

Generating Initial Data for Interacting Compact Binaries

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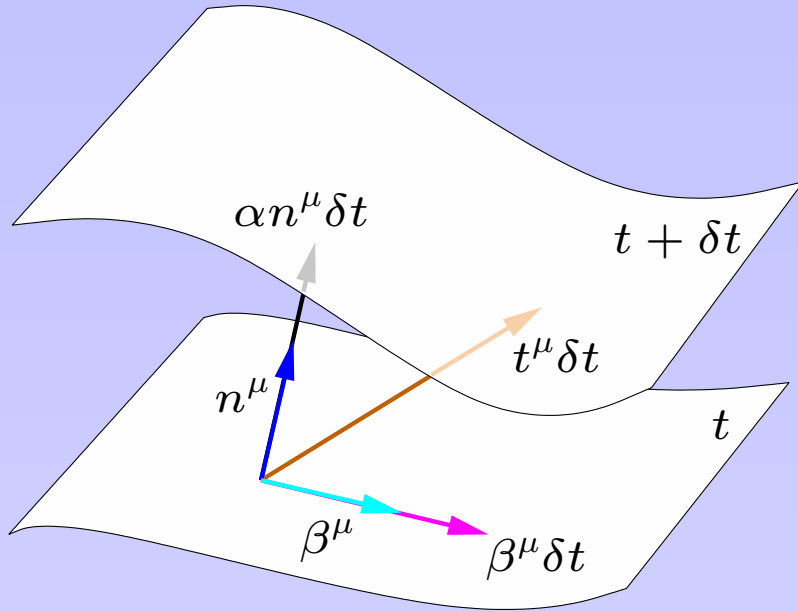
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Abstract

We will look at some of the general formalisms used to construct initial data and review the approaches that have been used in generating black-hole and neutron-star initial data. We will conclude with a brief look at some recent work in generating quasi-equilibrium black-hole binary initial data.

A related [review article](#) is online at [Living Review in Relativity](#)[18]

The 3 + 1 Decomposition



Lapse : α

Spatial metric : γ_{ij}

Shift vector : β^i

Extrinsic Curvature : K_{ij}

Time vector : $t^\mu = \alpha n^\mu + \beta^\mu$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \quad K_{\mu\nu} = -\frac{1}{2}\gamma_\mu^\alpha \gamma_\nu^\beta \mathcal{L}_n g_{\alpha\beta}$$

Constraint equations

$$\bar{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$\bar{\nabla}_j (K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

$$S_{\mu\nu} \equiv \gamma_\mu^\alpha \gamma_\nu^\beta T_{\alpha\beta}$$

$$j_\mu \equiv -\gamma_\mu^\nu n^\alpha T_{\nu\alpha}$$

$$\rho \equiv n^\mu n^\nu T_{\mu\nu}$$

$$T_{\mu\nu} = S_{\mu\nu} + 2n_{(\mu}j_{\nu)} + n_\mu n_\nu \rho$$

Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

$$\partial_t K_{ij} = -\bar{\nabla}_i \bar{\nabla}_j \alpha + \alpha \left[\bar{R}_{ij} - 2K_{il}K_j^l + K K_{ij} - 8\pi S_{ij} + 4\pi \gamma_{ij}(S - \rho) \right]$$

$$+ \beta^\ell \bar{\nabla}_\ell K_{ij} + K_{il} \bar{\nabla}_j \beta^\ell + K_{jl} \bar{\nabla}_i \beta^\ell$$

Generalized Conformal/TT Initial-Data Decomposition[46]

$$\begin{aligned}\gamma_{ij} &= \psi^4 \tilde{\gamma}_{ij} & : & & K^{ij} &= \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K & : & & \tilde{A}^{ij} &\equiv \frac{1}{\tilde{\sigma}} (\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij} \\ \tilde{\sigma} &\equiv \psi^{-6} \sigma & : & & \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} &= -2\pi \psi^5 \rho \\ \tilde{\nabla}_j (\tilde{\mathbb{L}}V)^{ij} - (\tilde{\mathbb{L}}V)^{ij} \tilde{\nabla}_j \ln \tilde{\sigma} - \frac{2}{3} \tilde{\sigma} \psi^6 \tilde{\nabla}^i K &= -\tilde{\sigma} \tilde{\nabla}_j \tilde{M}^{ij} + 8\pi \tilde{\sigma} \psi^{10} j^i\end{aligned}$$

\tilde{M}^{ij} is symmetric-tracefree, but not divergenceless. The variable V^i incorporates the solution of the constraints and the decomposition of \tilde{M}^{ij} into \tilde{M}_{TT}^{ij} .

$\tilde{\sigma} = 1 \Rightarrow$ Conf-TT Method (Method A)

$\sigma = 1 \Rightarrow$ Phys-TT Method (Method B)

$\tilde{M}^{ij} \Rightarrow -\frac{1}{2\tilde{\alpha}} \tilde{u}^{ij}$ Conf. Thin Sandwich
 $\tilde{\sigma} \Rightarrow 2\tilde{\alpha}$

Conformal Thin Sandwich(TS)[59]

$$[\tilde{u}^{ij} \equiv -\partial_t \tilde{\gamma}^{ij}]$$

$$\begin{aligned}\gamma_{ij} &= \psi^4 \tilde{\gamma}_{ij} & : & & K^{ij} &= \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K & : & & \tilde{A}^{ij} &\equiv \frac{1}{2\tilde{\alpha}} \left((\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij} \right) \\ \tilde{\alpha} &\equiv \psi^{-6} \alpha & : & & \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} &= -2\pi \psi^5 \rho \\ \tilde{\nabla}_j (\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \ln \tilde{\alpha} - \frac{4}{3} \tilde{\alpha} \psi^6 \tilde{\nabla}^i K &= \tilde{\alpha} \tilde{\nabla}_j \left(\frac{1}{\tilde{\alpha}} \tilde{u}^{ij} \right) + 16\pi \tilde{\alpha} \psi^{10} j^i \\ \tilde{\nabla}^2 (\psi^7 \tilde{\alpha}) - \frac{1}{8} \psi^7 \tilde{\alpha} \tilde{R} - \frac{5}{12} \psi^{11} \tilde{\alpha} K^2 - \frac{7}{8} \psi^{-1} \tilde{\alpha} \tilde{A}_{ij} \tilde{A}^{ij} - \psi^5 \beta^i \tilde{\nabla}_i K &= -2\pi \psi^{11} \tilde{\alpha} K (\rho + 2S) - \psi^5 \partial_t K\end{aligned}$$

α and β^i are the lapse and shift. \tilde{u}^{ij} is symmetric-tracefree.

Degrees of Freedom

Kinematical variables

- Lapse α : 1 degree of freedom
- Shift β^i : 3 degrees of freedom

Initial-data variables

- Metric γ_{ij} : 6 degrees of freedom
- Extrinsic curvature K_{ij} : 6 degrees of freedom

Decomposition of initial-data variables

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \left\{ \begin{array}{l} \psi : 1 \text{ constrained DOF} \\ \tilde{\gamma}_{ij} : \left\{ \begin{array}{l} 3 \text{ spatial gauge DOF} \\ 2 \text{ dynamical DOF} \end{array} \right\} \end{array} \right\} \text{Freely Specifiable}$$

$$K^{ij} = \psi^{-10} \left[\frac{1}{\tilde{\sigma}} (\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij} \right] + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K \left\{ \begin{array}{l} V^i : \left\{ \begin{array}{l} (\tilde{\mathbb{L}}V)^{ij} \equiv \tilde{\nabla}^i V^j + \tilde{\nabla}^j V^i - \frac{1}{3} \tilde{\gamma}^{ij} \tilde{\nabla}_k V^k \\ 3 \text{ constrained DOF} \end{array} \right\} \\ \tilde{M}^{ij} : \left\{ \begin{array}{l} \tilde{\nabla}_j \tilde{M}^{ij} = \tilde{\nabla}_j \left(\frac{1}{\tilde{\sigma}} (\tilde{\mathbb{L}}X)^{ij} \right) \\ 2 \text{ dynamical DOF} \end{array} \right\} \\ K : 1 \text{ temporal gauge DOF} \\ \tilde{\sigma} : \text{Defn. of TT decomp.} \end{array} \right\} \text{Freely Specifiable}$$

“Traditional” Black-Hole Data

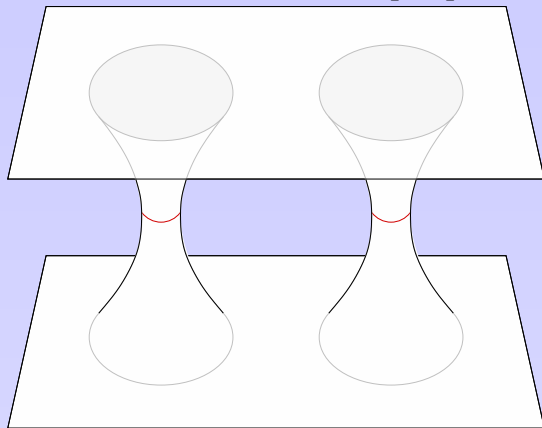
Conformal flatness and maximal slicing

$$\left. \begin{aligned} \tilde{\gamma}_{ij} &= f_{ij} \text{ (flat)} \\ \tilde{M}^{ij} &= 0 \\ K &= 0 \\ \tilde{\sigma} &= 1 \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \tilde{\nabla}_i (\tilde{\mathbb{L}}V)^{ij} &= 0 \Rightarrow \text{Bowen-York solution [11]} \\ &\text{Analytic solutions for } \tilde{A}^{ij} \end{aligned} \right.$$

$$\tilde{\nabla}^2 \psi + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0$$

