

Computing Initial Data I: Formalisms and History

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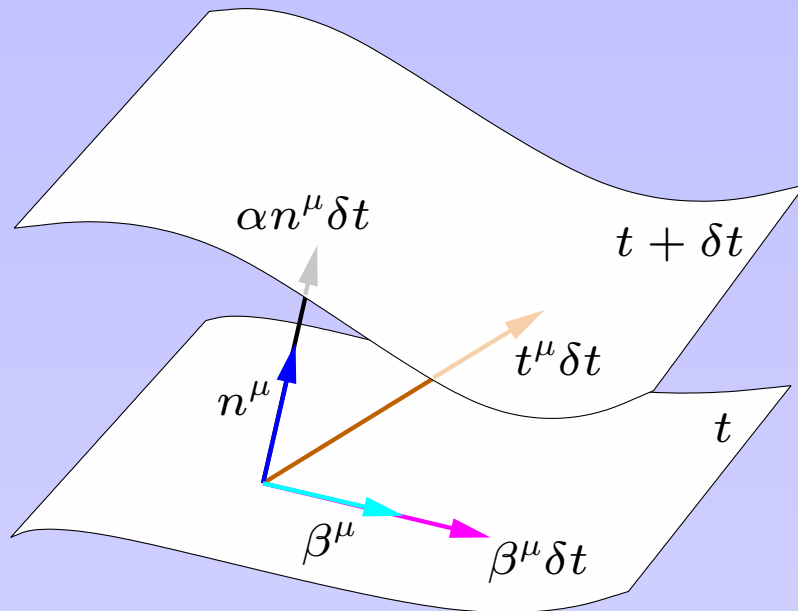
June. 27, 2002

Abstract

In the first of two talks on the computation of initial data, we will look at some of the formalisms used for posing the constraint equations of general relativity as a boundary-value problem. This process requires making well-motivated choices for which of the initial-data quantities are constrained and which can be freely specified. After looking at the general formalisms used to construct initial data, we will review the approaches that have been used to date in constructing black-hole and neutron-star initial data. Finally we will look at some of the current issues, from a physicist perspective, that are at the forefront of initial-data research.

A related [review article](#) is online at [Living Review in Relativity](#)[23]

The 3 + 1 Decomposition



Lapse : α

Spatial metric : γ_{ij}

Shift vector : β^i

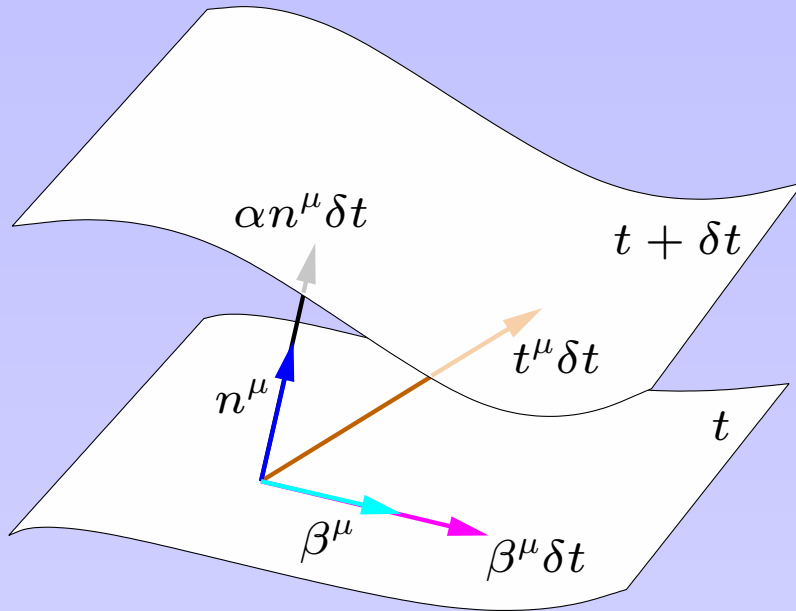
Extrinsic Curvature : K_{ij}

Time vector : $t^\mu = \alpha n^\mu + \beta^\mu$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \quad K_{\mu\nu} = -\frac{1}{2}\gamma_\mu^\alpha \gamma_\nu^\beta \mathcal{L}_n g_{\alpha\beta}$$

