A new attractor for the rate of profit

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Abstract

We propose a specific measure of the steady-state or long-run equilibrium rate of profit and argue on theoretical grounds that this rate serves as an attractor for the actual aggregate rate of profit. Empirical analysis using the Extended Penn World Tables suggests that the steady-state rate is a good predictor of the actual rate one or two years later. JEL classifications: O4 Growth, Population; P1 Capitalism; B5 Heterodox Economics

1 Introduction

In this note we propose a specific measure of the steady-state or long-run equilibrium rate of profit and argue that this rate serves as an attractor for the actual aggregate rate of profit. Our proposal differs from standard 'classical' accounts of the rate of profit in that it depends on neither the wage share of income nor the organic composition of capital. While theoretical analysis does not tell us how quickly to expect convergence of the actual rate of profit on the steady-state value, empirical analysis using the Extended Penn World Tables suggests that the steady-state rate is a good predictor of the actual rate one or two years later.

2 The steady-state rate of profit

We approach the time-evolution of the rate of profit from the standpoint of capital accumulation, as in Cottrell and Cockshott (2006), Zachariah (2008) and Cockshott *et al.* (2009). Initially we assume that all measurements are performed either in labour hours, or—what amounts to the same thing—in a monetary unit whose labour-time equivalent does not change from year to year. Using this approach we derive an equation for the time-evolution of the rate of profit and show that the rate of profit tracks towards a long-run value which depends on the rate of growth of the working population along with the fraction of profit that is reinvested.

Profit can be measured as a flow of labour value, in which case its units are person hours per annum, which in dimensional terms is just persons since the division hours/annum gives a scalar. Thus the annual flow of profit when measured in labour terms corresponds to a certain number of people—the number of people whose direct and indirect output is materialized in the goods purchased out of profits.

The capital stock of a nation is, in these terms, a quantity expressed in millions of person years. And the rate of profit is then:

$$R = \frac{\text{Millions of workers whose product is bought by profits}}{\text{Millions of worker years represented by the capital stock}}$$

The evolution of R then depends on how rapidly the capital stock is built up compared to the growth rate of the number of workers producing the surplus that corresponds to profit.

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Let us represent the total profit or surplus value as a given share of the economy's net output (we consider the effects of a change in this share later).[1] We will write this share as $(1-w)$, where w is the share of wages and salaries. Since we are working in terms of labour hours the net output itself is measured simply by the total current hours worked, which we'll write as L. (The gross output includes in addition labour-time previously embodied in means of production and "passed on" to the product in the current period.)

Writing S for total profit, we then have

$$S = (1-w)L \tag{1}$$

We next consider the growth of the capital stock. This can be thought of as 'source' (gross investment) minus 'sink'. The analysis of the sink requires some care. In labour-time terms, this is the total workers-hours required to maintain the existing capital stock. This includes physical depreciation, whereby part of the existing capital stock, K, wears out each year. We assume that such depreciation occurs at a constant rate, δ, so that δK hours must be spent in maintaining the capital stock in physical terms. However, if the productivity of labour is increasing over time, at some proportional rate g, then a given physical collection of commodities will come to embody a declining number of labour-hours. Call this effect *devaluation* of the capital stock. To maintain the capital stock in *value terms* (worker-years embodied), the physical capital stock must be expanded. The total sink is then $(g+\delta)K$. Gross investment we take to be a proportion, λ, of total profit. We then have

$$\dot{K} = \lambda S - (g+\delta)K \tag{2}$$

where $\dot{K}$ denotes the time-derivative of the capital stock, dK/dt.[2]

Now the rate of profit (in labour-time terms) is the ratio of the surplus, S, to the capital stock, K:

$$R = \frac{S}{K} \tag{3}$$

Using (1) to substitute for S in (3), we get

$$R = (1-w)\frac{L}{K} \tag{4}$$

If the wage-share in net output, w, remains constant then the time-derivative of the rate of profit is given by

$$\dot{R} = (1-w)\frac{d(L/K)}{dt} \tag{5}$$

That is, the change in the profit rate over time is a fraction of the change in the ratio of current labour to value of capital stock. Note that the rightmost term in (5) can be decomposed as

$$\frac{d(L/K)}{dt} = \frac{1}{K}\dot{L} - \frac{L}{K^2}\dot{K} = \frac{L}{K}\left(\frac{\dot{L}}{L} - \frac{\dot{K}}{K}\right)$$