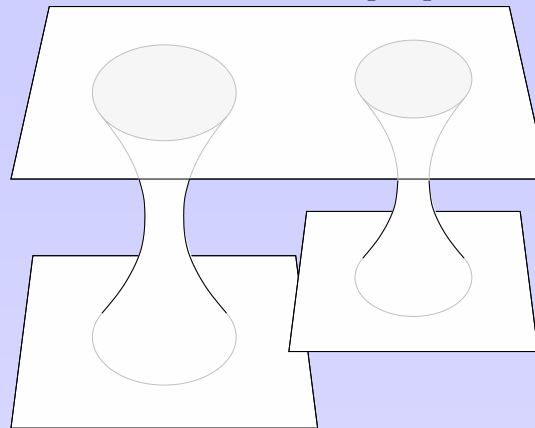
Three general solution schemes

Conformal Imaging-[21]



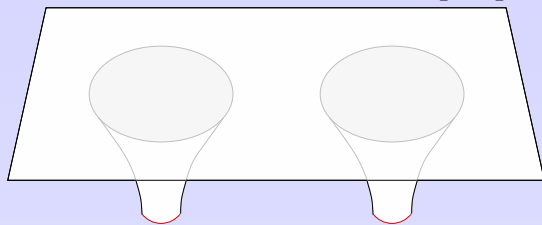
Inversion
symmetry
inner-BC

Puncture Method-[12]



No inner-BC:
singular
behavior
factored out

Apparent Horizon BC-[51]



Apparent
horizon
inner-BC

All methods can produce very general configurations of multiple black holes, but are fundamentally limited by choices for $\tilde{\gamma}_{ij}$ and Bowen-York \tilde{A}^{ij} .

Early work with BH and NS ID

- The “traditional” BH initial data approach was motivated by *computational convenience*, not by any strong physical arguments. Research focused on:
 - methods for solving the Hamiltonian constraint for one or two holes. [60, 15, 16, 21, 1, 37, 45, 27]
 - understanding the physical content (and limitations) of initial data containing one or two holes. [60, 25, 16, 20]
 - finding solutions that represented two black holes in nearly circular orbits. [17, 2, 45]
- Boosted Kerr — Use a superposition of analytic black hole metrics and extrinsic curvatures as trial data and solve the constraints to correct. [43, 42]
 - This approach lacks well motivated boundary conditions
- Single, stationary neutron stars have been studied extensively. The matter is assumed to be in hydrostatic equilibrium. Research has focused on:
 - various numerical methods. [10, 14, 13, 38, 22, 24, 48, 9, 32, 5, 7]
 - rigid and differential rotation. [56, 39, 22]
 - studies with various equations of state. [29, 23]

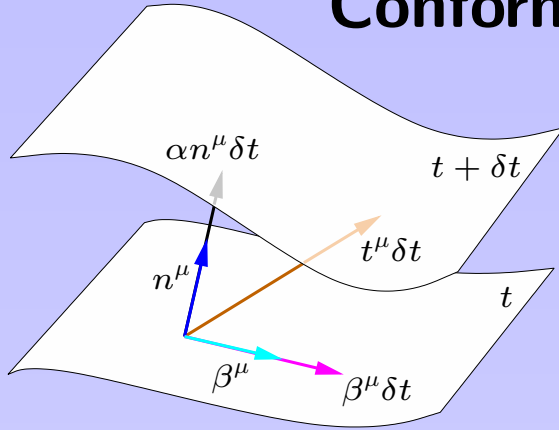
Recent Issues

- Current initial data schemes do not represent, with sufficient detail, the *physical configurations* we are most interested in — **astrophysical compact binary inspiral**.
 - The conformal-flatness assumption does not represent rapidly spinning objects well. It also contributes to unphysical gravity waves in orbiting binaries.[30, 45]
 - The Bowen-York analytic extrinsic curvature does not model the physics of binary inspiral well.[36, 44, 26]
 - Boosted Kerr extrinsic curvature does not do significantly better than the Bowen-York extrinsic curvature.[44]
- ★ Quasi-equilibrium — For sufficiently separated binaries, the timescale for orbital decay is much larger than the orbital period.
 - Binaries in quasi-circular orbits allow for the notion of an approximate helical Killing vector.

Compact Binary ID

- NS binaries were explored first. Research has focused on:
 - various numerical methods.[57, 58, 6, 3, 34, 35]
 - corotating (tidally locked) binaries.[6, 3, 4, 55, 52, 40, 55, 49]
 - irrotational binaries.[50, 47, 31, 8, 41, 52, 53, 49]
 - no conformal flatness assumption.[54]
- For black holes, use topology and quasi-equilibrium to fix boundary conditions on the constrained data.[33, 36] *Leads to overdetermined boundary conditions — only satisfied in true equilibrium.*
- ★ For black holes, use the principle of *quasi-equilibrium* to derive boundary conditions for the constrained data.[19, 28]

Conformal Thin-Sandwich Decomposition



$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

$$K^{ij} = \frac{\psi^{-10}}{2\tilde{\alpha}} \left[(\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij} \right] + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K$$

Hamiltonian Const. $\tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho$

Momentum Const. $\tilde{\nabla}_j (\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \tilde{\alpha} = \frac{4}{3} \tilde{\alpha} \psi^6 \tilde{\nabla}^i K + \tilde{\alpha} \tilde{\nabla}_j \left(\frac{1}{\tilde{\alpha}} \tilde{u}^{ij} \right) + 16\pi \tilde{\alpha} \psi^{10} j^i$

Const. Tr(K) eqn. $\tilde{\nabla}^2 (\psi^7 \tilde{\alpha}) - (\psi^7 \tilde{\alpha}) \left[\frac{1}{8} \psi \tilde{R} + \frac{5}{12} \psi^5 K^2 + \frac{7}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} - \psi^5 \beta^i \tilde{\nabla}_i K \right]$
 $= -2\pi \psi^5 K (\rho + 2S) - \psi^5 \partial_t K$

$$\tilde{A}^{ij} \equiv \frac{1}{2\tilde{\alpha}} \left[(\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij} \right]$$

Constrained vars : ψ and β^i

Freely specified : $\tilde{\gamma}_{ij}$, \tilde{u}^{ij} , K , and $\partial_t K$

\tilde{u}^{ij} and β^i have a simple physical interpretation, unlike \tilde{M}^{ij} and V^i .

$$\text{Quasi-equilibrium} \Rightarrow \begin{cases} \tilde{u}^{ij} = -\partial_t \tilde{\gamma}^{ij} = 0 \\ \partial_t K = 0 \end{cases}$$

Equations of Quasi-Equilibrium

$$\left. \begin{array}{l} \text{Ham. \& Mom. const.} \\ \text{eqns., \& Const Tr}(K) \\ \text{eqn. from Conf. TS} \\ + \tilde{u}^{ij} = \partial_t K = 0 \end{array} \right\} \Rightarrow \text{Eqns. of Quasi-Equilibrium}$$

With $\tilde{\gamma}_{ij} = f_{ij}$ and $K = 0$, these equations have been widely used to construct binary neutron star initial data [3, 40, 8, 52].

Binary neutron star initial data require:

- boundary conditions at infinity compatible with asymptotic flatness and corotation.

$$\psi|_{r \rightarrow \infty} = 1 \quad \beta^i|_{r \rightarrow \infty} = \Omega \left(\frac{\partial}{\partial \phi} \right)^i \quad \alpha|_{r \rightarrow \infty} = 1$$

- compatible solution of the equations of hydrostatic equilibrium. ($\Rightarrow \Omega$)

Binary black hole initial data require:

- a means for choosing the angular velocity of the orbit Ω .

★ *with excision*, inner boundary conditions are needed for ψ , β^i , and $\tilde{\alpha}$.

Gourgoulhon, Grandclément, & Bonazzola [33, 36]: Black-hole binaries with $\tilde{\gamma}_{ij} = f_{ij}$ & $K = 0$, “inversion-symmetry”, and “Killing-horizon” conditions on the excision boundaries.

“Solutions” require *constraint violating* regularity condition imposed on inner boundaries!