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Constraint equations

$$\bar{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$\bar{\nabla}_j (K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

$$S_{\mu\nu} \equiv \gamma_\mu^\alpha \gamma_\nu^\beta T_{\alpha\beta}$$

$$j_\mu \equiv -\gamma_\mu^\nu n^\alpha T_{\nu\alpha}$$

$$\rho \equiv n^\mu n^\nu T_{\mu\nu}$$

$$T_{\mu\nu} = S_{\mu\nu} + 2n_{(\mu}j_{\nu)} + n_\mu n_\nu \rho$$

Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

$$\partial_t K_{ij} = -\bar{\nabla}_i \bar{\nabla}_j \alpha + \alpha \left[\bar{R}_{ij} - 2K_{il}K_j^l + K K_{ij} \right. \\ \left. - 8\pi S_{ij} + 4\pi \gamma_{ij}(S - \rho) \right]$$

$$+ \beta^\ell \bar{\nabla}_\ell K_{ij} + K_{il} \bar{\nabla}_j \beta^\ell + K_{jl} \bar{\nabla}_i \beta^\ell$$

Degrees of Freedom

Kinematical variables

- Lapse α : 1 degree of freedom
- Shift β^i : 3 degrees of freedom

Initial-data variables

- Metric γ_{ij} : 6 degrees of freedom
- Extrinsic curv. K_{ij} : 6 degrees of freedom

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Simple counting arguments

Between γ_{ij} and K_{ij} , there are 12 degrees of freedom at each point in space.

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Constrained

We have 4 constraint equations and there are
 $12 - 4 - 4 = 4$
remaining *constrained* degrees of freedom.

Conformal Transverse-Traceless Decomposition

York–Lichnerowicz conformal decomposition of the 3-metric[46, 72, 54]:

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \left\{ \begin{array}{l} \psi : 1 \text{ constrained DOF} \\ \tilde{\gamma}_{ij} : \left\{ \begin{array}{l} 3 \text{ spatial gauge DOF} \\ 2 \text{ dynamical DOF} \end{array} \right. \end{array} \right.$$

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Conformal TT decomposition of extrinsic curvature[73, 74]:

$$K^{ij} = \psi^{-10} \left[(\tilde{\mathbb{L}}X)^{ij} + \tilde{Q}^{ij} \right] + \frac{1}{3} \gamma^{ij} K \left\{ \begin{array}{l} \tilde{Q}^{ij} : \left\{ \begin{array}{l} \tilde{\nabla}_j \tilde{Q}^{ij} = \tilde{Q}^i_i = 0 \\ 2 \text{ dynamical DOF} \end{array} \right. \\ X^i : \left\{ \begin{array}{l} (\tilde{\mathbb{L}}X)^{ij} \equiv \tilde{\nabla}^i X^j + \tilde{\nabla}^j X^i - \frac{2}{3} \tilde{\gamma}^{ij} \tilde{\nabla}_k X^k \\ 3 \text{ constrained DOF} \end{array} \right. \\ K : 1 \text{ temporal gauge DOF} \end{array} \right.$$

Specifying Initial Data

Freely specified degrees of freedom

$\tilde{\gamma}_{ij} \Leftarrow \left\{ \begin{array}{l} (2) \text{ initial dynamical ("wave") content} \\ (3) \text{ initial spatial gauge choices} \end{array} \right.$

$\tilde{Q}^{ij} \Leftarrow (2) \text{ initial dynamical ("wave") content}$

$K \Leftarrow (1) \text{ initial temporal gauge choice}$

Freely specified *dynamical* gauge freedom

$\beta^i \Leftarrow \text{how spatial coordinates evolve}$

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Constrained degrees of freedom (Conf. TT)

$$\psi \Leftarrow \begin{cases} (1) \text{ Hamiltonian constraint} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = -2\pi \psi^5 \rho \end{cases}$$

$$X^i \Leftarrow \begin{cases} (3) \text{ Momentum constraint} \\ \tilde{\Delta}_{\mathbb{L}} X^i = \frac{2}{3} \psi^6 \tilde{\nabla}^i K + 8\pi \psi^{10} j^i \end{cases}$$

$$\tilde{A}^{ij} \equiv (\tilde{\mathbb{L}} X)^{ij} + \tilde{Q}^{ij}$$

$$\tilde{\Delta}_{\mathbb{L}} X^i \equiv \tilde{\nabla}_j (\tilde{\mathbb{L}} X)^{ij}$$

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Boundary conditions

The constraints form a set of 4 coupled nonlinear PDEs for (ψ, X^i) that require the specification of boundary conditions at spatial infinity and any interior boundaries.