We will assume that the total labour performed per year changes at a proportional rate n (that is, $\dot{L}/L = n$). In addition we infer from (2) that

$$\frac{\dot{K}}{K} = \frac{\lambda S - (g+\delta)K}{K} = \lambda\frac{S}{K} - (g+\delta) = \lambda R - (g+\delta)$$

It follows that

$$\dot{R} = (1-w)\frac{L}{K}\left[n - \lambda R + (g+\delta)\right] \tag{6}$$

[1] At the level of abstraction of this argument, we are not distinguishing the components of surplus value—profit of enterprise, rent and interest—but rather treating the entire surplus as 'profit'.

[2] Note that since we have defined λ as the ratio of *gross* investment to S, and S is defined as a fraction of net output, it is possible in principle to have $\lambda > 1$. (This could happen if the capitalists fully cover depreciation and devaluation, and at the same time plough all of the surplus into new investment.)

Here we have an expression for the time-evolution of the value-rate of profit in terms of the basic parameters of the system. Under what condition is the rate of profit unchanging (i.e. $\dot{R} = 0$)? Given that the wage share, w, must lie between 0 and 1, and that total hours worked, L, must be positive, the required condition is that $n - \lambda R + (g + \delta) = 0$. That is,

$$R^\star = \frac{n + g + \delta}{\lambda} \tag{7}$$

where $R^\star$ is the value of R that yields $\dot{R} = 0$. This is the *steady-state* rate of profit—the rate which, once attained, will persist over time in the absence of disturbances to the parameters.

It is easy to show that the steady-state rate of profit, $R^\star$, is an *attractor* for the actual rate of profit. Suppose that at some point in time R is greater than $R^\star$. Since R enters equation (6) with a negative coefficient, namely $-\lambda(1-w)L/K$, this means that $\dot{R}$ will be negative, or in other words the rate of profit will be falling. By the same token, if $R < R^\star$ the rate of profit will be rising.

Equation (7) shows that the long-run rate of profit is positively related to the growth rate of population (strictly speaking, total hours worked), the growth rate of labour productivity, and the rate of depreciation of the capital stock. It is inversely related to the proportion of the surplus that is reinvested. Note, however, this this long-run rate does not depend on the wage share in net output, w. If w were changing continuously over time this would require a modification to equation (5), but

- the long-run rate of profit is independent of w so long as w is constant (that is, it makes no difference to $R^\star$ whether w equals 0.1 or 0.9); and
- if w does change progressively over time, its change is nonetheless bounded. If the wage share falls over time (the rate of surplus value rises) it can't fall below 0, so the ultimate effect is limited.

To get a sense of what the above analysis implies, consider the simplified case where both depreciation and the growth of labour productivity are zero. Then equation (7) becomes

$$R^\star = \frac{n}{\lambda}$$

That is, the long-run rate of profit is simply a multiple of the population growth rate, n, that multiple being larger, the smaller is the fraction of the surplus that is reinvested. If 100 per cent of the surplus is reinvested, the steady-state rate of profit just equals the population growth rate. Historical demography therefore plays a key role in the long-run evolution of the rate of profit.

3 Empirical results

The Extended Penn World Tables (EPWT, currently at version 3.0) provide suitable data for investigating the empirical counterpart to the theory set out above—see Marquetti (2009). These tables build on version 6.2 of the Penn World Table (Heston *et al.*, 2006), adding estimates of national capital stocks and other related series.

The first use of the EPWT for this purpose was made by David Zachariah (2008). He computed the steady-state rate of profit for the UK and Japan on the basis of the theory described here. He defines the profit rate as $R = S/K$, as above. Total net profit, S, was computed by subtracting wages, W, and depreciation, δK, from the Gross Domestic Product, Y. That is, $S = Y - W - \delta K$. His steady-state rate of profit is as in equation (7). Figure 1 shows plots of five-year moving averages of the actual and steady-state rates taken from Zachariah's paper. It can be seen that the actual rate of profit closely follows the steady-state rate—with a time lag, as one would expect.