AH and QE Conditions on the Inner Boundary

The quasi-equilibrium inner boundary conditions start with the following assumptions:

1. The inner boundary \mathcal{S} is a (MOTS):
marginally outer-trapped surface

$$\rightarrow \theta = 0$$

2. The inner boundary \mathcal{S} doesn't move:

$$\rightarrow \mathcal{L}_\zeta \tau = 0 \text{ and } D_i \mathcal{L}_\zeta \tau \equiv h_i^j \bar{\nabla}_j \mathcal{L}_\zeta \tau = 0$$

$$t^\mu = \alpha n^\mu + \beta^\mu \quad \zeta^\mu \equiv \alpha n^\mu + \beta_\perp s^\mu$$

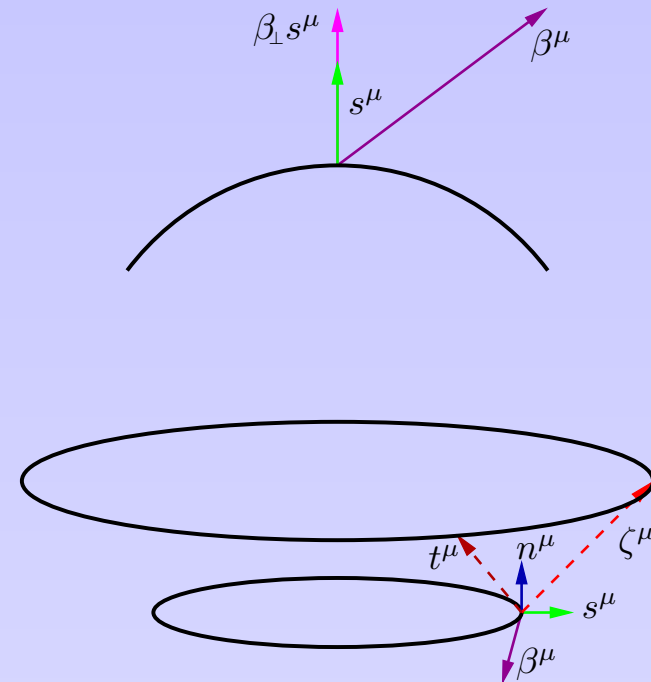
$$\beta_\perp \equiv \beta^i s_i$$

3. The inner boundary \mathcal{S} remains a MOTS:

$$\rightarrow \mathcal{L}_\zeta \theta = 0$$

4. The horizons are in quasi-equilibrium:

$$\rightarrow \sigma_{ij} = 0 \text{ and no matter is on } \mathcal{S}$$



AH/Quasi-Equilibrium Boundary Conditions

$$\theta = \frac{\psi^{-2}}{\sqrt{2}} \left[\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j + 4\tilde{s}^k \tilde{\nabla}_k \ln \psi - \psi^2 J \right]$$

$$\mathcal{L}_\zeta \theta = -\frac{1}{\sqrt{2}} \left[\theta(\theta + \frac{1}{2}\dot{\theta} + \frac{1}{\sqrt{2}}K) + \mathcal{E} \right] (\beta_\perp + \alpha)$$

$$-\frac{1}{\sqrt{2}} \left[\theta(\frac{1}{2}\theta - \frac{1}{2}\dot{\theta} + \frac{1}{\sqrt{2}}K) + \mathcal{D} + 8\pi T_{\mu\nu} k^\mu \acute{k}^\nu \right] (\beta_\perp - \alpha)$$

$$+ \theta s^i \bar{\nabla}_i \alpha$$

$$\mathcal{D} \equiv h^{ij} (D_i + J_i)(D_j + J_j) - \frac{1}{2} R$$

$$\mathcal{E} \equiv \sigma_{ij} \sigma^{ij} + 8\pi T_{\mu\nu} k^\mu k^\nu$$

$$\sigma_{ij} = \frac{1}{\sqrt{2}} (H_{ij} - \frac{1}{2} h_{ij} H) \left(1 - \frac{\beta_\perp}{\alpha} \right)$$

$$-\frac{1}{\sqrt{2}} \frac{\psi^4}{\alpha} \left\{ \tilde{h}_{k(i} \tilde{D}_{j)} \beta_{\parallel}^k - \frac{1}{2} \tilde{h}_{ij} \tilde{D}_k \beta_{\parallel}^k - \frac{1}{2} [\tilde{h}_{ik} \tilde{h}_{jl} \tilde{u}^{kl} - \frac{1}{2} \tilde{h}_{ij} \tilde{h}_{kl} \tilde{u}^{kl}] \right\}$$

$$\tilde{s}^k \tilde{\nabla}_k \ln \psi = -\frac{1}{4} (\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j - \psi^2 J)$$

$$\beta^i = \alpha \psi^{-2} \tilde{s}^i + \beta_{\parallel}^i$$

$$0 = \tilde{D}^{(i} \beta_{\parallel}^{j)} - \frac{1}{2} \tilde{h}^{ij} \tilde{D}_k \beta_{\parallel}^k$$

Summary of BH QE Formalism

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \gamma^{ij} K \quad \tilde{A}^{ij} = \frac{\psi^6}{2\alpha} (\tilde{\mathbb{L}}\beta)^{ij} \quad \partial_t \tilde{\gamma}_{ij} = 0$$

$$\tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0$$

$$\tilde{\nabla}_j (\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \ln \alpha \psi^{-6} = \frac{4}{3} \alpha \tilde{\nabla}^i K$$

$$\tilde{\nabla}^2 (\alpha \psi) - (\alpha \psi) \left[\frac{1}{8} \tilde{R} + \frac{5}{12} \psi^4 K^2 + \frac{7}{8} \psi^{-8} A_{ij} A^{ij} \right] = \psi^5 \beta^i \tilde{\nabla}_i K \quad \partial_t K = 0$$

$$\tilde{s}^k \tilde{\nabla}_k \ln \psi|_S = -\frac{1}{4} (\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j - \psi^2 J)|_S \quad \theta = 0$$

$$\beta^i|_S = \begin{cases} \alpha \psi^{-2} \tilde{s}^i|_S & \text{corotation} \\ \alpha \psi^{-2} \tilde{s}^i|_S + \Omega \xi^i|_S & \text{irrotation} \end{cases}$$

$$\mathcal{L}_\zeta \theta = 0$$

$$\sigma_{ij} = 0$$

$$\alpha|_S = \text{unspecified by QE! ?}$$

$$\psi|_{r \rightarrow \infty} = 1$$

$$\beta^i|_{r \rightarrow \infty} = \Omega \left(\frac{\partial}{\partial \phi} \right)^i$$

$$\alpha|_{r \rightarrow \infty} = 1$$

The only remaining freedom in the system is the choice of the lapse boundary condition, the initial spatial and temporal gauge, and the initial dynamical (“wave”) content found in

$$\alpha|_S, \tilde{\gamma}_{ij} \text{ and } K.$$

New Results

with H. Pfeiffer

Corotation

$\tilde{\gamma}_{ij} = f_{ij}$: Maximal Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \frac{\alpha\psi}{2r}$

$\tilde{\gamma}_{ij} = f_{ij}$: Eddington-Finkelstein Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \alpha\psi$

Compared with

- Effective-One-Body PN[26]
- Inversion-Symmetric HKV[36]

Irrotation

$\tilde{\gamma}_{ij} = f_{ij}$: Maximal Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \frac{\alpha\psi}{2r}$

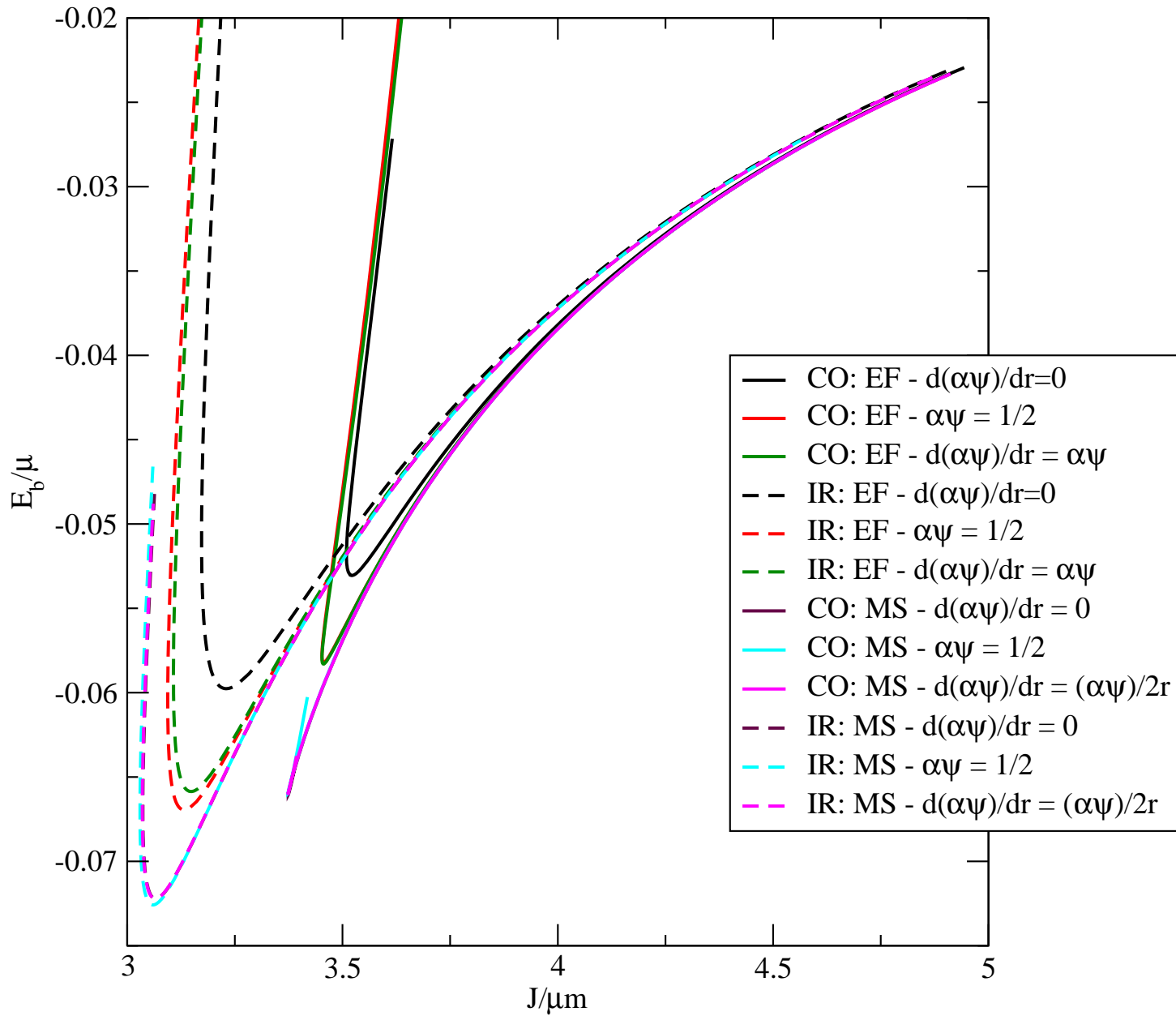
$\tilde{\gamma}_{ij} = f_{ij}$: Eddington-Finkelstein Slicing:

