Economic Growth: Practice Problems Key

Intermediate Macroeconomics

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Question 1

a) We want to show that $f(\lambda K, \lambda L) = \lambda f(K, L)\ \forall\ \lambda > 0$:

$$Y(\lambda K, \lambda L) = A(\lambda K)^{\frac{1}{3}}(\lambda L)^{\frac{2}{3}} = A\lambda^{\frac{1}{3}}K^{\frac{1}{3}}\lambda^{\frac{2}{3}}L^{\frac{2}{3}} = \lambda AK^{\frac{1}{3}}L^{\frac{2}{3}} = \lambda Y$$

b)

$$MPK = \frac{\partial Y}{\partial K} = \frac{1}{3}AK^{-\frac{2}{3}}L^{\frac{2}{3}}$$

$$MPL = \frac{\partial Y}{\partial L} = \frac{2}{3}AK^{\frac{1}{3}}L^{-\frac{1}{3}}$$

c)

$$\frac{\partial MPK}{\partial K} = -\frac{2}{9}AK^{-\frac{5}{3}}L^{\frac{2}{3}} < 0$$

$$\frac{\partial MPL}{\partial L} = -\frac{2}{9}AK^{\frac{1}{3}}L^{-\frac{4}{3}} < 0$$

Question 2

a) We want to show that $f_t(\lambda K_t, \lambda L_t) = \lambda f_t(K_t, L_t)\ \forall\ \lambda > 0$:

$$Y_t(\lambda K_t, \lambda L_t) = \left(A(\lambda K_t)^{\frac{1}{2}} + \frac{3}{2}(\lambda L_t)^{\frac{1}{2}}\right)^2 = \left(A\lambda^{\frac{1}{2}}K_t^{\frac{1}{2}} + \frac{3}{2}\lambda^{\frac{1}{2}}L_t^{\frac{1}{2}}\right)^2$$

$$= \left(\lambda^{\frac{1}{2}}\left(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\right)\right)^2 = \lambda\left(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\right)^2 = \lambda Y_t$$

And, the marginal product of capital and marginal product of labor are given by

$$MPK = \frac{\partial Y_t}{\partial K_t} = 2\Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)\frac{1}{2}AK_t^{-\frac{1}{2}} = \Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)AK_t^{-\frac{1}{2}}$$

$$MPL = \frac{\partial Y_t}{\partial L_t} = 2\Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)\frac{3}{2}\frac{1}{2}L_t^{-\frac{1}{2}} = \frac{3}{2}\Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)L_t^{-\frac{1}{2}}$$

b) Dividing both sides of the production function by N_t,

$$\frac{Y_t}{N_t} = \frac{\Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)^2}{N_t} = \frac{\Big(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}\Big)^2}{(N_t^{\frac{1}{2}})^2}$$

$$= \left(\frac{AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}}}{N_t^{\frac{1}{2}}}\right)^2 = \Big(A\left(\frac{K_t}{N_t}\right)^{\frac{1}{2}} + \frac{3}{2}\left(\frac{L_t}{N_t}\right)^{\frac{1}{2}}\Big)^2$$

$$\Rightarrow y_t = \left(Ak_t^{\frac{1}{2}} + \frac{3}{2}\right)^2$$

where the last step follows from $A1$ and using the per capita variables y_t and k_t.

c) Using the per capita production function from part b) and the identities provided in the problem,

$$k_{t+1} = (1-\delta)k_t + x_t = (1-\delta)k_t + s_t = (1-\delta)k_t + sy_t$$

$$\Rightarrow k_{t+1} = (1-\delta)k_t + s\left(Ak_t^{\frac{1}{2}} + \frac{3}{2}\right)^2$$

Question 3

a) Plugging the per capita production function into the per capita neoclassical growth equation given in the problem, we have the following:

$$k_{t+1} = (1-\delta)k_t + sy_t$$

Assuming a steady state,

$$k_{ss} = (1-\delta)k_{ss} + sy_{ss}$$

$$\delta k_{ss} = sy_{ss}$$

$$\Rightarrow \frac{k_{ss}}{y_{ss}} = \frac{s}{\delta}$$

b)

$$\frac{\partial \frac{k_{ss}}{y_{ss}}}{\partial s} = \frac{1}{\delta} > 0$$

c)

$$\frac{\partial \frac{k_{ss}}{y_{ss}}}{\partial \delta} = -s\delta^{-2} < 0$$

d)

$$k_{ss} = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{.20(5)}{.05}\right)^{\frac{1}{1-.33}} = 87.4653$$

$$y_{ss} = Ak_{ss}^{\alpha} = 5(87.4653)^{.33} = 21.8663$$

$$x_{ss} = \delta k_{ss} = .05(87.4653) = 4.3733$$

$$\frac{k_{ss}}{y_{ss}} = \frac{s}{\delta} = \frac{.20}{.05} = 4$$

Question 4

a) Using the per capita endogenous growth equation,

$$k_{t+1} = (1 - \delta + sA)k_t$$

$$\frac{k_{t+1}}{k_t} = 1 - \delta + sA$$

$$1 + g_{k_t} = 1 - \delta + sA$$

$$\Rightarrow g_{k_t} = sA - \delta$$

b)Using the per capita social technology function,

$$y_t = Ak_t$$

$$\Rightarrow \frac{y_{t+1}}{y_t} = \frac{Ak_{t+1}}{Ak_t}$$

$$1 + g_{y_t} = 1 + g_{k_t}$$

$$\Rightarrow g_{y_t} = g_{k_t}$$

c) The growth rate of per capita capital is ten percent:

$$g_{k_t} = .20(1) - .10 = .10$$