General Initial-Data Decompositions

Conformal TT[75, 77]:

$$\begin{aligned} \gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \psi^{-4} \tilde{\gamma}^{ij} K \quad : \quad \tilde{A}^{ij} &\equiv (\tilde{\mathbb{L}}V)^{ij} + \tilde{M}^{ij} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} &= -2\pi \psi^5 \rho \\ \tilde{\Delta}_{\mathbb{L}} V^i - \frac{2}{3} \psi^6 \tilde{\nabla}^i K &= -\tilde{\nabla}_j \tilde{M}^{ij} + 8\pi \psi^{10} j^i \end{aligned}$$

Physical TT[53, 54, 55]:

$$\begin{aligned} \gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad : \quad K^{ij} = \psi^{-4} \left(\tilde{A}^{ij} + \frac{1}{3} \tilde{\gamma}^{ij} K \right) \quad : \quad \tilde{A}^{ij} &\equiv (\tilde{\mathbb{L}}V)^{ij} + \psi^{-6} \tilde{M}^{ij} \\ \tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^5 \tilde{A}_{ij} \tilde{A}^{ij} &= -2\pi \psi^5 \rho \\ \tilde{\Delta}_{\mathbb{L}} V^i + 6(\tilde{\mathbb{L}}V)^{ij} \tilde{\nabla}_j \ln \psi + \psi^{-6} \tilde{\nabla}_j \tilde{M}^{ij} &= \frac{2}{3} \tilde{\nabla}^i K + 8\pi \psi^4 j^i \end{aligned}$$

\tilde{M}^{ij} is symmetric-tracefree, but not divergenceless. The new variable V^i incorporates the solution of the constraints and the decomposition of \tilde{M}^{ij} into \tilde{Q}^{ij} .

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Conformal Thin Sandwich(TS)[76]:

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α and β^i are the lapse and shift. \tilde{u}^{ij} is symmetric-tracefree.

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α and β^i are the lapse and shift. \tilde{u}^{ij} is symmetric-tracefree.

Special Initial-Data Decompositions

- The Conformal TS, Conformal TT, & Physical TT decompositions are widely used, but are not unique. In particular, different conformal weightings of the extrinsic curvature are used.
- For stationary situation (i.e. axisymmetric, time independent) like isolated neutron stars, different decompositions are used.

$$ds^2 = -\psi^{-4}dt^2 + \psi^4[A^2(dr^2 + r^2d\theta^2) + B^2r^2 \sin^2 \theta(d\phi + \beta^\phi dt)^2]$$

- ★ The 4 constraints yield 2 independent equations that fix ψ and β^ϕ .
- ★ Stationarity ($\partial_t \gamma_{ij} = 0, \partial_t K_{ij} = 0$) yields 2 independent equations that fix A and B .

“Traditional” Black-Hole Data

Conformal TT with conformal flatness and maximal slicing

$$\left. \begin{array}{l} \tilde{\gamma}_{ij} = f_{ij} \text{ (flat)} \\ \tilde{M}^{ij} = 0 \\ K = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \tilde{\Delta}_{\mathbb{L}} X^i = 0 \Rightarrow \\ \tilde{\nabla}^2 \psi + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0 \end{array} \right.$$

Bowen-York solution [14, 45]
Analytic solutions for \tilde{A}^{ij}

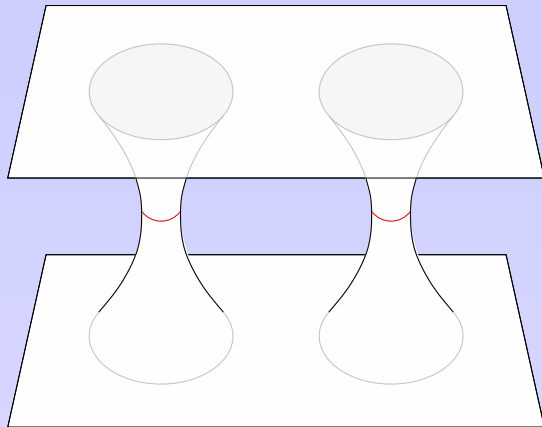
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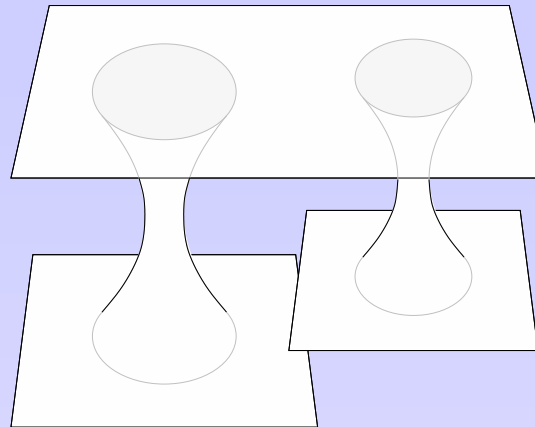
Three general solution schemes

Conformal Imaging-[52, 14, 21, 26]