- $\frac{\partial(\alpha\psi)}{\partial r} = 0$
- $\alpha\psi = \frac{1}{2}$
- $\frac{\partial(\alpha\psi)}{\partial r} = \alpha\psi$

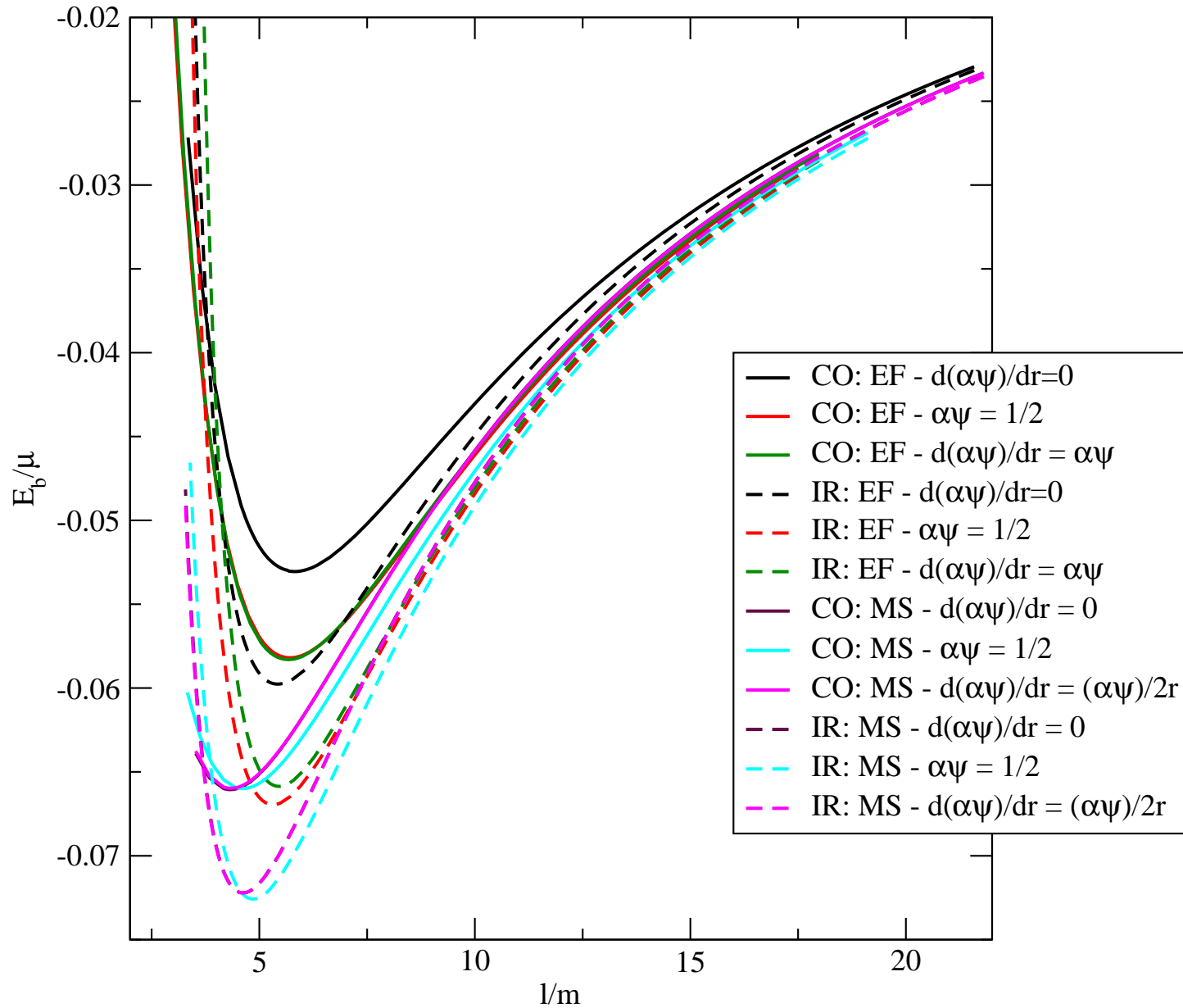
Compared with

- Effective-One-Body PN[26]
- Conformal Imaging[17]
- Puncture Method[2]