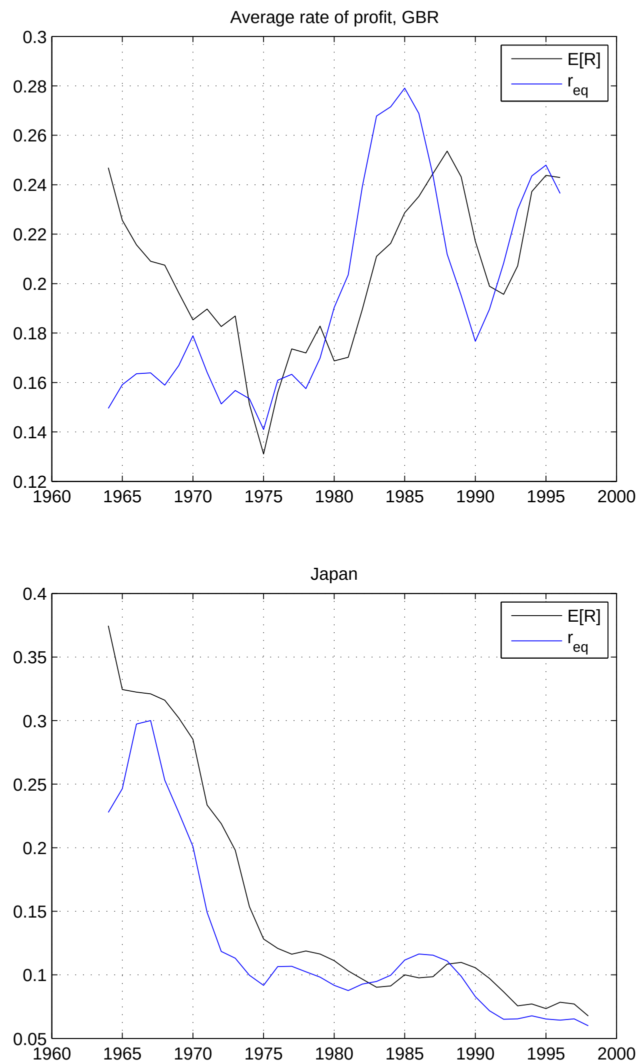


Figure 1: The evolution of the steady-state rate of profit predicted by the theory in this note, r_{eq}, and the observed average rate of profit, $E[R]$, in the UK and Japan (Zachariah, 2008)

We have extended Zachariah's analysis to all countries in the EPWT. Graphs for the 18 countries with the highest data quality are shown in Figures 2, 3 and 4. Once again, it seems clear that the steady state profit rate acts as an attractor for the actual rate. These results and others can be viewed via an interactive online system developed by one of the authors (Tajaddinov, 2009). In addition to graphs, this system generates output of the sort shown in Table 1, which quantifies the effectiveness of the steady-state rate as a predictor of the actual rate.

Table 1 contains two pairs of columns, one based on the raw or unfiltered data and one based on a filtered version of the data. The filtering has two aspects. First, the EPWT contains two variant measures of the growth of labour productivity: one unadjusted and one smoothed via a locally weighted regression of the 'loess' type. The latter is intended to purge fluctuations in labour productivity at business cycle frequencies. In our 'filtered' calculations we use the smoothed productivity series. Secondly, the filtered results employ a moving average for both the actual and the steady-state rate of profit. Specifically, the final filter is a three-year weighted moving average as shown in equation (8), where x_t denotes the raw data and $f(x_t)$ the filtered series.

$$f(x_t) = 0.5x_t + 0.3x_{t-1} + 0.2x_{t-2} \tag{8}$$

As the filter involves no look-ahead, we assume it is suitable for the purposes of forecasting. Additionally, in the filtered case any data values missing at source are interpolated using the straight line method. The unfiltered columns offer results that have been subjected to no such filtering. Note that the plots in Figures 2, 3 and 4 employ the filtered data.

The basic rationale for filtering is that the mechanism we describe in section 2 is, in the real world, overlaid by business cycle fluctuations and other noise, partially obscuring the relationship that we seek to highlight.

The 'Attractor ratio' in Table 1 measures the proportion of annual observations for which the equilibrium rate is an effective leading indicator for the actual rate—that is, the motion of the actual rate is towards the equilibrium rate. More formally, the criterion is that either of the biconditionals 9 and 10 is satisfied in year t (where $R^\star$ denotes the equilibrium rate and R the actual rate).

$$R_t^\star \geq R_t \iff R_{t+1} \geq R_t \tag{9}$$

$$R_t^\star \leq R_t \iff R_{t+1} \leq R_t \tag{10}$$

The 'Lag' value in Table 1 gives the optimal lag length for predicting the actual profit rate based on the steady-state rate. This is found by sliding the two series against each other in search of the best fit. For this table 'best fit' is defined by minimization of the Mean Absolute Prediction Error (MAPE). Results based on alternative criteria (maximization of the correlation coefficient, minimization of the Mean Square Error) can be viewed by accessing the online system mentioned above (Tajaddinov, 2009).

