Physics 310/610 – Cosmology

# Homework Set P

1. In class we showed that the average photon, at present, does not hit any electrons. In this problem, you will determine if the average electron is hit by a photon. The cross section is still the Thomson cross-section given in class. The density is the density of *photons*, since that's what an electron is trying to hit. The relative speed is still $c$. In the current age of the universe, how many collisions will a free electron have? Will a typical electron have been hit by at least one photon?

2. For each of the following, estimate the thermal energy $k_B T$ of the universe. Use $g_{\text{eff}} = 3.36$.
   (a) When primordial tritium decays ($t$ = 17.8 y).
   (b) When primordial free neutrons decay ($t$ = 886 s).

3. For each of the following, find $g_{\text{eff}}$, and estimate the age of the universe in seconds.
   (a) At nucleosynthesis, when $k_B T = 80\text{ keV}$.
   (b) When the thermal energy is the same as the electron rest energy, $k_B T = mc^2$. All particles are at the same temperature. In addition to photons and neutrinos, there are also electrons and positrons ($g$ = 4 extra fermions).
   (c) At the electroweak scale, $k_B T = 100\text{ GeV}$. At this time, everything is at the same temperature, and there are $g$ = 28 total spin states for bosons and $g$ = 90 total spin states for fermions.

**Graduate Problem**: Do this problem only if you are in PHY 610.

4. Consider a particle moving at the speed of light in a flat universe, so $ds = 0$, where

$$ds^2 = -c^2 dt^2 + a^2(t)\left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right]$$

   Assume that the particle starts at $r = 0$ at time $t = 0$ and travels radially.
   (a) Assume first that the universe is radiation dominated, so that $a(t) \propto t^{1/2}$. Show that at time $t$ the distance the particle has traveled $d = ra(t)$ is at most $k_r ct$, and determine the pure numerical constant $k_{r,}$, independent of $t$.
   (b) Assume second that the universe is matter dominated, so that $a(t) \propto t^{2/3}$. Show that at time $t$, the distance the particle has traveled is at most $k_m ct$, and determine the pure numerical constant $k_{m,}$, independent of $t$.
   (c) Assume third that the universe is cosmological constant dominated, so that $a(t) \propto \exp(H_1 t)$. Show that in this case, for sufficient time, the distance traveled is greater than any multiple of $ct$.