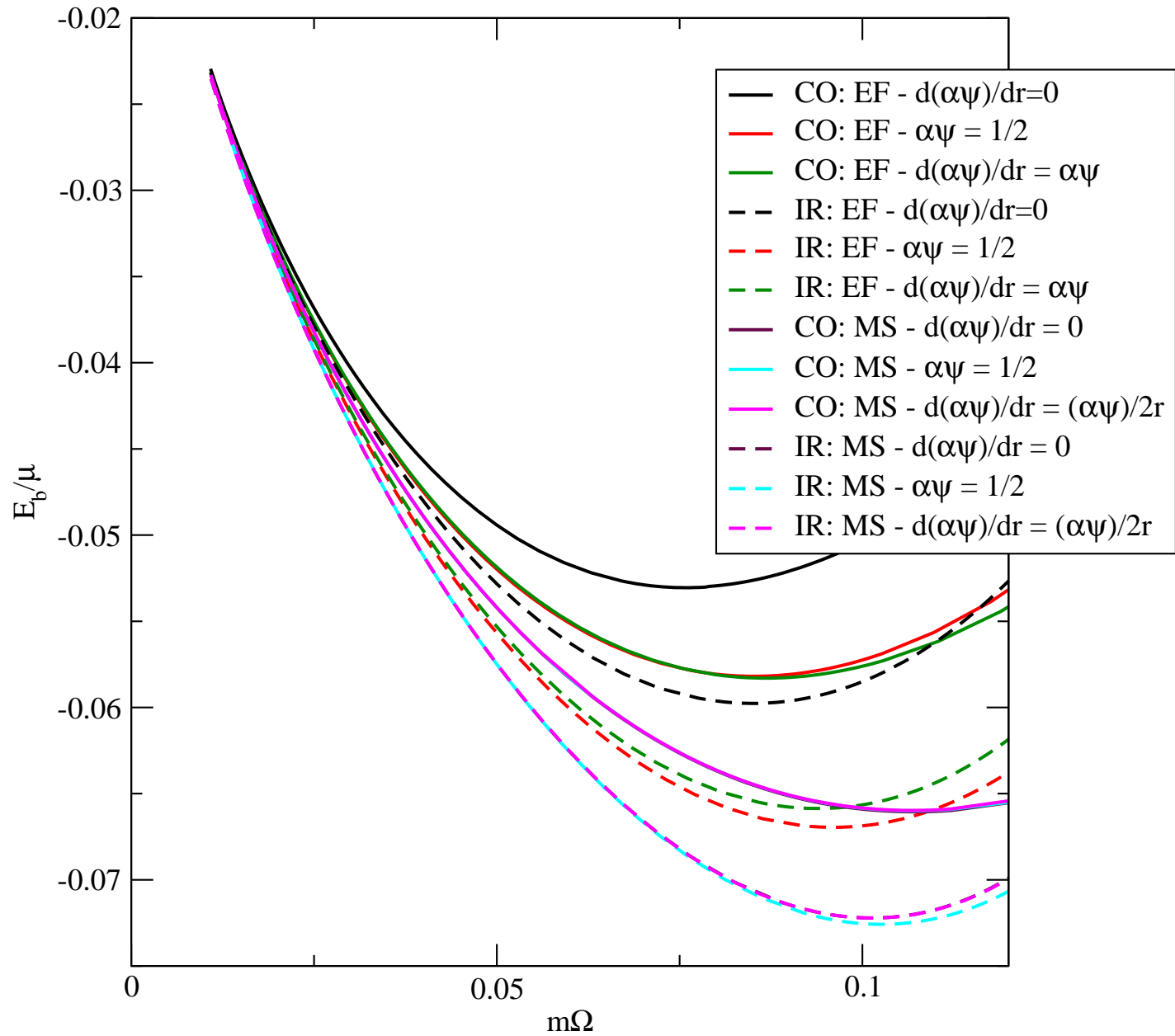
Results: E_b/μ vs $J/\mu m$



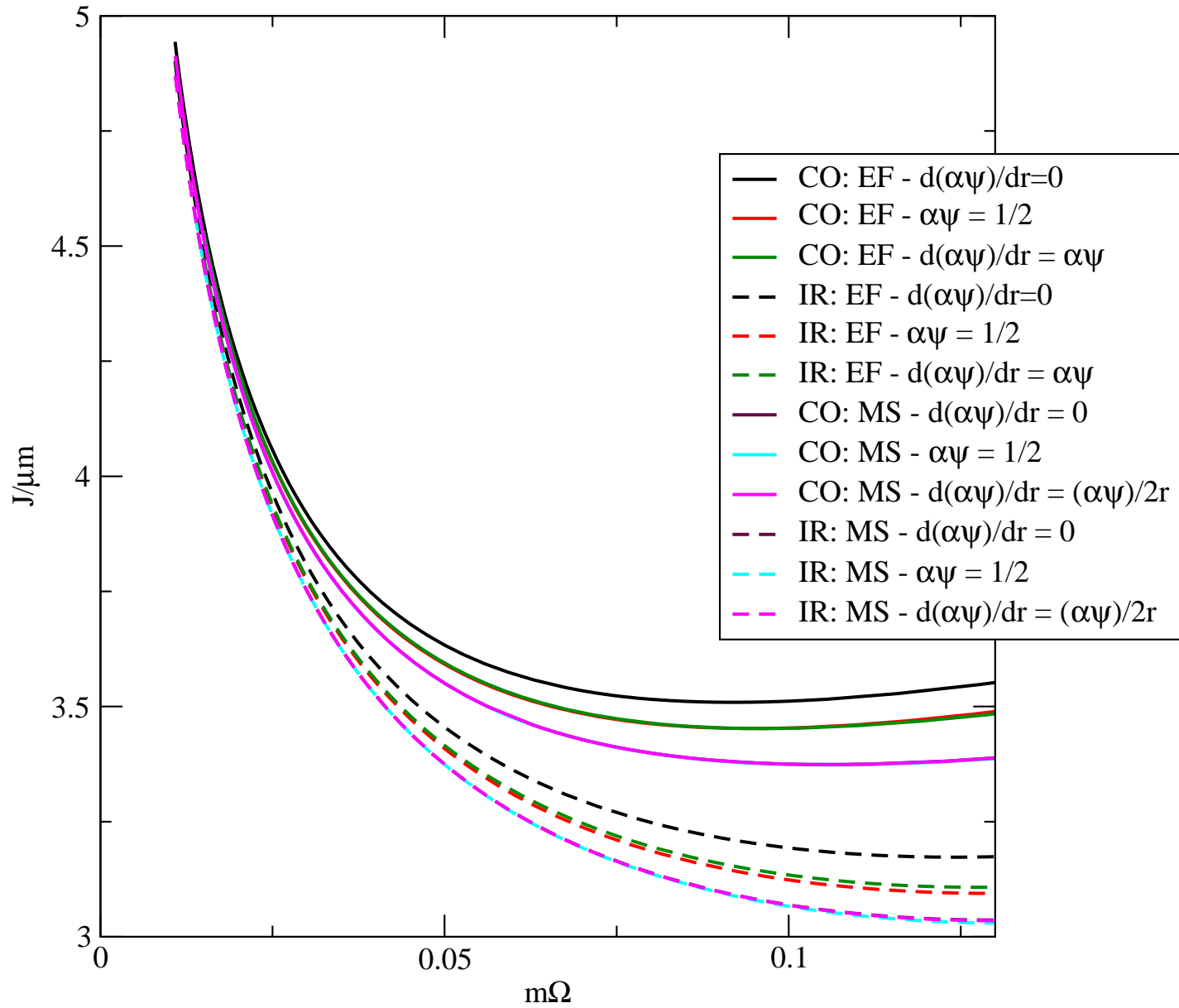
Results: E_b/μ vs ℓ/m



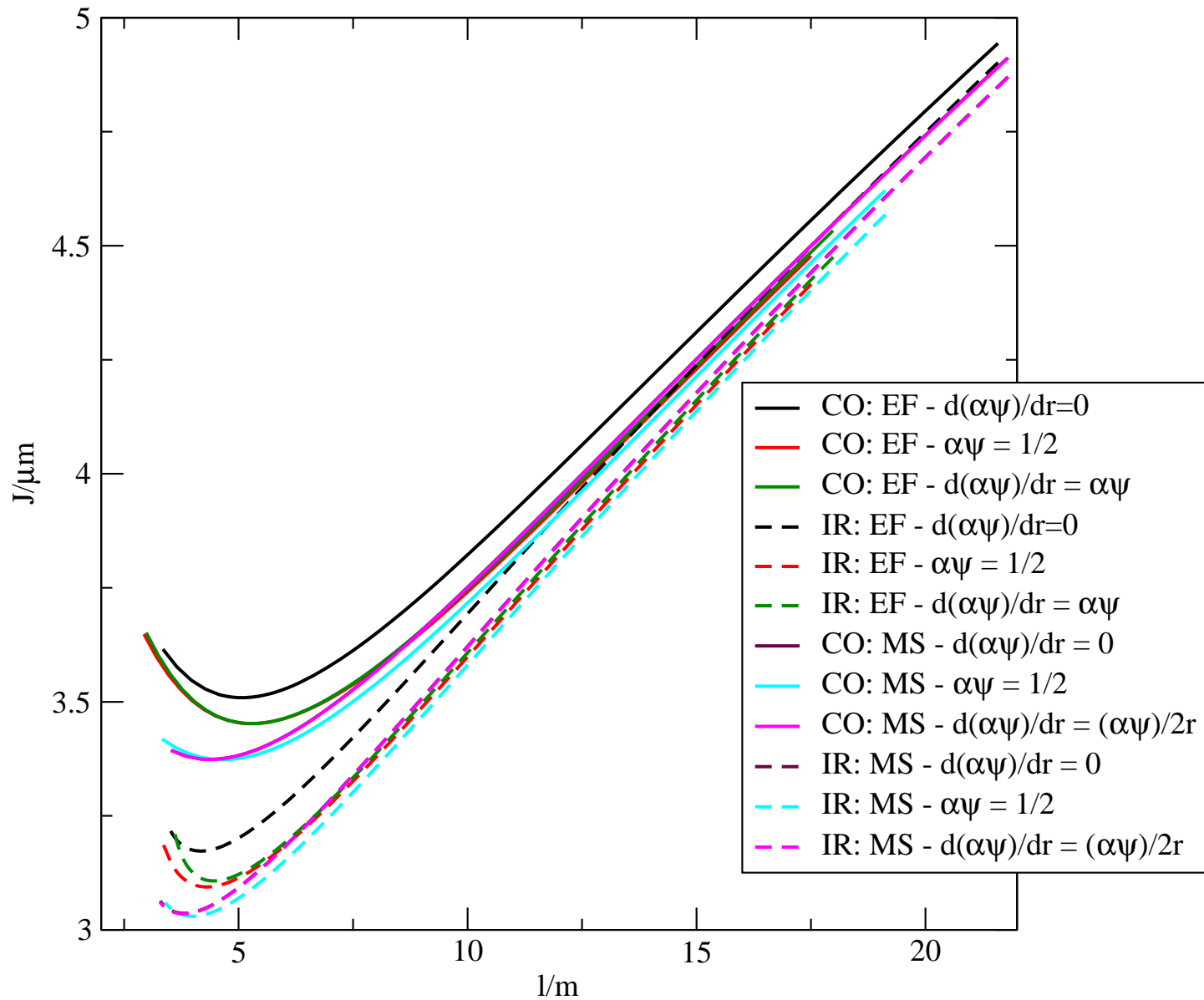
Results: E_b/μ vs $m\Omega$



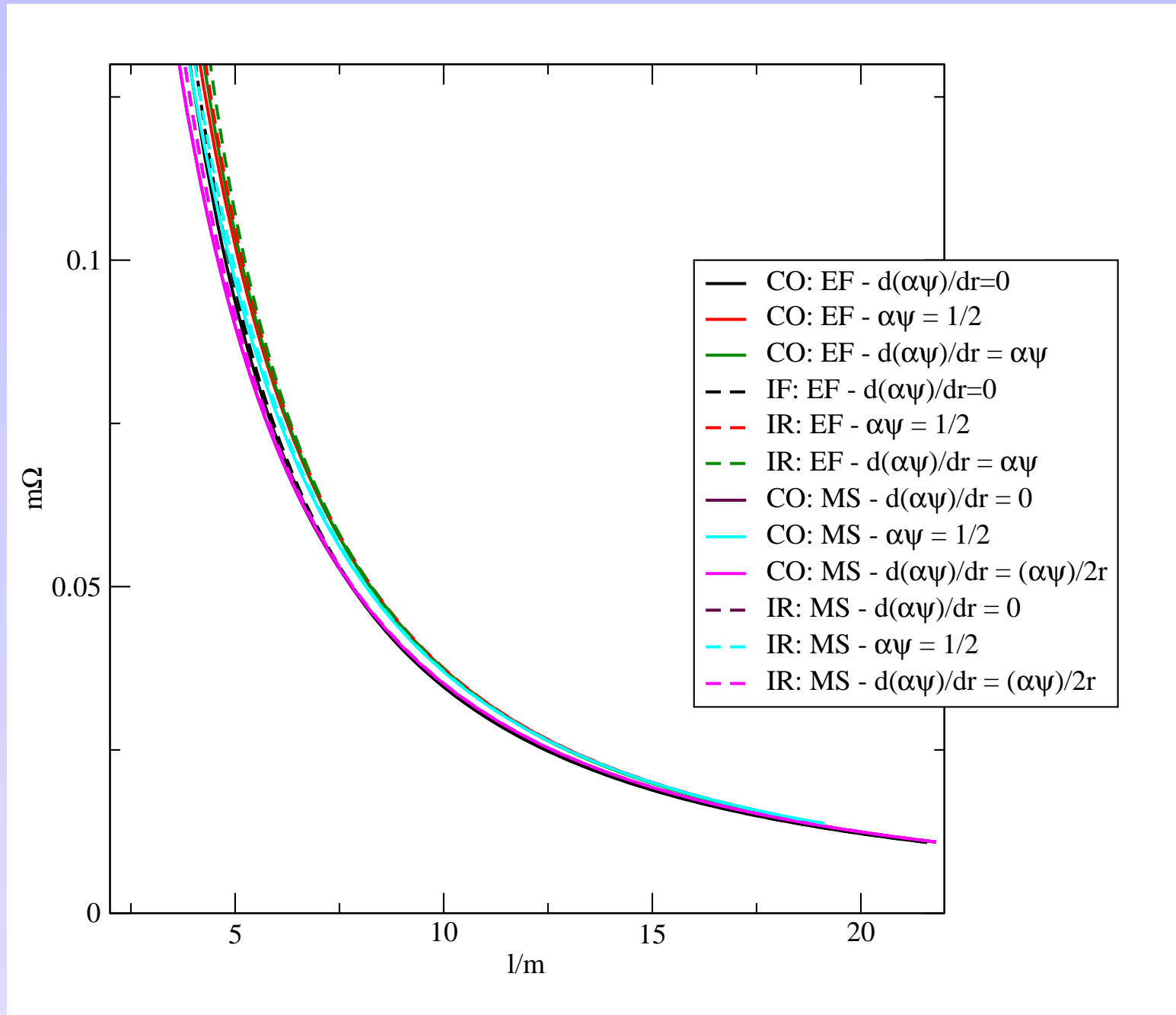
Results: $J/\mu m$ vs $m\Omega$



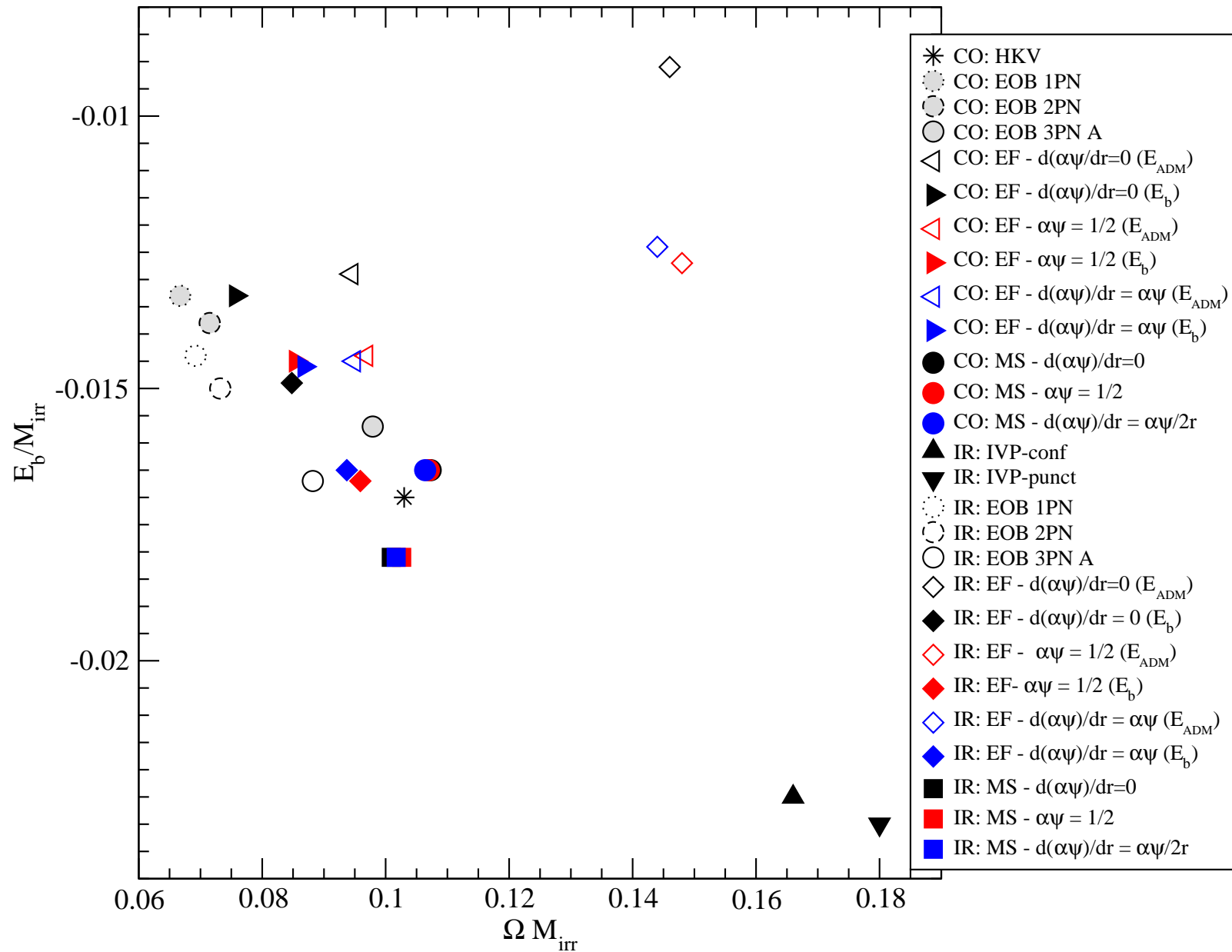
Results: $J/\mu m$ vs ℓ/m



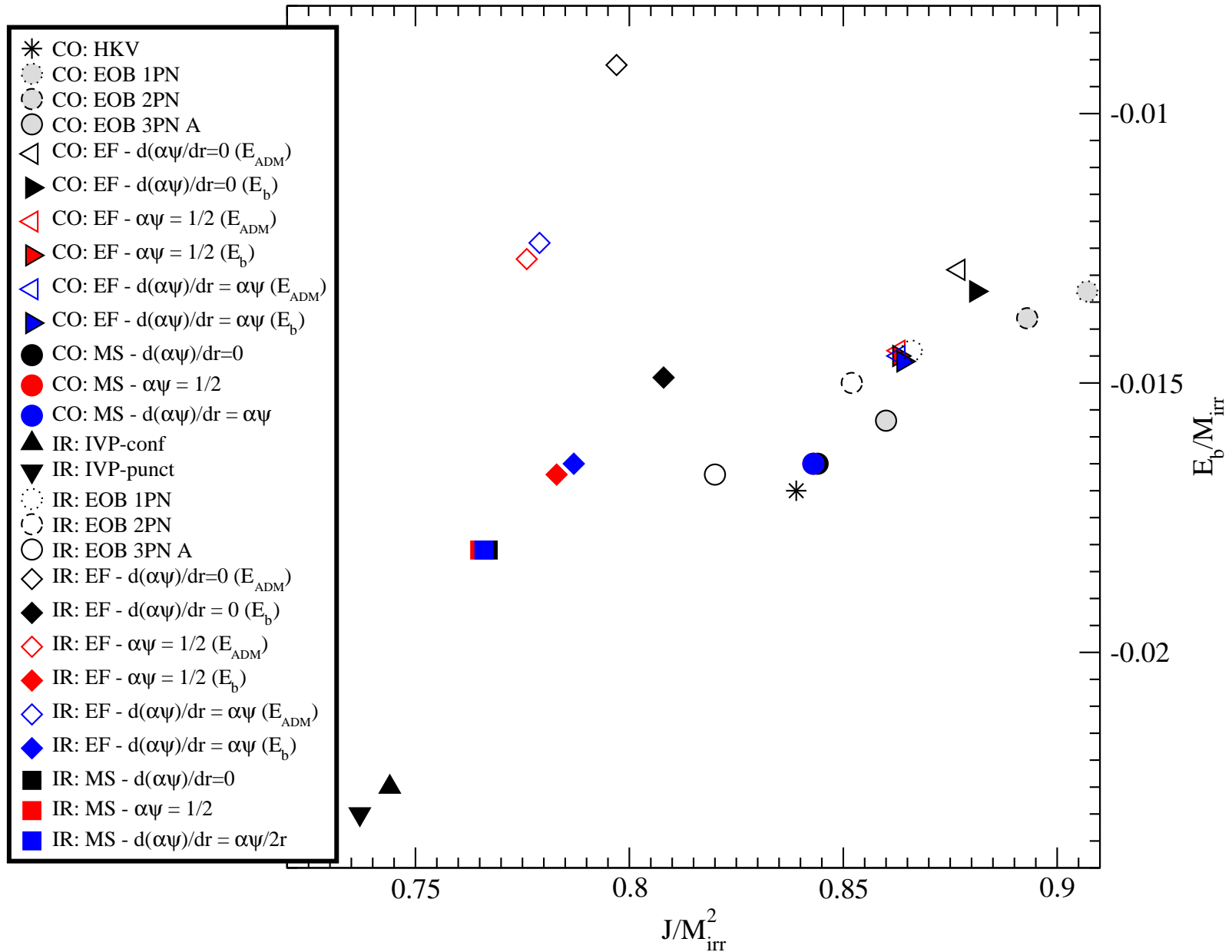
Results: $m\Omega$ vs l/m



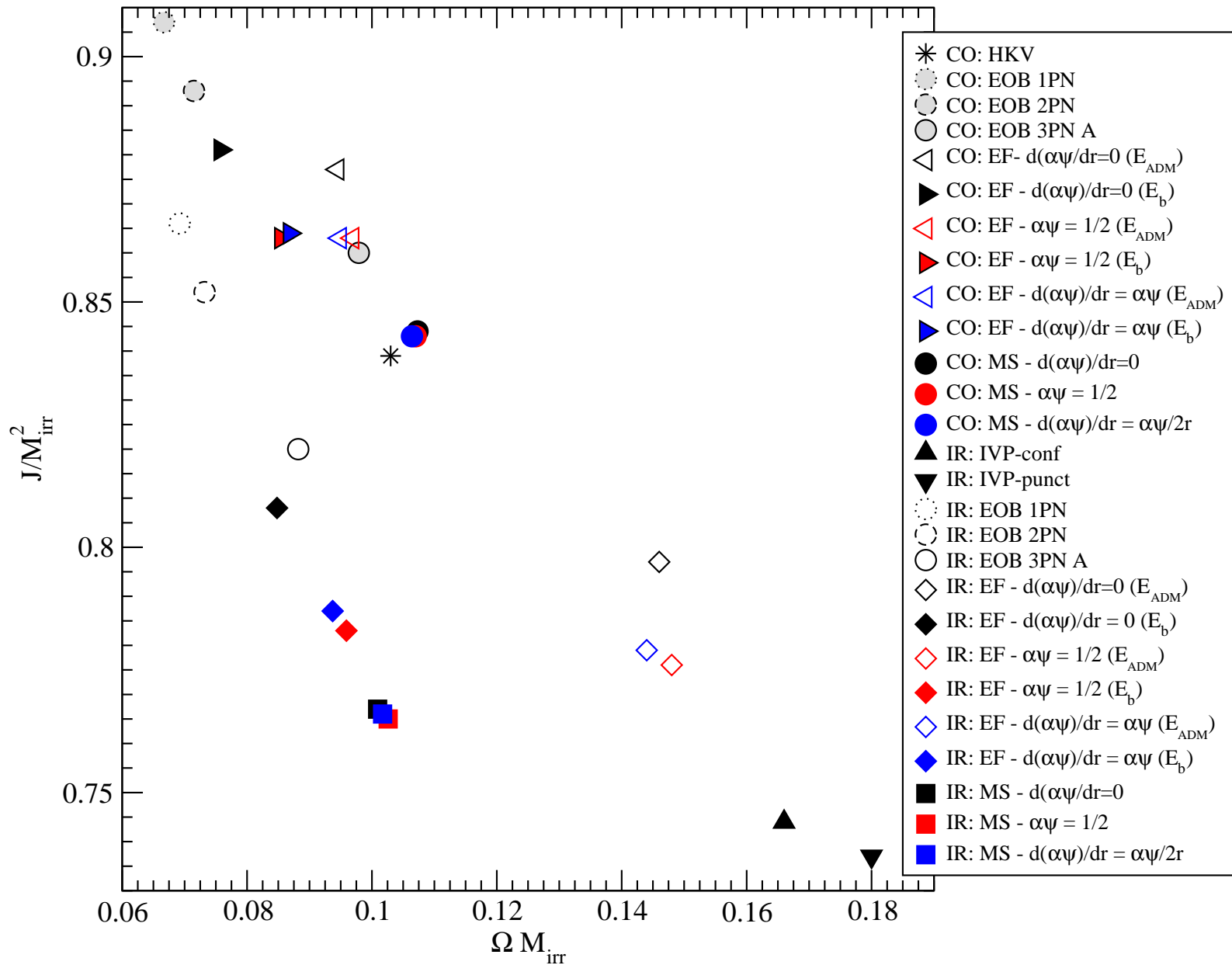
Results: ISCO — E_b/M_{irr} vs ΩM_{irr}



Results: ISCO — E_b/M_{irr} vs J/M_{irr}^2



Results: ISCO — J/M_{irr}^2 vs ΩM_{irr}



Future Issues

- How do we choose $\tilde{\gamma}_{ij}$? [55, 44, 54]
 - Can we use post-Newtonian metrics to improve the choice of $\tilde{\gamma}_{ij}$?
 - Can we use a perturbative evolution to iteratively improve the choice of $\tilde{\gamma}_{ij}$?
- How do we incorporate non-vanishing radial motion for close binaries?
- What can we do for elliptical orbits?
- . . . ?

References

- [1] D. N. Arnold, A. Mukherjee, and L. Pouly. Adaptive finite elements and colliding black holes. In D. F. Griffiths, D. J. Higham, and G. A. Watson, editors, *Numerical Analysis 1997: Proceedings of the 17th Dundee Biennial Conference*, pages 1–15, Essex, England, 1998. Addison Wesley Longman. 5
- [2] T. W. Baumgarte. Innermost stable circular orbit of binary black holes. *Phys. Rev. D*, 62:024018/1–8, July 2000. 5, 13