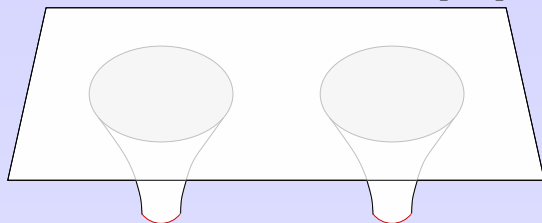
Inversion
symmetry
inner-BC

Puncture Method-[47, 17, 15]



No inner-BC:
singular
behavior
factored out

Apparent Horizon BC-[62]



Apparent
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inner-BC

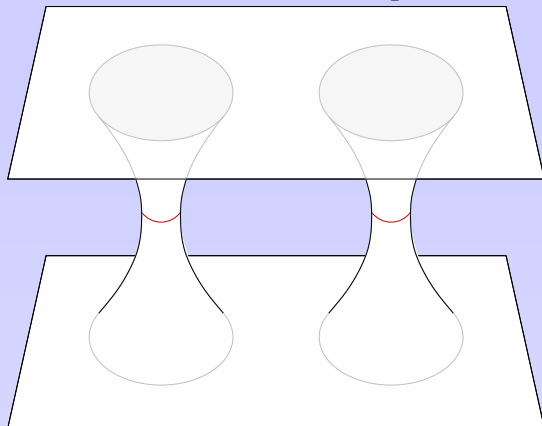
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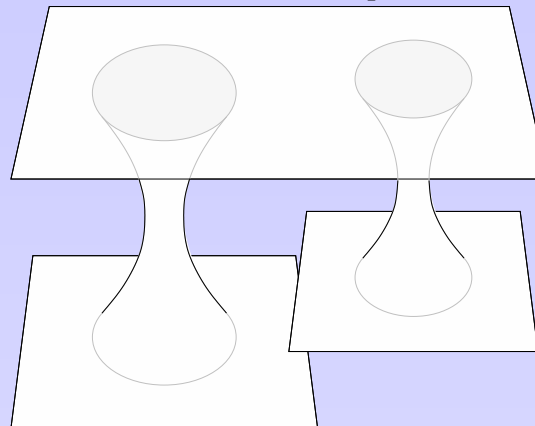
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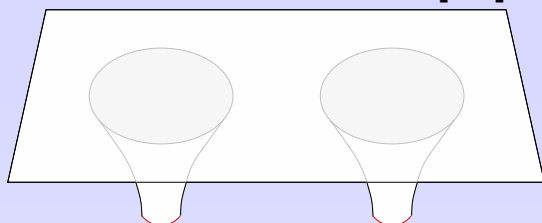
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Apparent
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All methods can produce very general configurations of multiple black holes, but are fundamentally limited by choices of $\tilde{\gamma}_{ij} = f_{ij}$ and Bowen-York \tilde{A}^{ij} .

Black-Hole Data

- The “traditional” black hole initial data approach was motivated by *computational convenience*, not by any strong physical arguments. Research focused on:
 - methods for solving the Hamiltonian constraint for one or two holes. [77, 20, 21, 26, 1, 42, 57, 33]
 - understanding the physical content (and limitations) of initial data containing one or two holes. [77, 30, 21, 25]
 - finding solutions that represented two black holes in nearly circular orbits. [22, 2, 57]

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- Recent work has generalized many of these assumption and requires solving the full set of 4 (or more) coupled, non-linear elliptic PDEs. [51, 6, 50, 38, 41, 56]

Neutron-Star Data[59]

- Single, stationary neutron stars have been studied extensively. The matter is assumed to be in hydrostatic equilibrium. Research has focused on:
 - various numerical methods.[13, 19, 18, 43, 27, 29, 60, 12, 37, 8, 10]
 - rigid and differential rotation.[69, 44, 27]
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- Binary systems have also been studied. These are usually based on the assumption that the binary is nearly in equilibrium (quasi-equilibrium). Research has focused on:
 - various numerical methods.[70, 71, 9, 3, 39, 40]
 - corotating (tidally locked) binaries.[9, 3, 4, 48, 65]
 - irrotational binaries.[61, 58, 36, 11, 49, 63, 64]

Current Issues

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- ★ Forced-equilibrium — Enforce a “helical Killing-vector” yielding data with equal amounts of ingoing and outgoing radiation – then subtract away the unphysical ingoing radiation. ($\partial_t \gamma_{ij} = \partial_t K_{ij} = 0$)[7, 32, 68, 67, 66]
 - Questions about boundary conditions, etc. . .

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 - How do we choose $\tilde{\gamma}_{ij}$?[65, 56]
 - * Can we use post-Newtonian metrics to improve the choice of $\tilde{\gamma}_{ij}$?
 - * Can we use a perturbative evolution to iteratively improve the choice of $\tilde{\gamma}_{ij}$?

