

Approximate Killing Vectors for Computing Spin in Black-Hole Initial Data and Evolutions

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Measuring the Spin of a Black Hole

- Spin is only rigorously defined at spatial/null infinity.
- Must use *quasi-local* definition: e.g. Brown & York[2] or Ashtekar & Krishnan[1]

$$S = -\frac{1}{8\pi} \oint_{BH} K_{ij} \xi^i s^j \sqrt{h} d^2x$$

$$\xi^i = \begin{cases} \xi_{\text{SCK}}^i & : \text{ Killing vector of } \tilde{h}_{ij} \Rightarrow \text{conformal Killing vector of } h_{ij} \\ \xi_{\text{SAKV}}^i & : \text{ Approximate Killing vector of } h_{ij} \end{cases}$$

Approximate Killing Vectors

Killing Transport on $S^2[3]$

$$D_i \xi_j = \epsilon_{ij} L$$

$$D_i L = -\frac{1}{2} R \epsilon_i{}^j \xi_j$$

- Equations assume KV exists!
- Solution is “path dependent”
- Solution violates equations
- $\xi^i \neq \epsilon^{ij} D_j v$

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New AKV Method

$$\xi^i = \epsilon^{ij} D_j v + D^i d$$

$$D_i \xi_j = \epsilon_{ij} L + S_{ij} + h_{ij} \Lambda$$

- Find solutions that minimize S^{ij} !
- $\xi^i = \epsilon^{ij} D_j v$ by construction
- $L = \frac{1}{2} \epsilon_{ij} D^i \xi^j$ by construction

New Approximate Killing Vectors

- Minimize

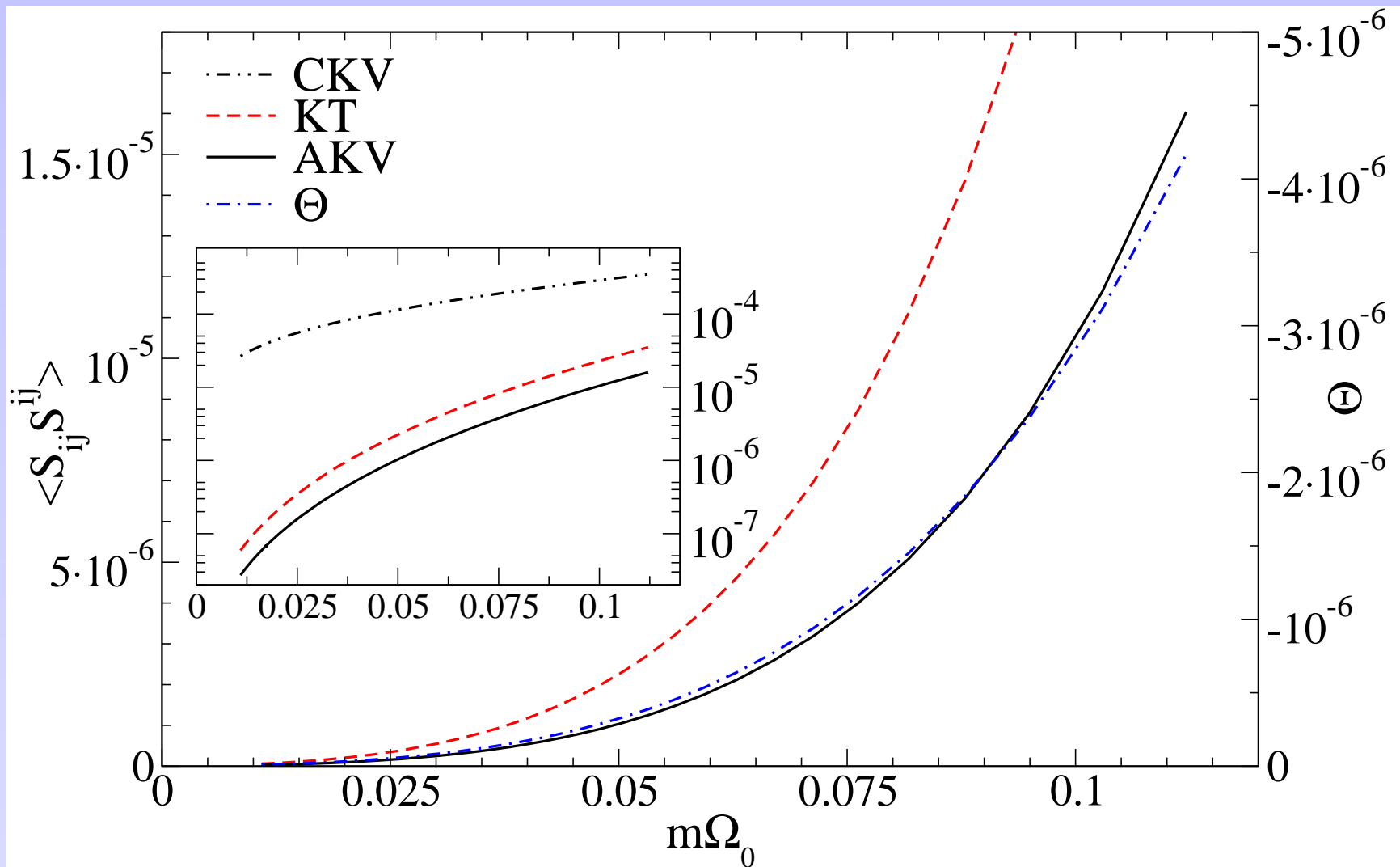
$$S_{ij}S^{ij} = (D_i D_j v)(D^i D^j v) - \frac{1}{2}(D^k D_k v)^2$$