- [3] T. W. Baumgarte, G. B. Cook, M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky. General relativistic models of binary neutron stars in quasiequilibrium. *Phys. Rev. D*, 57:7299–7311, June 1998. 7, 9
- [4] T. W. Baumgarte, G. B. Cook, M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky. The stability of relativistic neutron stars in binary orbit. *Phys. Rev. D*, 57:6181–6184, May 1998. 7
- [5] M. Bocquet, S. Bonazzola, E. Gourgoulhon, and J. Novak. Rotating neutron star models with magnetic field. *Astron. Astrophys.*, 301:757–775, Sept. 1995. 5
- [6] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. A relativistic formalism to compute quasi-equilibrium configurations of non-synchronized neutron star binaries. *Phys. Rev. D*, 56:7740–7749, Dec. 1997. 7
- [7] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. Numerical approach for high precision 3-D relativistic star models. *Phys. Rev. D*, 58:104020/1–14, Nov. 1998. 5
- [8] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. Numerical models of irrotational binary neutron stars in general relativity. *Phys. Rev. Lett.*, 82:892–895, Feb. 1999. 7, 9
- [9] S. Bonazzola, E. Gourgoulhon, M. Salgado, and J.-A. Marck. Axisymmetric rotating relativistic

- bodies: A new numerical approach for “exact” solutions. *Astron. Astrophys.*, 278:421–443, Nov. 1993. 5
- [10] S. Bonazzola and J. Schneider. An exact study of rigidly and rapidly rotating stars in general relativity with application to the crab pulsar. *Astrophys. J.*, 191:273–286, July 1974. 5
