Physics 310/610 – Cosmology

Homework Set M

1. The critical density is the density required to have $\Omega = 1$. Assuming Hubble's constant is $H_0 = 67.8\ \text{km/s/Mpc}$,
   (a) Find the critical density. Write your answer in kg/m^3 and in $M_\odot/\text{kpc}^3$.
   (b) The actual value of Ω for ordinary matter is only $\Omega_b = 0.0484$. If this is all in the form of hydrogen atoms, what is the number density of hydrogen atoms per cubic meter?

2. We have mostly been neglecting the photons. As we will discover shortly, the universe is filled with electromagnetic radiation at a temperature $T_r = 2.725\ \text{K}$.
   (a) Find the energy density u. Also find the mass density $\rho_r = u/c^2$.
   (b) What is the contribution Ω_r to the total energy density of the universe?

3. In class I claimed that any point on a 3-sphere of radius a could be written as

$$x = a\sin\psi\sin\theta\cos\phi, \quad y = a\sin\psi\sin\theta\sin\phi, \quad z = a\sin\psi\cos\theta, \quad w = a\cos\psi.$$

   Show that these points do, in fact, constitute a 3-sphere of radius a.

Graduate problem: Only do this problem if you are in PHY 610

4. A closed universe has space distance formula

$$ds^2 = a^2\left[d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

   Our goal in this problem is to find the volume of the universe. The *metric* g_{ij} is just the 3×3 matrix defined by $ds^2 = \sum_i \sum_j g_{ij} dx^i dx^j$.
   (a) Find the volume of the universe, which is given by $V = \int \sqrt{\det\left(g_{ij}\right)}\, d^3x$. Note the determinant $\det\left(g_{ij}\right)$ takes care of any necessary factors in the integral. You may have to think a bit (or ask) about the limits on all the angular variables.
   (b) Using the Friedman equation with $k = +1$ (closed universe), find an expression for a_0 in terms of Ω and H_0.
   (c) Experimentally, $H_0 = 67.8$ km/s/Mpc, and $\Omega = 1.0023 \pm 0.0055$. Assuming $1 < \Omega < 1.01$, find a minimum size for the universe a_0 in Gpc and a minimum volume in Gpc^3.