In Table 1 lag zero of the steady-state rate is never the optimal predictor of the actual rate when using the the filtered data, but it gives the lowest MAPE for half of the countries when using the raw data. This reflects the common impact of high-frequency noise on both series and supports the rationale for filtering.

Panel regression

There remains the question of whether the steady-state profit rate, $R^\star$, really gives additional explanatory power over the actual rate, after taking into account the history of the actual rate itself. We tackled this by means of a panel regression, using data from 16 countries—those with the highest data quality minus Hong Kong and Luxembourg, which we set aside on account of their exceptionally small size and high degree of openness. In this context we used the smoothed series for growth of labour productivity from the EPWT but performed no further filtering of the data.

Table 4 shows the results of pooled OLS estimates of the current aggregate rate, R_t, using three yearly lags of both R and $R^\star$ as regressors, along with a constant and, for good measure, a linear time trend. The minimum time-series length of the country samples was 31 years and the maximum 37. The standard errors reported are of the heteroskedasticity- and autocorrelation-robust (HAC) type, as proposed for panel data by Arellano (2003).

Pooled OLS imposes the assumption of a common intercept. As it happens this hypothesis is not rejected: the test statistic, derived from a fixed-effects regression that allows the intercept to differ by country, is $F(15, 555) = 0.660$ with a P-value of 0.824. Furthermore, the R^2 value of 0.973 suggests that we are not doing too much violence to the data in imposing a common specification across the 16 countries in the sample.

It can be seen that $R^\star$ is strongly significant even in the presence of the lags of R_t. The individual P-value for $R^\star_{t-1}$ is 3.96×10^{-17} and the Wald test statistic for all lags of $R^\star$ is $\chi^2(3) = 436.5$, with P-value 2.77×10^{-94}.[3] The panel regression therefore amply confirms the impression given by the per-country plots, namely that the steady-state rate of profit has significant predictive value for the actual rate.

4 Conclusion

We have shown that a parsimonious representation of the steady-state rate of profit serves as a remarkably good predictor of the actual rate, over a horizon of a year or two. It would appear that a small set of fundamental factors—population growth, growth of labour productivity, the fraction of the economic surplus that is reinvested—effectively governs the evolution of actual national profit rates over time. A useful extension of the research presented here would be to investigate the connection between the “actual rate of profit” as we have defined it—which is admittedly a very broad construct—and some measure of the average of the rates of profit accruing, in ordinary accounting terms, to the particular capitals in each national economy.

References

Arellano, M. 2003. *Panel Data Econometrics*, Oxford: Oxford University Press

Cockshott, P., A. Cottrell, G. Michaelson, I. Wright and V. Yakovenko 2009. *Classical Econophysics*, London: Routledge

Cottrell, A. and P. Cockshott 2006. ‘Demography and the falling rate of profit’, *Indian Development Review* 4(1): 39–59

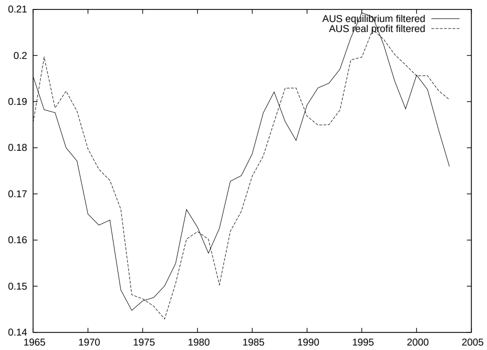
Heston, A., R. Summers and B. Aten 2006. ‘Penn World Table Version 6.2’. Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania

Marquetti, A. 2009. ‘Extended Penn World Tables: Economic Growth Data on 118 Countries’. `http://homepage.newschool.edu/~foleyd/epwt/`