Quasi-Equilibrium with Black Hole Excision[24]

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \quad K^{ij} = \psi^{-10} \tilde{A}^{ij} + \frac{1}{3} \gamma^{ij} K \quad \tilde{A}^{ij} = \frac{\psi^6}{2\alpha} (\tilde{\mathbb{L}}\beta)^{ij} \quad \partial_t \tilde{\gamma}_{ij} = 0$$

$$\tilde{\nabla}^2 \psi - \frac{1}{8} \psi \tilde{R} - \frac{1}{12} \psi^5 K^2 + \frac{1}{8} \psi^{-7} \tilde{A}_{ij} \tilde{A}^{ij} = 0$$

$$\tilde{\Delta}_{\mathbb{L}} \beta^i - (\tilde{\mathbb{L}}\beta)^{ij} \tilde{\nabla}_j \ln \alpha \psi^{-6} = \frac{4}{3} \alpha \tilde{\nabla}^i K$$

$$\tilde{\nabla}^2 (\alpha \psi) - (\alpha \psi) \left[\frac{1}{8} \tilde{R} + \frac{5}{12} \psi^4 K^2 + \frac{7}{8} \psi^{-8} \tilde{A}_{ij} \tilde{A}^{ij} \right] = \psi^5 \beta^i \tilde{\nabla}_i K \quad \partial_t K = 0$$

$$\tilde{s}^k \tilde{\nabla}_k \ln \psi|_S = -\frac{1}{4} (\tilde{h}^{ij} \tilde{\nabla}_i \tilde{s}_j - \psi^2 J)|_S \quad \theta = 0$$

$$\beta^i|_S = \begin{cases} \alpha \psi^{-2} \tilde{s}^i|_S & \text{corotation} \\ \alpha \psi^{-2} \tilde{s}^i|_S + \Omega \tilde{h}^i_j \left(\frac{\partial}{\partial \phi} \right)^j \Big|_S & \text{irrotation} \end{cases} \quad \begin{matrix} \mathcal{L}_\zeta \theta = 0 \\ \sigma_{ij} = 0 \end{matrix}$$

$$J \tilde{s}^i \tilde{\nabla}_i \alpha|_S = -\psi^2 (J^2 - JK + \tilde{\mathcal{D}}) \alpha|_S \quad \mathcal{L}_\zeta \theta = 0$$

$$\begin{aligned} \psi|_{r \rightarrow \infty} &= 1 \\ \beta^i|_{r \rightarrow \infty} &= \Omega \left(\frac{\partial}{\partial \phi} \right)^i \\ \alpha|_{r \rightarrow \infty} &= 1 \end{aligned}$$

The only remaining freedom in the system is the choice of the orbital angular velocity Ω , the initial spatial gauge and the initial dynamical (“wave”) content found in $\tilde{\gamma}_{ij}$, and the initial temporal gauge in K .

References

- [1] D. N. Arnold, A. Mukherjee, and L. Pouly. Adaptive finite elements and colliding black holes. In D. F. Griffiths, D. J. Higham, and G. A. Watson, editors, *Numerical Analysis 1997: Proceedings of the 17th Dundee Biennial Conference*, pages 1–15, Essex, England, 1998. Addison Wesley Longman. 8
- [2] T. W. Baumgarte. Innermost stable circular orbit of binary black holes. *Phys. Rev. D*, 62:024018/1–8, July 2000. 8
- [3] T. W. Baumgarte, G. B. Cook, M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky. General relativistic models of binary neutron stars in quasiequilibrium. *Phys. Rev. D*, 57:7299–7311, June 1998. 9
- [4] T. W. Baumgarte, G. B. Cook, M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky. The stability of relativistic neutron stars in binary orbit. *Phys. Rev. D*, 57:6181–6184, May 1998. 9
- [5] D. Bernstein, D. W. Hobill, E. Seidel, and L. Smarr. Initial data for the black hole plus brill wave spacetime. *Phys. Rev. D*, 50:3760–3782, Sept. 1994. 8
- [6] N. T. Bishop, R. Isaacson, M. Maharaj, and J. Winicour. Black hole data via a Kerr-Schild approach. *Phys. Rev. D*, 57:6113–6118, May 1998. 8
- [7] J. K. Blackburn and S. Detweiler. Close black-hole binary systems. *Phys. Rev. D*, 46:2318–2333, Sept. 1992. 10
- [8] M. Bocquet, S. Bonazzola, E. Gourgoulhon, and J. Novak. Rotating neutron star models with magnetic field. *Astron. Astrophys.*, 301:757–775, Sept. 1995. 9
- [9] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. A relativistic formalism to compute quasi-equilibrium configurations of non-synchronized neutron star binaries. *Phys. Rev. D*, 56:7740–7749, Dec. 1997. 9

