Physics 310/610 – Cosmology

Homework Set K

1. A distant star is lensed by the Sun. The Sun is at d_L = 1.00 AU, and the star is so distant that effectively $d_S \approx d_{LS}$
   (a) Find the Einstein radius θ_E in arc-seconds for distant stars lensed by the Sun.
   (b) A star is positioned such that it would normally be just at the edge of the Sun, $\beta = 30' = 1800''$. There will be two images, $\theta_\pm$. Show that the inner one is invisible (because it is less than the Sun's radius away), and find or approximate how much the outer one is displaced, $\theta_+ - \beta$, in arc-seconds.

2. Suppose a lensing object has mass $M = 1.00 M_\odot$ is exactly half way to the source, so $d_S = 2d_L$. Show that the Einstein radius θ_E as a function of the distance d_L take the form $\theta_E = C\sqrt{\text{kpc}/d_L}$, and determine the constant C in milli-arc seconds. It is very difficult to measure variations in direction at the milli-arc second scale.

3. Show that the amplification of a star by gravitational microlensing is an amplification; that is, show

$$A_{tot} = \frac{1}{2}\left(\frac{\beta}{\sqrt{\beta^2 + 4\theta_E^2}} + \frac{\sqrt{\beta^2 + 4\theta_E^2}}{\beta}\right) > 1.$$

Physics 610: Only do this problem if you are in the graduate version of this course

4. The amplification is always greater than one. So why do we have to get things lined up pretty well before we get lensing events? Taylor expand the formula for amplification in powers of θ_E/β to at least fourth order in θ_E. Based on your Taylor expansion, estimate the largest angle β you can miss by to have even a 0.1% amplification.