- [11] J. M. Bowen and J. W. York, Jr. Time-asymmetric initial data for black holes and black-hole collisions. *Phys. Rev. D*, 21:2047–2056, Apr. 1980. 4
- [12] S. Brandt and B. Brügmann. A simple construction of initial data for multiple black holes. *Phys. Rev. Lett.*, 78:3606–3609, May 1997. 4
- [13] E. M. Butterworth. On the structure and stability of rapidly rotating fluid bodies in general relativity. II. the structure of uniformly rotating pseudopolytropes. *Astrophys. J.*, 204:561–572, Mar. 1976. 5
- [14] E. M. Butterworth and J. R. Ipser. On the structure and stability of rapidly rotating fluid bodies in general relativity. I. The numerical method for computing structure and its application to uniformly rotating homogeneous bodies. *Astrophys. J.*, 204:200–233, Feb. 1976. 5
- [15] M. W. Choptuik and W. G. Unruh. An introduction to the multi-grid method for numerical relativists. *Gen. Relativ. Gravit.*, 18:813–843, Aug. 1986. 5
- [16] G. B. Cook. Initial data for axisymmetric black-hole collisions. *Phys. Rev. D*, 44:2983–3000, Nov. 1991. 5
- [17] G. B. Cook. Three-dimensional initial data for the collision of two black holes II: Quasicircular orbits for equal mass black holes. *Phys. Rev. D*, 50:5025–5032, Oct. 1994. 5, 13
- [18] G. B. Cook. Initial data for numerical relativity. Article in online journal Living Reviews in Relativity, 2000. <http://www.livingreviews.org/Articles/Volume3/2000-5cook>. 0
- [19] G. B. Cook. Corotating and irrotational binary black holes in quasi-circular orbit. *Phys. Rev. D*, 65:084003/1–13, Apr. 2002. 7