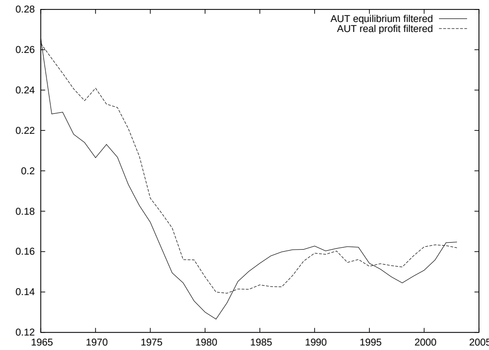
Tajaddinov, T. 2009. ‘Equilibrium rate of profit’. `http://compbio.dcs.gla.ac.uk/profits`

Zachariah, D. 2008. ‘Determinants of the average profit rate and the trajectory of capitalist economies’. Presented at the conference on Probabilistic Political Economy, Kingston University, July 2008

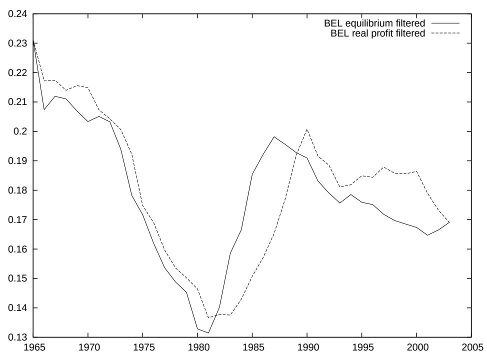
[3] The Wald χ^2 statistic tests the restriction that the true coefficients are zero for all lags of $R^\star$. It is calculated from the robust estimate of the covariance matrix of the coefficients.