subject to constraint that $|\vec{\xi}|^2 = (D_i v)(D^i v) = \text{const.}$

- $\mathcal{L} \equiv S_{ij}S^{ij} + \frac{1}{2}{}^2R\Theta(D_k v)(D^k v)$

$$\frac{\delta \mathcal{L}}{\delta v} = 0 \quad \Rightarrow \quad \begin{aligned} \xi^i &\equiv \epsilon^{ij} D_j v \\ D^i D_i v + 2L &= 0 \\ D^i D_i L - (1 - \Theta) \left[\frac{1}{2}(D^{i2}R)D_i v - {}^2RL \right] &= 0 \end{aligned}$$

Corotating BBH “Spin AKVs”



Corotating BBH AKVs

In addition to expected
“spin AKV,” we find
2 more AKVs.

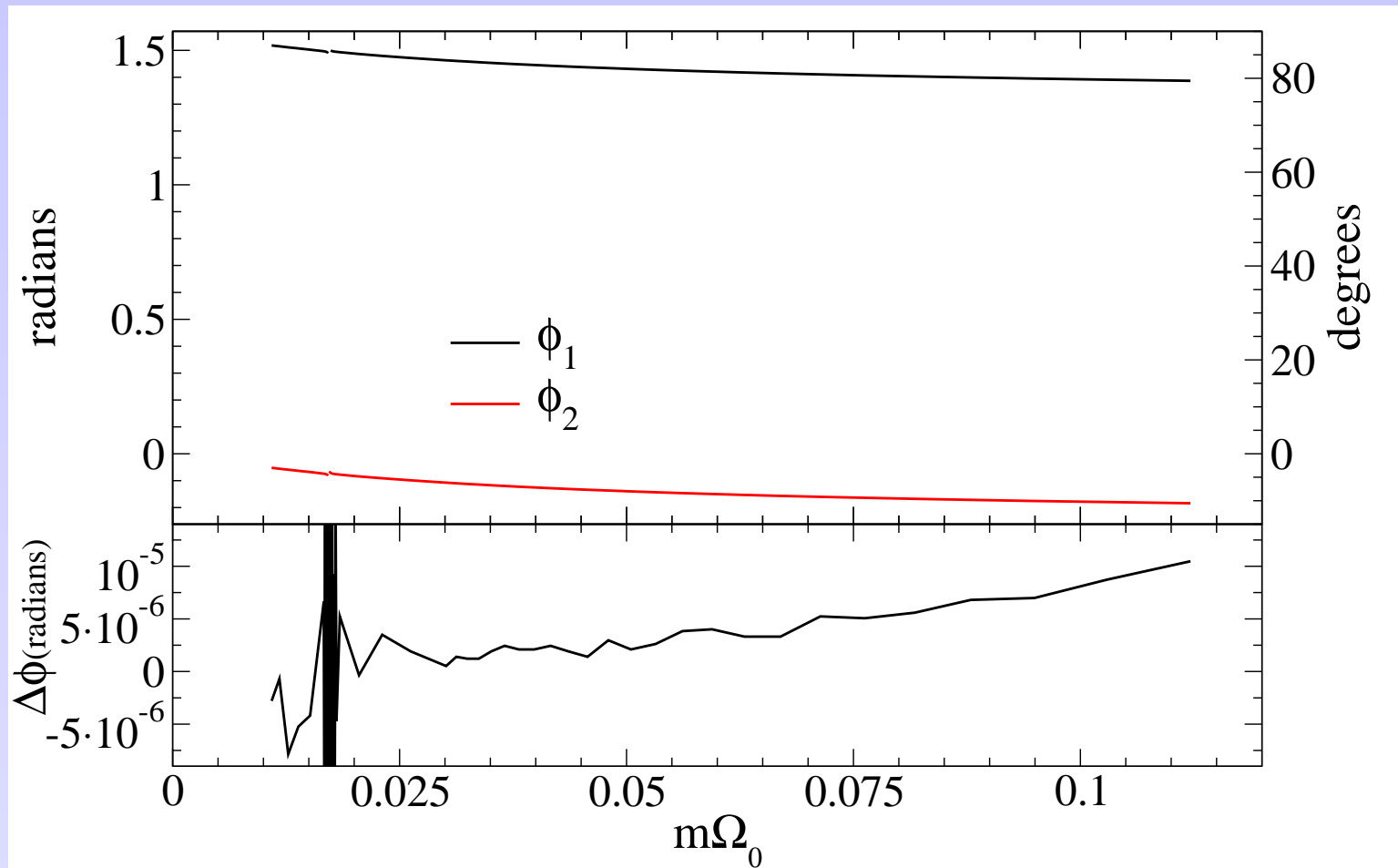
No spin!