- [10] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. Numerical approach for high precision 3-D relativistic star models. *Phys. Rev. D*, 58:104020/1–14, Nov. 1998. 9
- [11] S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. Numerical models of irrotational binary neutron stars in general relativity. *Phys. Rev. Lett.*, 82:892–895, Feb. 1999. 9
- [12] S. Bonazzola, E. Gourgoulhon, M. Salgado, and J.-A. Marck. Axisymmetric rotating relativistic bodies: A new numerical approach for “exact” solutions. *Astron. Astrophys.*, 278:421–443, Nov. 1993. 9
- [13] S. Bonazzola and J. Schneider. An exact study of rigidly and rapidly rotating stars in general relativity with application to the crab pulsar. *Astrophys. J.*, 191:273–286, July 1974. 9
- [14] J. M. Bowen and J. W. York, Jr. Time-asymmetric initial data for black holes and black-hole collisions. *Phys. Rev. D*, 21:2047–2056, Apr. 1980. 7
- [15] S. Brandt and B. Brügmann. A simple construction of initial data for multiple black holes. *Phys. Rev. Lett.*, 78:3606–3609, May 1997. 7
- [16] S. R. Brandt and E. Seidel. Evolution of distorted rotating black holes. III: Initial data. *Phys. Rev. D*, 54:1403–1416, July 1996. 8
- [17] D. R. Brill and R. W. Lindquist. Interaction energy in geometrostatics. *Phys. Rev.*, 131:471–476, July 1963. 7
- [18] E. M. Butterworth. On the structure and stability of rapidly rotating fluid bodies in general relativity. II. the structure of uniformly rotating pseudopolytropes. *Astrophys. J.*, 204:561–572, Mar. 1976. 9
- [19] E. M. Butterworth and J. R. Ipser. On the structure and stability of rapidly rotating fluid bodies in general relativity. I. The numerical method for computing structure and its application to uniformly rotating homogeneous bodies. *Astrophys. J.*, 204:200–233, Feb. 1976. 9

- [20] M. W. Choptuik and W. G. Unruh. An introduction to the multi-grid method for numerical relativists. *Gen. Relativ. Gravit.*, 18:813–843, Aug. 1986. 8
- [21] G. B. Cook. Initial data for axisymmetric black-hole collisions. *Phys. Rev. D*, 44:2983–3000, Nov. 1991. 7, 8
- [22] G. B. Cook. Three-dimensional initial data for the collision of two black holes II: Quasicircular orbits for equal mass black holes. *Phys. Rev. D*, 50:5025–5032, Oct. 1994. 8
- [23] G. B. Cook. Initial data for numerical relativity. Article in online journal Living Reviews in Relativity, 2000. <http://www.livingreviews.org/Articles/Volume3/2000-5cook>. 0
- [24] G. B. Cook. Corotating and irrotational binary black holes in quasi-circular orbit. *Phys. Rev. D*, 65:084003/1–13, Apr. 2002. 11, 12
- [25] G. B. Cook and A. M. Abrahams. Horizon structure of initial-data sets for axisymmetric two-black-hole collisions. *Phys. Rev. D*, 46:702–713, July 1992. 8
- [26] G. B. Cook, M. W. Choptuik, M. R. Dubal, S. Klasky, R. A. Matzner, and S. R. Oliveira. Three-dimensional initial data for the collision of two black holes. *Phys. Rev. D*, 47:1471–1490, Feb. 1993. 7, 8
- [27] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Spin-up of a rapidly rotating star by angular momentum loss: Effects of general relativity. *Astrophys. J.*, 398:203–223, Oct. 1992. 9
- [28] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Rapidly rotating neutron stars in general relativity: Realistic equations of state. *Astrophys. J.*, 424:823–845, Apr. 1994. 9
- [29] G. B. Cook, S. L. Shapiro, and S. A. Teukolsky. Rapidly rotating polytropes in general relativity. *Astrophys. J.*, 422:227–242, Feb. 1994. 9

