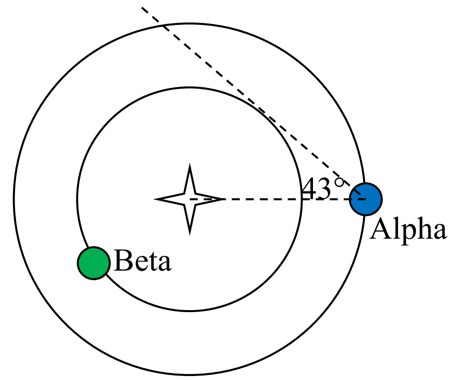
Physics 310 – Cosmology

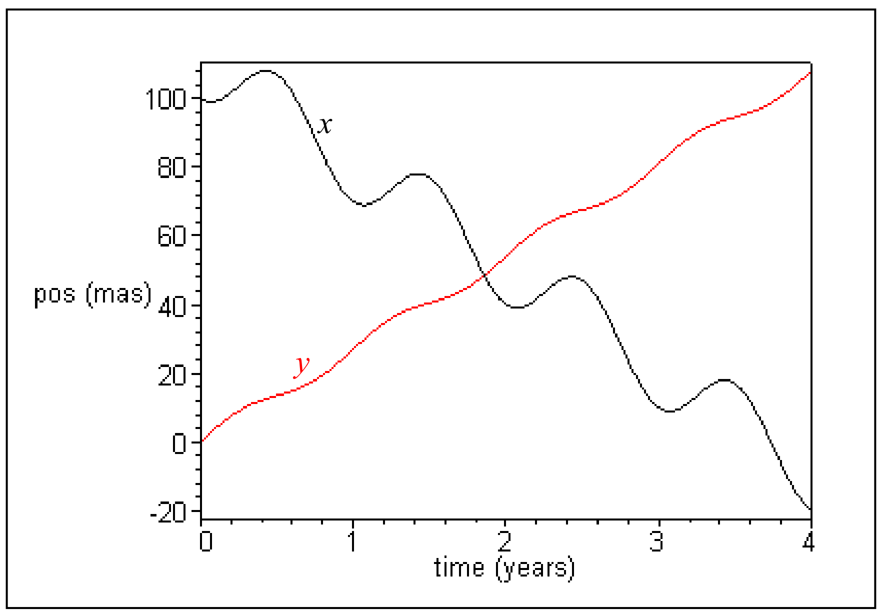
Homework Set E

1. The occupants of planet Alpha have determined that both they, and another planet Beta orbit their star in circular orbits, as sketched at right. Over the course of the year, they have discovered that planet Beta is never more than 43.0 degrees from their star.



   (a) What angle of the triangle Alpha-Beta-Star is a right angle when Beta is in the right position to be at this maximum angle?
   (b) When Beta is at this maximum angle, a radar signal is sent to Beta which returns exactly 782 seconds later. How far is Beta from Alpha at this moment, in AU?
   (c) What is the distance of Alpha from the star, and of Beta from the star, in AU?
   (d) Observations indicate that at this same point, Beta has an angular *diameter* of 37.0" (37 arc-seconds). What is the *radius* of the planet Beta, in km?

2. A star at $\beta = 12.0°$ above the ecliptic has its position accurately marked in the sky over a four-year period. The position as a function of time ends up looking like the accompanying graphs.



   (a) Find the angular velocity of proper motion in the x- and y-directions
   (b) Find the parallax in mas, and the distance to the star in pc. The x-direction corresponds to the major axis of the parallax ellipse.
   (c) Find the transverse velocity components v_x and v_y.
   (d) The spectrum of the star is measured, and it is found that a spectral line that should be at 710.43 nm is actually at 710.61 nm. What is the radial velocity of the star, and is it towards us or away from us?
   (e) What is the total speed of the star relative to the solar system?

Graduate Problems: Only do this problem if you are in PHY 610

3. A star at distance d is moving directly away from us at velocity v, which is related to the red-shift z as described in class. To simplify things, assume that the star has radius R and a uniform temperature T as viewed in its own frame
   (a) Because of Lorentz contraction, the star *as viewed by us* will no longer be a sphere. Can we nonetheless use the formula $\alpha = 2R/d$ to get the angular size as viewed by us, or would it have to be modified?
   (b) The expected number of photons occupying any particular photon state at temperature T is given by

$$n(E) = \frac{1}{e^{E/k_B T} - 1}$$

   For a photon moving towards us, what would be the energy E' that we observe, in terms of E and the red-shift z? Show that the distribution is still thermal, but with different temperature T' as observed by us.
   (c) Argue that the apparent brightness of a star moving away from us will simply be reduced by a factor of $(1+z)^4$ compared to a star that is not moving away from us.