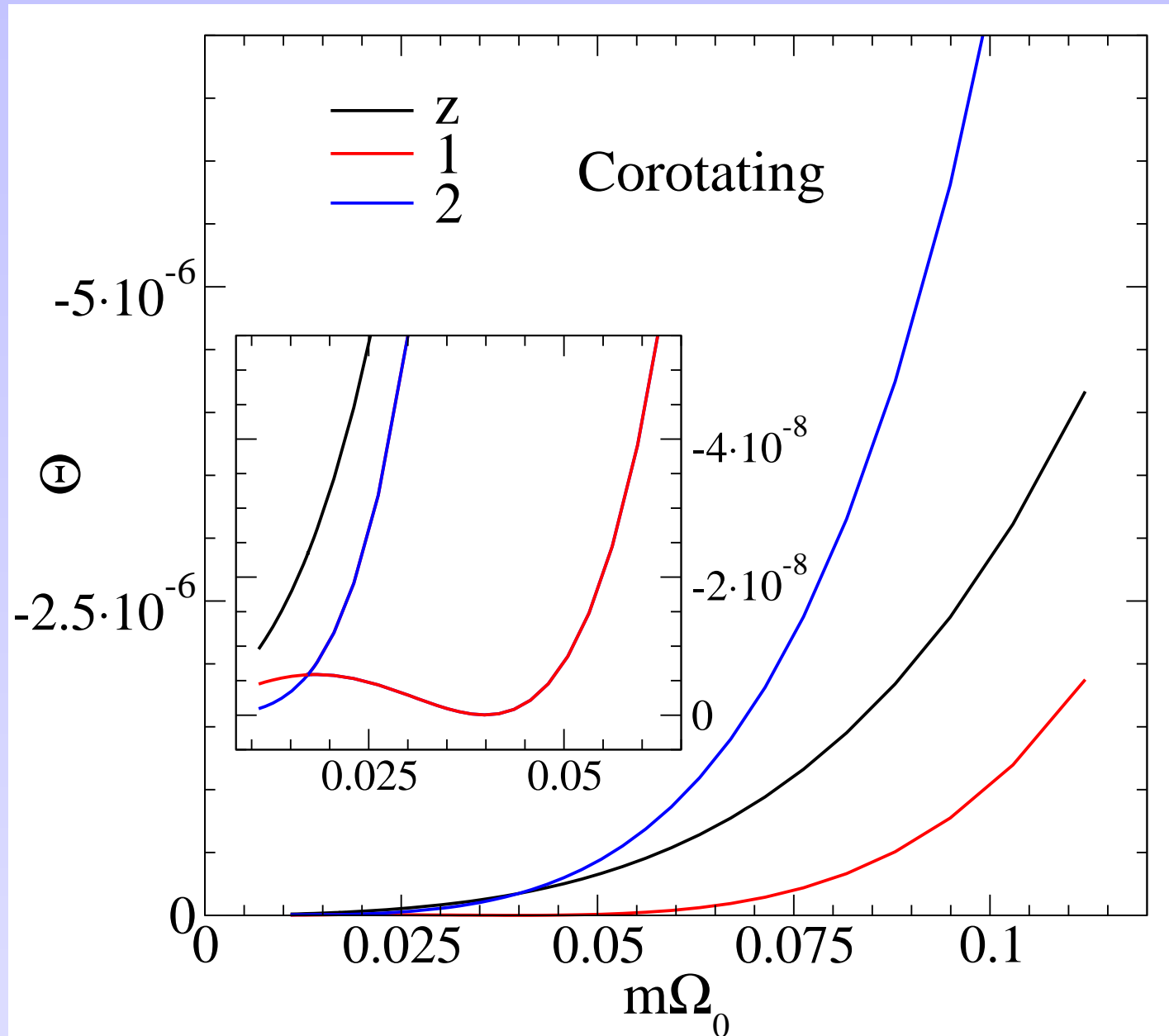
Corotating BBH AKVs