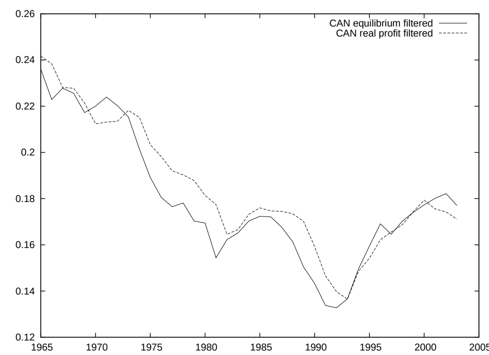
(a) Australia



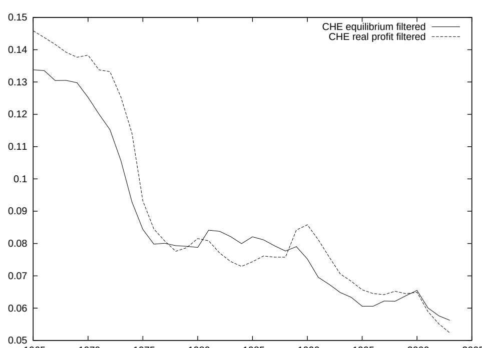
(b) Austria



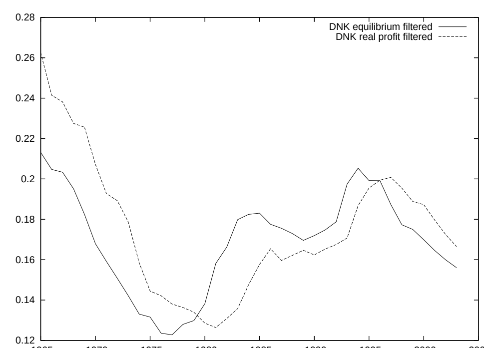
(c) Belgium



(d) Canada

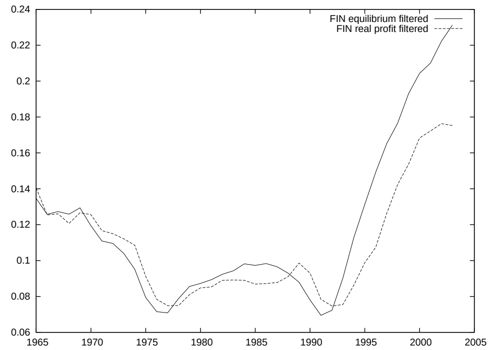


(e) Switzerland

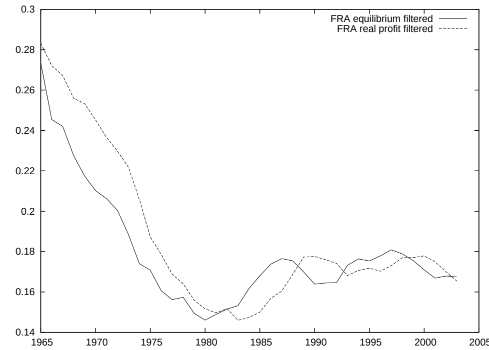


(f) Denmark

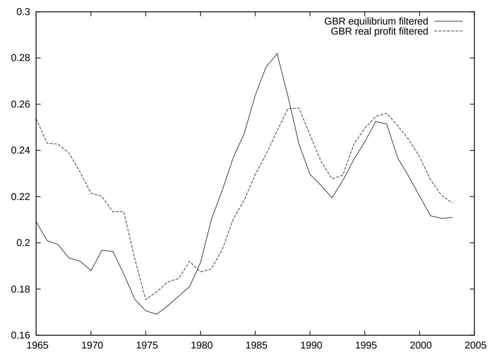
Figure 2: Trajectories of the steady-state and the actual rate of profit



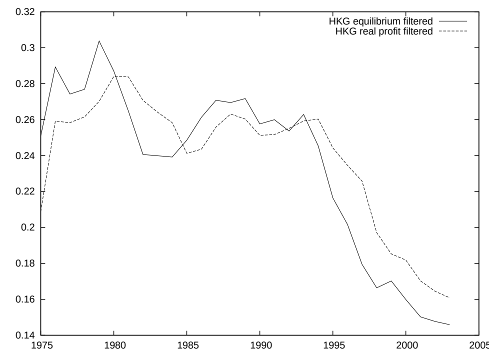
(a) Finland



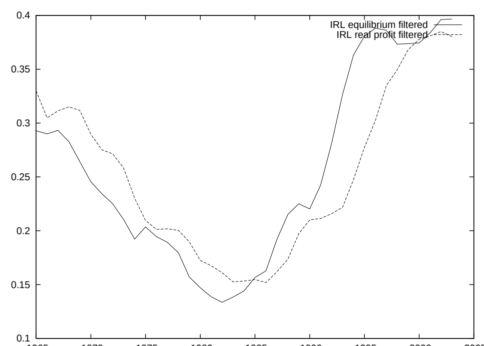
(b) France



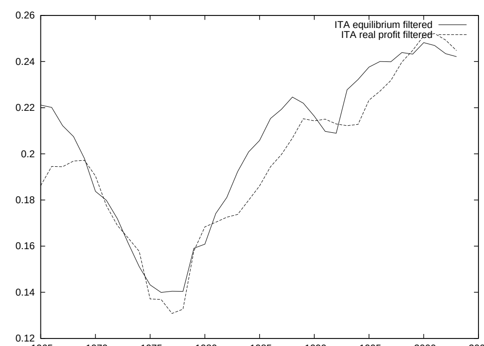
(c) Great Britain



(d) Hong Kong

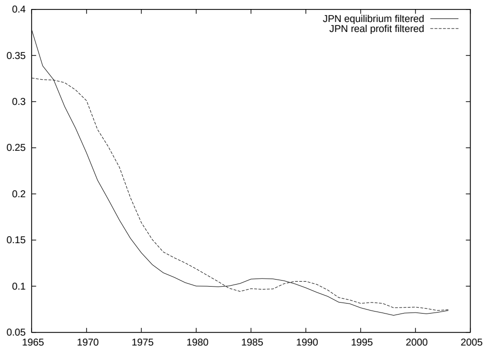


(e) Ireland

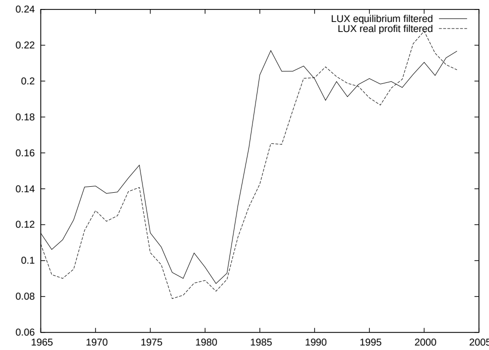


(f) Italy

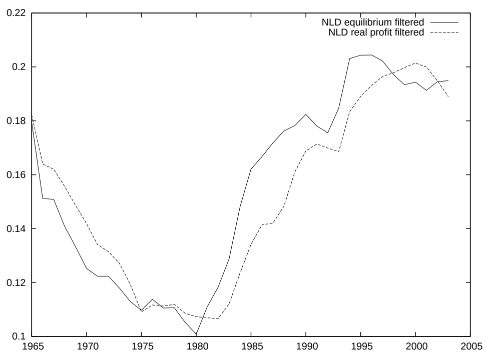
Figure 3: Trajectories of the steady-state and the actual rate of profit



