out so close, it is clear we are doing something right.

(d) [4] Based on the angular size of the ellipse and its actual radius, what is the distance to the supernova, and hence to M33?

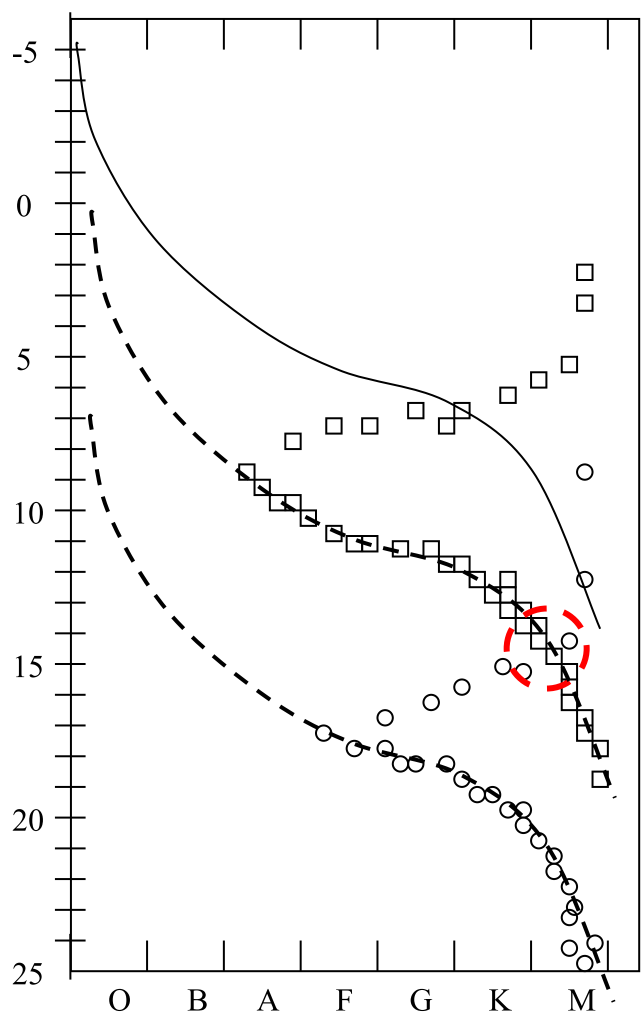
The angular radius of the ellipse is half of the semi-major axis, which I measured as 2.5 cm, or 25 mas. Using the formula $s = d\theta$, where we must work in radians as usual, we have

$$d = \frac{s}{\theta} = \frac{21{,}600\text{ AU}}{0.025''}\cdot\left(\frac{\text{pc/AU}}{1\text{ rad}/1''}\right) = 8.64\times10^5\text{ pc} = 864\text{ kpc}.$$

Wikipedia gives the distance as 970 kpc, so we are a bit off.

2. [15] A cluster of stars is discovered, and the spectral type and apparent magnitude *m* of several member stars is plotted (circles). As a comparison, the standard main sequence is shown as the solid line (with absolute magnitudes *M*, instead of apparent).

(a) [5] What is the distance to the cluster represented by circles?



I have added two more copies of the main sequence curve to the given curve. Measuring as carefully as possible, I find that the new curve is 11.8 magnitudes below the original curve. Assuming the squares near the bottom curve represent main sequence stars, and remembering that the upper curve is the absolute magnitude M while the middle curve is the apparent magnitude m (and also remembering that larger numbers are near the bottom), we estimate $m - M = 11.8$. Plugging this into the distance formula, we have

$$d = 10^{[1+(m-M)/5]} = 2300\text{ pc}.$$

(b) [2] Why are there no stars on the upper part of this main sequence?

The uppermost main sequence stars represent the most massive stars. These are the first ones to die, because the more massive a star is, the faster it dies.

(c) [3] Another cluster is represented by the squares. What is the distance to this cluster?

Measuring as carefully as possible, I find that the new curve is 5.1 magnitudes below the original curve. Plugging this into the distance formula, we have

$$d = 10^{[1+(m-M)/5]} = 105 \text{ pc}.$$

(d) [2] How does the age of these two clusters compare?

We note that more of the main sequence stars for the circle cluster have turned off of the main sequence. The A stars are still main sequence stars for the square cluster, but they are gone for the circle cluster. So the circle cluster is older.

(e) [3] A couple of the circles are very close to some of the squares on the graph. If we could go and visit these stars, would these stars be very similar? Why or why not?

The stars that are in similar places (circled region) have the same temperature and the same apparent magnitude. If, for example, they were both main sequence stars, they would be similar. But whereas the squares are main sequence, the circles are giant stars. They would be very different. They only look the same to us because the much brighter giant stars (circles) are much more distant than the relatively nearby main sequence stars (squares).

There are no graduate problems for this homework