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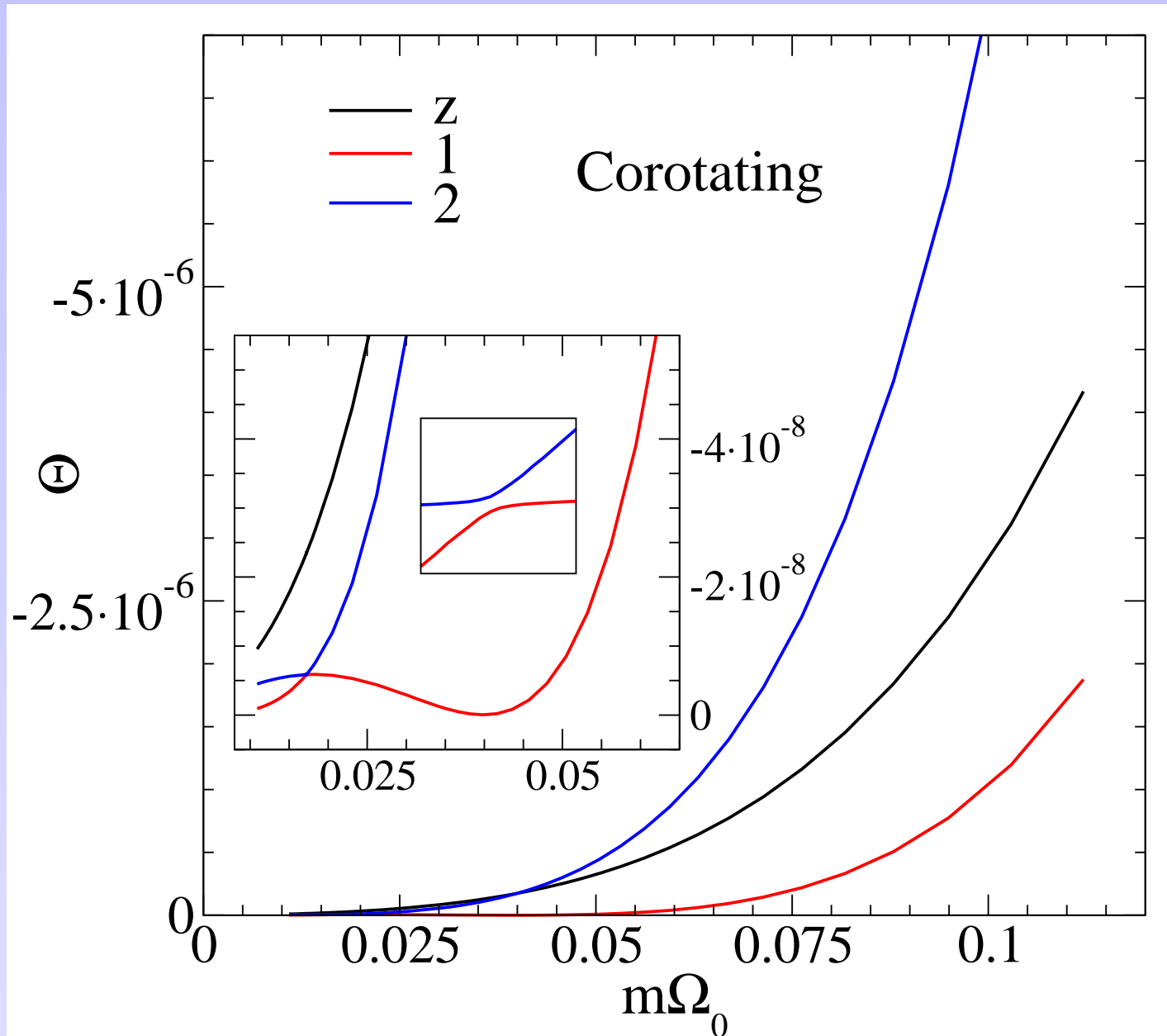
No spin!