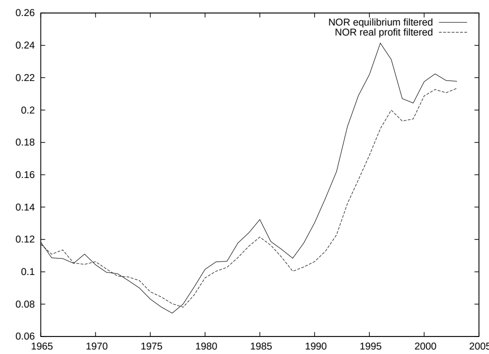
(a) Japan



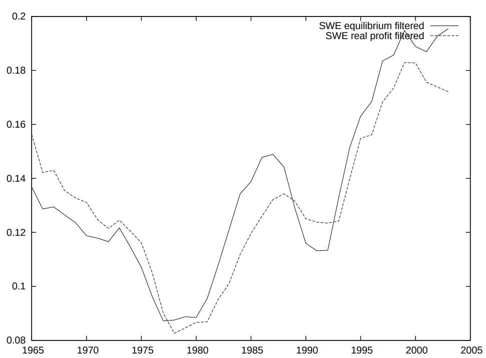
(b) Luxembourg



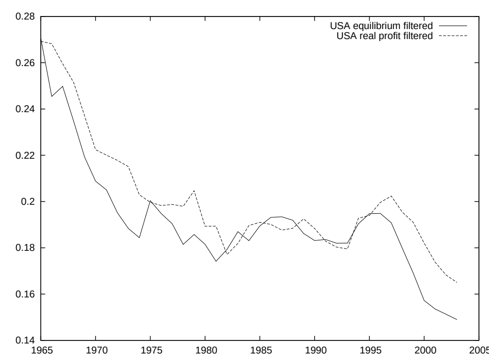
(c) Netherlands



(d) Norway



(e) Sweden



(f) USA

Figure 4: Trajectories of the steady-state and the actual rate of profit

Country	Filtered data Attractor Ratio	Lag	Unfiltered data Attractor Ratio	Lag
Australia	0.82	1	0.59	0
Austria	0.64	2	0.62	1
Belgium	0.77	2	0.62	1
Canada	0.79	1	0.64	1
Switzerland	0.67	2	0.67	1
Denmark	0.87	3	0.69	3
Finland	0.79	2	0.46	0
France	0.87	3	0.46	0
Great Britain	0.72	2	0.72	1
Hong Kong	0.79	2	0.62	0
Ireland	0.77	3	0.62	1
Italy	0.72	1	0.64	0
Japan	0.74	2	0.62	1
Luxembourg	0.62	1	0.51	0
Netherlands	0.79	2	0.60	1
Norway	0.77	2	0.57	0
Sweden	0.77	1	0.54	0
USA	0.69	2	0.56	0

Table 1: Shows the proportion of years for which the steady-state rate of profit is a leading indicator for the actual rate, and the lag (in years) at which the series match most closely, for 18 countries with 'grade A' data quality in EPWT.

Pooled OLS for R_t, total observations used = 578
Robust (HAC) standard errors

	Coefficient	Std. Error	t-ratio	P-value
const	−0.0010706	0.00046264	−2.314	0.0210
R_{t-1}	0.69237	0.044327	15.620	0.0000
R_{t-2}	−0.12339	0.047770	−2.583	0.0100
R_{t-3}	0.084726	0.028862	2.936	0.0035
$R^{\star}_{t-1}$	0.43344	0.049901	8.686	0.0000
$R^{\star}_{t-2}$	−0.12430	0.055806	−2.227	0.0263
$R^{\star}_{t-3}$	0.034550	0.051676	0.669	0.5040
time	3.9702e–05	2.1096e–05	1.882	0.0604

Mean dependent var	0.164924	S.D. dependent var	0.057493
Sum squared resid	0.051316	S.E. of regression	0.009488
R^2	0.973094	Adjusted R^2	0.972764
$F(7, 570)$	2945.036	P-value(F)	0.000000
$\hat{\rho}$	−0.000961	Durbin–Watson	1.949943

Table 2: Panel-data regression of the current aggregate profit rate on three lags of the current rate and three lags of the steady-state rate, plus a time trend