- [30] G. B. Cook and J. W. York, Jr. Apparent horizons for boosted or spinning black holes. *Phys. Rev. D*, 41:1077–1085, Feb. 1990. 8
- [31] T. Damour, E. Gourgoulhon, and P. Grandclément. Circular orbits of corotating binary black holes: comparison between analytical and numerical results. In press, preprint gr-qc/0204011, 2002. 10
- [32] S. Detweiler. Periodic solutions of the Einstein equations for binary systems. *Phys. Rev. D*, 50:4929–4943, Oct. 1994. 10
- [33] P. Diener, N. Jansen, A. Khokhlov, and I. Novikov. Adaptive mesh refinement approach to the construction of initial data for black hole collisions. *Class. Quantum Gravit.*, 17:435–451, Jan. 2000. 8
- [34] J. L. Friedman, J. R. Ipser, and L. Parker. Rapidly rotating neutron star models. *Astrophys. J.*, 304:115–139, May 1986. 9
- [35] A. Garat and R. H. Price. Nonexistence of conformally flat slices of the Kerr spacetime. *Phys. Rev. D*, 61:124011/1–4, June 2000. 10
- [36] E. Gourgoulhon. Relations between three formalisms for irrotational binary neutron stars in general relativity. Los Alamos Archive Preprint, 1998. <http://xxx.lanl.gov/abs/gr-qc/9804054>. 9
- [37] E. Gourgoulhon and S. Bonazzola. Noncircular axisymmetric stationary spacetimes. *Phys. Rev. D*, 48:2635–2652, Sept. 1993. 9
- [38] E. Gourgoulhon, P. Grandclément, and S. Bonazzola. Binary black holes in circular orbits. I. A global spacetime approach. *Phys. Rev. D*, 65:044020/1–19, Feb. 2002. 8, 11
- [39] E. Gourgoulhon, P. Grandclément, K. Taniguchi, J.-A. Marck, and S. Bonazzola. Quasiequilibrium sequences of synchronized and irrotational binary neutron stars in general relativity: Method and tests. *Phys. Rev. D*, 63:064029/1–27, Mar. 2001. 9

- [40] P. Grandclément, S. Bonazzola, E. Gourgoulhon, and J.-A. Marck. A multi-domain spectral method for scalar and vectorial Poisson equations with non-compact sources. *J. Comp. Phys.*, 170:231–260, 2001. 9
- [41] P. Grandclément, E. Gourgoulhon, and S. Bonazzola. Binary black holes in circular orbits. II. Numerical methods and first results. *Phys. Rev. D*, 65:044021/1–18, Feb. 2002. 8, 10, 11
- [42] L. E. Kidder and L. S. Finn. Spectral methods for numerical relativity: The initial data problem. *Phys. Rev. D*, 62:084026/1–13, Oct. 2000. 8
- [43] H. Komatsu, Y. Eriguchi, and I. Hachisu. Rapidly rotating general relativistic stars – I. Numerical method and its application to uniformly rotating polytropes. *Mon. Not. R. astr. Soc.*, 237:355–379, Mar. 1989. 9
- [44] H. Komatsu, Y. Eriguchi, and I. Hachisu. Rapidly rotating general relativistic stars – II. differentially rotating polytropes. *Mon. Not. R. astr. Soc.*, 239:153–171, July 1989. 9
- [45] A. D. Kulkarni. Time-asymmetric initial data for the N black hole problem in general relativity. *J. Math. Phys.*, 25:1028–1034, Apr. 1984. 7
- [46] A. Lichnerowicz. L'intégration des équations de la gravitation relativiste et le problème des n corps. *J. Math. Pures et Appl.*, 23:37–63, 1944. 3
- [47] R. W. Lindquist. Initial-value problem on Einstein-Rosen manifolds. *J. Math. Phys.*, 4:938–950, July 1963. 7
- [48] P. Marronetti, G. J. Mathews, and J. R. Wilson. Binary neutron-star systems: From the Newtonian regime to the last stable orbit. *Phys. Rev. D*, 58:107503/1–4, Nov. 1998. 9
- [49] P. Marronetti, G. J. Mathews, and J. R. Wilson. Irrotational binary neutron stars in quasiequilibrium. *Phys. Rev. D*, 60:087301/1–4, Oct. 1999. 9

- [50] P. Marronetti and R. A. Matzner. Solving the initial value problem of two black holes. *Phys. Rev. Lett.*, 85:5500–5503, Dec. 2000. 8, 10
- [51] R. A. Matzner, M. F. Huq, and D. Shoemaker. Initial data and coordinates for multiple black hole systems. *Phys. Rev. D*, 59:024015/1–6, Jan. 1999. 8, 10
- [52] C. W. Misner. The method of images in geometrostatics. *Ann. Phys.*, 24:102–117, Oct. 1963. 7
- [53] N. Ó Murchadha and J. W. York, Jr. Initial-value problem of general relativity. I. General formulation and physical interpretation. *Phys. Rev. D*, 10:428–436, July 1974. 5
- [54] N. Ó Murchadha and J. W. York, Jr. Initial-value problem of general relativity. II. Stability of solution of the initial-value equations. *Phys. Rev. D*, 10:437–446, July 1974. 3, 5
- [55] N. Ó Murchadha and J. W. York, Jr. Gravitational potentials: A constructive approach to general relativity. *Gen. Relativ. Gravit.*, 7:257–261, Mar. 1976. 5
- [56] H. P. Pfeiffer, G. B. Cook, and S. A. Teukolsky. Comparing initial-data sets for binary black holes. In press, preprint gr-qc/0203085, 2002. 8, 10, 11
- [57] H. P. Pfeiffer, S. A. Teukolsky, and G. B. Cook. Quasicircular orbits for spinning binary black holes. *Phys. Rev. D*, 62:104018/1–11, Nov. 2000. 8, 10
- [58] M. Shibata. A relativistic formalism for computation of irrotational binary stars in quasi-equilibrium states. *Phys. Rev. D*, 58:024012/1–5, July 1998. 9
- [59] N. Stergioulas. Rotating stars in relativity. Article in Online Journal Living Reviews in Relativity, 1998. <http://www.livingreviews.org/Articles/Volume1/1998-8stergio>. 9
- [60] N. Stergioulas and J. L. Friedman. Comparing models of rapidly rotating relativistic stars constructed by two numerical methods. *Astrophys. J.*, 444:306–311, May 1995. 9