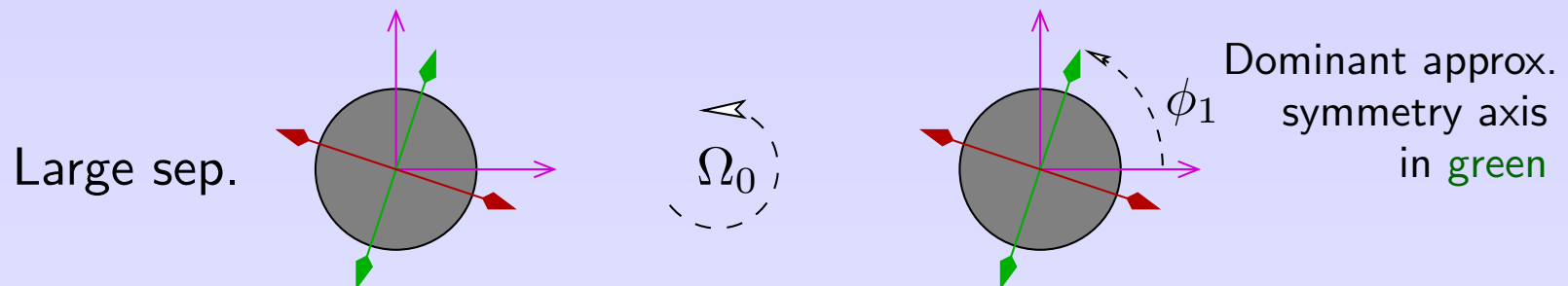
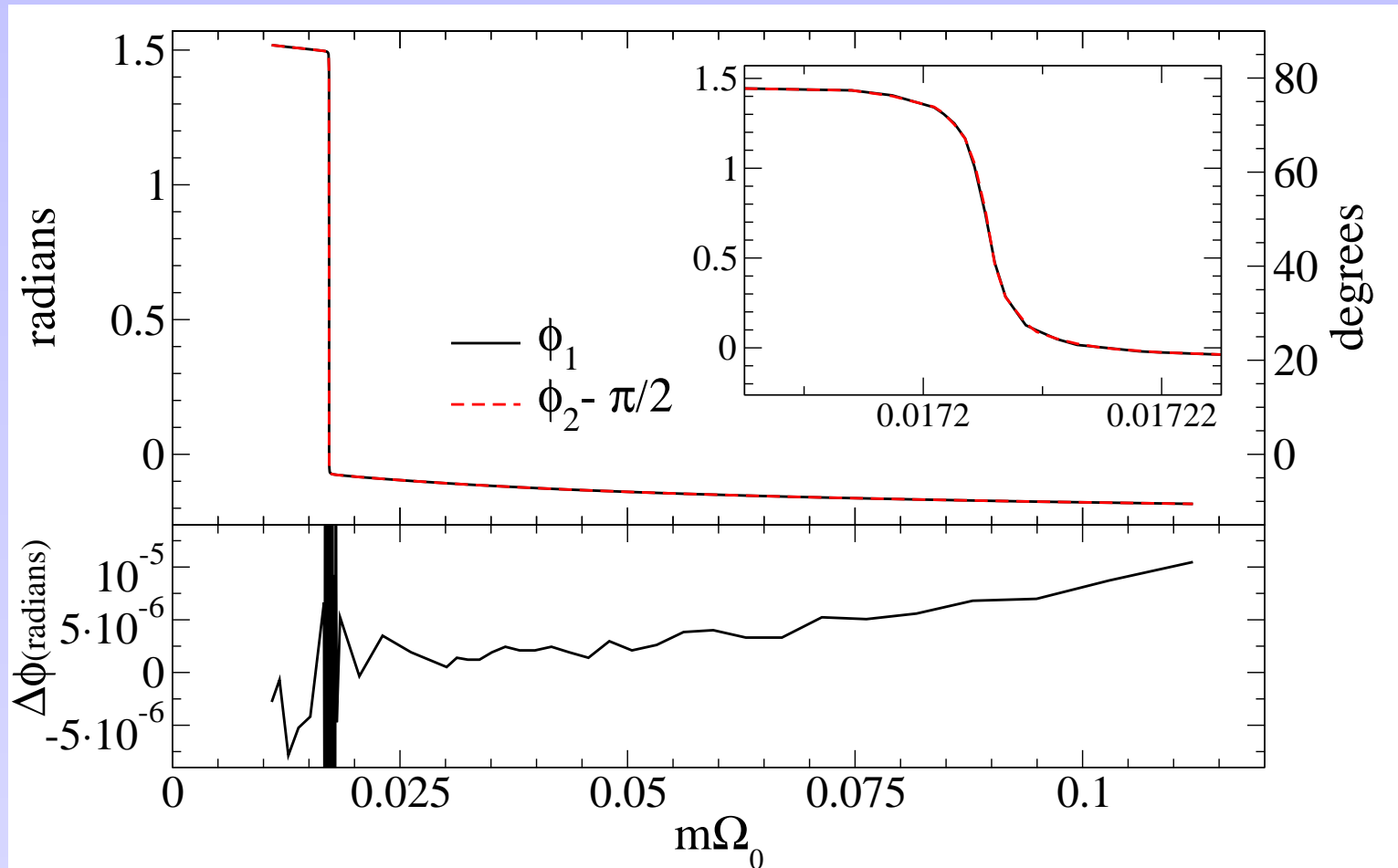
Corotating BBH AKVs