- [20] G. B. Cook and A. M. Abrahams. Horizon structure of initial-data sets for axisymmetric two-black-hole collisions. *Phys. Rev. D*, 46:702–713, July 1992. 5
- [21] G. B. Cook, M. W. Choptuik, M. R. Dubal, S. Klasky, R. A. Matzner, and S. R. Oliveira. Three-dimensional initial data for the collision of two black holes. *Phys. Rev. D*, 47:1471–1490, Feb. 1993. 4, 5
- [22] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Spin-up of a rapidly rotating star by angular momentum loss: Effects of general relativity. *Astrophys. J.*, 398:203–223, Oct. 1992. 5
- [23] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Rapidly rotating neutron stars in general relativity: Realistic equations of state. *Astrophys. J.*, 424:823–845, Apr. 1994. 5
- [24] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Rapidly rotating polytropes in general relativity. *Astrophys. J.*, 422:227–242, Feb. 1994. 5
- [25] G. B. Cook and J. W. York, Jr. Apparent horizons for boosted or spinning black holes. *Phys. Rev. D*, 41:1077–1085, Feb. 1990. 5
- [26] T. Damour, E. Gourgoulhon, and P. Grandclément. Circular orbits of corotating binary black holes: Comparison between analytical and numerical results. *Phys. Rev. D*, 66:024007/1–15, July 2002. 6, 13
- [27] P. Diener, N. Jansen, A. Khokhlov, and I. Novikov. Adaptive mesh refinement approach to the construction of initial data for black hole collisions. *Class. Quantum Gravit.*, 17:435–451, Jan. 2000. 5
- [28] D. M. Eardley. Black hole boundary conditions and coordinate conditions. *Phys. Rev. D*, 57:2299–2304, Feb. 1998. 7
- [29] J. L. Friedman, J. R. Ipser, and L. Parker. Rapidly rotating neutron star models. *Astrophys. J.*, 304:115–139, May 1986. 5

- [30] A. Garat and R. H. Price. Nonexistence of conformally flat slices of the Kerr spacetime. *Phys. Rev. D*, 61:124011/1–4, June 2000. 6
- [31] E. Gourgoulhon. Relations between three formalisms for irrotational binary neutron stars in general relativity. Los Alamos Archive Preprint, 1998. <http://xxx.lanl.gov/abs/gr-qc/9804054>. 7
- [32] E. Gourgoulhon and S. Bonazzola. Noncircular axisymmetric stationary spacetimes. *Phys. Rev. D*, 48:2635–2652, Sept. 1993. 5
- [33] E. Gourgoulhon, P. Grandclément, and S. Bonazzola. Binary black holes in circular orbits. I. A global spacetime approach. *Phys. Rev. D*, 65:044020/1–19, Feb. 2002. 7, 9
- [34] E. Gourgoulhon, P. Grandclément, K. Taniguchi, J.-A. Marck, and S. Bonazzola. Quasiequilibrium sequences of synchronized and irrotational binary neutron stars in general relativity: Method and tests. *Phys. Rev. D*, 63:064029/1–27, Mar. 2001. 7
- [35] P. Grandclément, S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. A multi-domain spectral method for scalar and vectorial Poisson equations with non-compact sources. *J. Comp. Phys.*, 170:231–260, 2001. 7
- [36] P. Grandclément, E. Gourgoulhon, and S. Bonazzola. Binary black holes in circular orbits. II. Numerical methods and first results. *Phys. Rev. D*, 65:044021/1–18, Feb. 2002. 6, 7, 9, 13
- [37] L. E. Kidder and L. S. Finn. Spectral methods for numerical relativity: The initial data problem. *Phys. Rev. D*, 62:084026/1–13, Oct. 2000. 5
- [38] H. Komatsu, Y. Eriguchi, and I. Hachisu. Rapidly rotating general relativistic stars – I. Numerical method and its application to uniformly rotating polytropes. *Mon. Not. R. astr. Soc.*, 237:355–379, Mar. 1989. 5
- [39] H. Komatsu, Y. Eriguchi, and I. Hachisu. Rapidly rotating general relativistic stars – II. differentially rotating polytropes. *Mon. Not. R. astr. Soc.*, 239:153–171, July 1989. 5

- [40] P. Marronetti, G. J. Mathews, and J. R. Wilson. Binary neutron-star systems: From the Newtonian regime to the last stable orbit. *Phys. Rev. D*, 58:107503/1–4, Nov. 1998. 7, 9
- [41] P. Marronetti, G. J. Mathews, and J. R. Wilson. Irrotational binary neutron stars in quasiequilibrium. *Phys. Rev. D*, 60:087301/1–4, Oct. 1999. 7
- [42] P. Marronetti and R. A. Matzner. Solving the initial value problem of two black holes. *Phys. Rev. Lett.*, 85:5500–5503, Dec. 2000. 5
- [43] R. A. Matzner, M. F. Huq, and D. Shoemaker. Initial data and coordinates for multiple black hole systems. *Phys. Rev. D*, 59:024015/1–6, Jan. 1999. 5
- [44] H. P. Pfeiffer, G. B. Cook, and S. A. Teukolsky. Comparing initial-data sets for binary black holes. In press, preprint gr-qc/0203085, 2002. 6, 23
- [45] H. P. Pfeiffer, S. A. Teukolsky, and G. B. Cook. Quasicircular orbits for spinning binary black holes. *Phys. Rev. D*, 62:104018/1–11, Nov. 2000. 5, 6
- [46] H. P. Pfeiffer and J. W. York, Jr. Extrinsic curvature and the Einstein constraints. *Phys. Rev. D*, 67:044022/1–8, Feb. 2003. 2
- [47] M. Shibata. A relativistic formalism for computation of irrotational binary stars in quasi-equilibrium states. *Phys. Rev. D*, 58:024012/1–5, July 1998. 7
- [48] N. Stergioulas and J. L. Friedman. Comparing models of rapidly rotating relativistic stars constructed by two numerical methods. *Astrophys. J.*, 444:306–311, May 1995. 5
- [49] K. Taniguchi and E. Gourgoulhon. Quasiequilibrium sequences of synchronized and irrotational binary neutron stars in general relativity. III Identical and different mass stars with $\gamma = 2$. *Phys. Rev. D*, 66:104019/1–14, Nov. 2002. 7

- [50] S. A. Teukolsky. Irrotational binary neutron stars in quasiequilibrium in general relativity. *Astrophys. J.*, 504:442–449, Sept. 1998. 7
- [51] J. Thornburg. Coordinate and boundary conditions for the general relativistic initial data problem. *Class. Quantum Gravit.*, 4:1119–1131, Sept. 1987. 4
- [52] K. Uryū and Y. Eriguchi. New numerical method for constructing quasiequilibrium sequences of irrotational binary neutron stars in general relativity. *Phys. Rev. D*, 61:124023/1–19, June 2000. 7, 9
- [53] K. Uryū, M. Shibata, and Y. Eriguchi. Properties of general relativistic, irrotational binary neutron stars in close quasiequilibrium orbits: Polytropic equations of state. *Phys. Rev. D*, 62:104015/1–15, Nov. 2000. 7
- [54] F. Usui and Y. Eriguchi. Quasiequilibrium sequences of synchronously rotating binary neutron stars with constant rest masses in general relativity: Another approach without using the conformally flat condition. *Phys. Rev. D*, 65:064030/1–9, Mar. 2002. 7, 23
- [55] F. Usui, K. Uryu, and Y. Eriguchi. New numerical scheme to compute three-dimensional configurations of quasiequilibrium compact stars in general relativity: Application to synchronously rotating binary star systems. *Phys. Rev. D*, 61:024039/1–23, Jan. 2000. 7, 23
- [56] J. R. Wilson. Models of differentially rotating stars. *Astrophys. J.*, 176:273–286, Aug. 1972. 5
- [57] J. R. Wilson and G. J. Mathews. Relativistic hydrodynamics. In C. R. Evans, L. S. Finn, and D. W. Hobill, editors, *Frontiers in Numerical Relativity*, pages 306–314. Cambridge University Press, Cambridge, England, 1989. 7
- [58] J. R. Wilson, G. J. Mathews, and P. Marronetti. Relativistic numerical method for close neutron star binaries. *Phys. Rev. D*, 54:1317–1331, July 1996. 7

- [59] J. W. York, Jr. Conformal 'thin-sandwich' data for the initial-value problem of general relativity. *Phys. Rev. Lett.*, 82:1350–1353, Feb. 1999. 2
- [60] J. W. York, Jr. and T. Piran. The initial value problem and beyond. In R. A. Matzner and L. C. Shepley, editors, *Spacetime and Geometry*, pages 147–176. University of Texas, Austin, 1982. 5