Physics 310/610 – Cosmology
# Homework Set W

1.  This question concerns the relative strength of electric and gravitational forces.
    (a) Write a formula for the gravitational force between two electrons. Find the ratio of the gravitational force to the Coulomb force, $F = ke^2/r^2$, where $k$ is Coulomb's constant and $e$ is the electron charge (which you can go look up). Show that the ratio is constant and evaluate it.
    (b) For very energetic particles, the mass $m$ for the electron is replaced by $E/c^2$. For what energy $E$ in GeV will the gravitational force between a pair of electrons be as strong as the electric force?
    (c) A typical energy of a particle at temperature $T$ is given by $3k_BT$. What is $k_BT$ when gravity is the same strength as the other forces? How old is the universe at this time? Assume the universe is radiation dominated at this time, with $g_{\text{eff}} = 200$ for definiteness.

2.  We have recently been discussing things like the *Planck mass*, the *Planck time*, and the *Planck length*. What are all these quantities? Any quantum theory of gravity must involve the speed of light $c$, the reduced Planck constant $\hbar$ and the gravitational constant $G$. Start by looking up the units of each of these quantities (unless you know them).
    (a) Using only dimensional analysis, find expressions for the Planck time $t_P$, the Planck distance $l_P$ and the Planck mass $m_P$, where in each case the formula will be of the form $x_P = G^\alpha \hbar^\beta c^\gamma$, where $\alpha$, $\beta$, and $\gamma$ are simple rational numbers.
    (b) Evaluate each of the quantities in part (a) in standard SI units.
    (c) Using simple combinations from parts (a) and (b), find the Planck energy $E_P$ in both J and GeV, and find the Planck mass density $\rho_P$ in kg/m$^3$. Naively, the mass density of empty space should be about $\rho_P$. What is the ratio of $\rho_P$ to the actual mass density of empty space, $\rho_\Lambda = 5.65 \times 10^{-27}$ kg/m$^3$.
    (d) Suppose it actually takes an energy $E_P$ to create a universe. Given that the cost of electricity in the United States in 2023 is about $0.169 per kilowatt hour, how much would it cost you to make a universe?