Physics 310/610 – Cosmology

Solution Set A

1. [15] Using the small-angle approximation, find the angular size of the

(a) [5] Sun, with a distance 1.000 AU and diameter $2.000 R_\odot$, in arc-minutes.

We apply the formula for small angles, and use the values for the radius of the Sun and the distance to it from the "units" handout.

$$\alpha = \frac{2.000 \times 6.957 \times 10^8 \text{ m}}{1.496 \times 10^{11} \text{ m}} = \left(0.009301 \text{ rad}\right) \frac{360^\circ}{2\pi \text{ rad}} \cdot \frac{60'}{1^\circ} = 31.97'.$$

Because the Earth-Sun distance varies, this angle varies slightly.

(b) [5] The nearest star (α Cen A), at a distance of 1.325 pc and diameter $2.446 R_\odot$, in mas.

We perform a similar computation.

$$\alpha = \frac{2.446 \times 6.957 \times 10^8 \text{ m}}{1.325 \times 3.086 \times 10^{16} \text{ m}} = \left(4.162 \times 10^{-8} \text{ rad}\right) \frac{360^\circ}{2\pi \text{ rad}} \cdot \frac{60'}{1^\circ} \cdot \frac{60''}{1'} \cdot \frac{1000 \text{ mas}}{1''} = 8.58 \text{ mas}.$$

This is a very small angle.

(c) [5] A telescope with diameter *d* observing light of wavelength λ can resolve objects with angular size $\alpha \sim \lambda/d$. Explain why even the largest telescopes on Earth (*d* ~ 10 m) have difficulty directly measuring the diameter of even the nearest stars (besides the Sun) using visible light (λ ~ 5,000 Å).

Using waves about 5,000 Å with a 10 m telescope, the smallest angles you can see would be about

$$\alpha = \frac{5000 \times 10^{-10} \text{ m}}{10 \text{ m}} = \left(5.00 \times 10^{-8} \text{ rad}\right) \frac{360^\circ}{2\pi \text{ rad}} \cdot \frac{60'}{1^\circ} \cdot \frac{60''}{1'} \cdot \frac{1000 \text{ mas}}{1''} = 10.3 \text{ mas}.$$

Since this is *larger* than the actual size, effectively the star would appear as one blurry pixel. Some stars like Betelgeuse do a little better, but it is practically impossible to directly image a star's diameter.

2. [15] When an electron in hydrogen falls from level *n* to level *m*, the energy emitted is

Wavelength	Classification
7,000 – 10^7 Å	Infrared
4,000 – 7,000 Å	Visible
100 – 4,000 Å	Ultraviolet

$$\Delta E = (13.6\text{ eV})\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

Assuming the light is emitted as a single photon of light, find the energy (in eV), frequency (Hz), wavelength (Å) and classification of the energy for each of the following transitions:
(a) 2 → 1 (b) 3 → 2 (c) 4 → 3 [5 each]

To find the energy, we just plug and chug in the relevant equation:

$$\Delta E_{21} = (13.6\text{ eV})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.20\text{ eV},$$

$$\Delta E_{32} = (13.6\text{ eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.889\text{ eV},$$

$$\Delta E_{43} = (13.6\text{ eV})\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.661\text{ eV}.$$

We then use $\nu = E/h$ and then $\lambda = c/\nu$ to get the frequencies and wavelengths.

$$\nu_{21} = \frac{10.20\text{ eV}}{4.136\times10^{-15}\text{ eV}\cdot\text{s}} = 2.466\times10^{15}\text{ Hz},$$

$$\lambda_{21} = \frac{2.998\times10^{8}\text{ m/s}}{2.466\times10^{15}\text{ s}^{-1}} = 1.216\times10^{-7}\text{m} = 1216\text{Å},$$

$$\nu_{32} = \frac{1.889\text{ eV}}{4.136\times10^{-15}\text{ eV}\cdot\text{s}} = 4.567\times10^{14}\text{ Hz},$$

$$\lambda_{32} = \frac{2.998\times10^{8}\text{ m/s}}{4.567\times10^{14}\text{ s}^{-1}} = 6.564\times10^{-7}\text{m} = 6564\text{Å},$$

$$\nu_{43} = \frac{0.661\text{ eV}}{4.136\times10^{-15}\text{ eV}\cdot\text{s}} = 1.598\times10^{14}\text{ Hz},$$

$$\lambda_{43} = \frac{2.998\times10^{8}\text{ m/s}}{1.598\times10^{14}\text{ s}^{-1}} = 1.876\times10^{-6}\text{m} = 18,760\text{Å}.$$

Looking at the final numbers, we see that these are ultraviolet, visible, and infrared light respectively.