- [61] S. A. Teukolsky. Irrotational binary neutron stars in quasiequilibrium in general relativity. *Astrophys. J.*, 504:442–449, Sept. 1998. 9
- [62] J. Thornburg. Coordinate and boundary conditions for the general relativistic initial data problem. *Class. Quantum Gravit.*, 4:1119–1131, Sept. 1987. 7
- [63] K. Uryū and Y. Eriguchi. New numerical method for constructing quasiequilibrium sequences of irrotational binary neutron stars in general relativity. *Phys. Rev. D*, 61:124023/1–19, June 2000. 9
- [64] K. Uryū, M. Shibata, and Y. Eriguchi. Properties of general relativistic, irrotational binary neutron stars in close quasiequilibrium orbits: Polytropic equations of state. *Phys. Rev. D*, 62:104015/1–15, Nov. 2000. 9
- [65] F. Usui, K. Uryu, and Y. Eriguchi. New numerical scheme to compute three-dimensional configurations of quasiequilibrium compact stars in general relativity: Application to synchronously rotating binary star systems. *Phys. Rev. D*, 61:024039/1–23, Jan. 2000. 9, 11
- [66] J. T. Whelan, C. Beetle, W. Landry, and R. H. Price. Radiation-balanced simulations for binary inspiral. *Class. Quantum Gravit.*, 19:1285–1290, Apr. 2002. 10
- [67] J. T. Whelan, W. Krivan, and R. H. Price. Quasistationary binary inspiral: II. Radiation-balanced boundary conditions. *Class. Quantum Gravit.*, 17:4895–4912, Dec. 2000. 10
- [68] J. T. Whelan and J. D. Romano. Quasistationary binary inspiral. I. Einstein equations for the two Killing vector spacetime. *Phys. Rev. D*, 60:084009/1–20, Oct. 1999. 10
- [69] J. R. Wilson. Models of differentially rotating stars. *Astrophys. J.*, 176:273–286, Aug. 1972. 9
- [70] J. R. Wilson and G. J. Mathews. Relativistic hydrodynamics. In C. R. Evans, L. S. Finn, and D. W. Hobill, editors, *Frontiers in Numerical Relativity*, pages 306–314. Cambridge University Press, Cambridge, England, 1989. 9

- [71] J. R. Wilson, G. J. Mathews, and P. Marronetti. Relativistic numerical method for close neutron star binaries. *Phys. Rev. D*, 54:1317–1331, July 1996. 9
- [72] J. W. York, Jr. Role of conformal three-geometry in the dynamics of gravitation. *Phys. Rev. Lett.*, 28:1082–1085, Apr. 1972. 3
- [73] J. W. York, Jr. Conformally invariant orthogonal decomposition of symmetric tensors on Riemannian manifolds and the initial-value problem of general relativity. *J. Math. Phys.*, 14:456–464, Apr. 1973. 3
- [74] J. W. York, Jr. Covariant decompositions of symmetric tensors in the theory of gravitation. *Ann. Inst. Henri Poincaré, A*, 21:319–332, 1974. 3
- [75] J. W. York, Jr. Kinematics and dynamics of general relativity. In L. L. Smarr, editor, *Sources of Gravitational Radiation*, pages 83–126. Cambridge University Press, Cambridge, England, 1979. 5
- [76] J. W. York, Jr. Conformal ‘thin-sandwich’ data for the initial-value problem of general relativity. *Phys. Rev. Lett.*, 82:1350–1353, Feb. 1999. 5
- [77] J. W. York, Jr. and T. Piran. The initial value problem and beyond. In R. A. Matzner and L. C. Shepley, editors, *Spacetime and Geometry*, pages 147–176. University of Texas, Austin, 1982. 5, 8