Economic Growth: Practice Problems

Intermediate Macroeconomics

John T. Dalton

Question 1 Consider an economy with a Cobb Douglas aggregate production function given by $Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$.

a) Prove that this production function exhibits constant returns to scale.

b) Solve for the marginal product of capital and the marginal product of labor.

c) Prove that this production function exhibits diminishing marginal product of capital and diminishing marginal product of labor.

Question 2 Consider an economy with an aggregate production function given by $Y_t = \left(AK_t^{\frac{1}{2}} + \frac{3}{2}L_t^{\frac{1}{2}} \right)^2$.

a) Repeat parts a) and b) from Question 1 for this production function.

b) Assume A1: labor is inelastically supplied, $L_t=N_t$. Derive the per capita form of this production function, that is, find y_t in terms of k_t and any remaining parameters, where $y_t = \frac{Y_t}{N_t}$ and $k_t = \frac{K_t}{N_t}$.

c) Assume A2: population is constant, $N_{t+1}=N_t$. Along with the per capita production function derived in part b), use the following per capita identities to derive the per capita neoclassical growth equation in terms of k_{t+1}, k_t, and any remaining parameters:

Per capita savings equals per capita investment: $s_t = x_t$

Savings rate is constant: $\frac{s_t}{y_t} = s$

Per capita law of motion of capital: $k_{t+1} = (1-\delta)k_t + x_t$

Question 3 Consider the economy studied in lecture using the assumptions A1, A2, and A3. The per capita production function is given by $y_t = Ak_t^{\alpha}$, and the per capita neoclassical growth equation is given by $k_{t+1} = (1-\delta)k_t + sAk_t^{\alpha}$.

a) Derive an expression for the steady state capital-to-output ratio, $\frac{k_{ss}}{y_{ss}}$, in terms of s and δ.

b) What happens to $\frac{k_{ss}}{y_{ss}}$ as s increases?

c) What happens to $\frac{k_{ss}}{y_{ss}}$ as δ increases?

Now, let $\delta = .05$, $A = 5$, $s = .20$, and $\alpha = .33$.

d) Solve for k_{ss}, y_{ss}, x_{ss}, and $\frac{k_{ss}}{y_{ss}}$.

Question 4 Consider an endogenous growth economy.

a) Derive an expression for the growth rate of per capita capital, g_{kt}, in terms of s, A, and δ.

b) Show that the growth rate of per capita output, g_{yt}, equals the growth rate of per capita capital, g_{kt}.

c) Let $\delta = .10$, $A = 1$, and $s = .20$. What is the growth rate of per capita capital, g_{kt}?