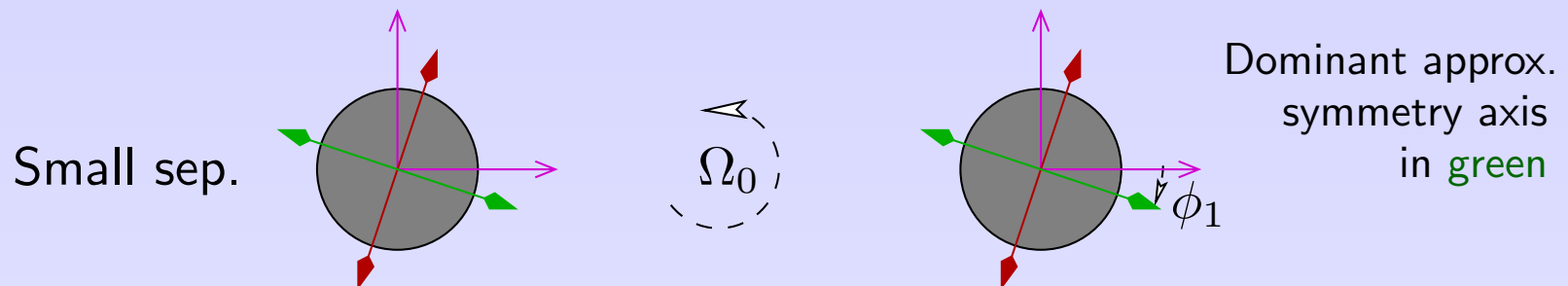
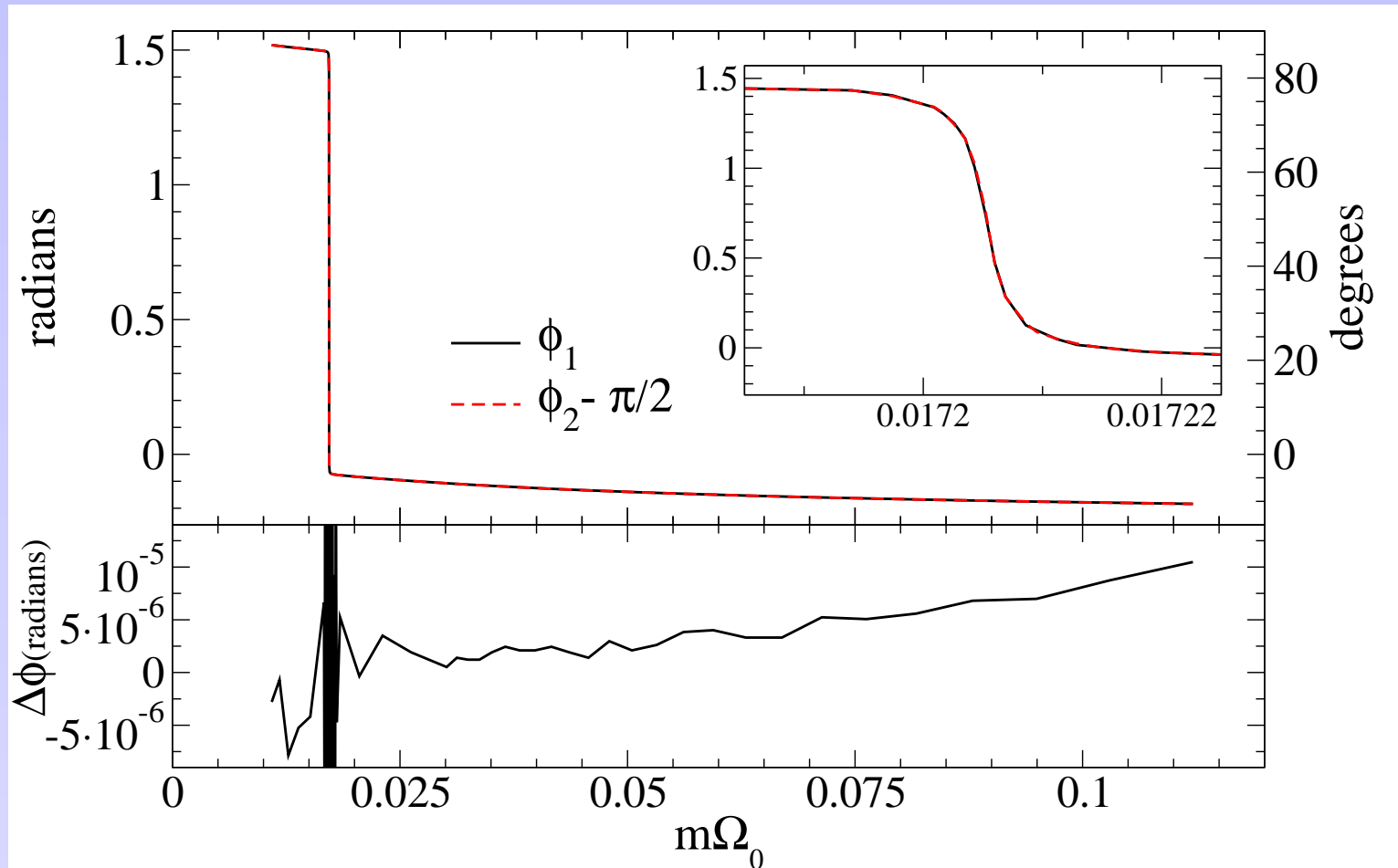
Corotating BBH AKVs



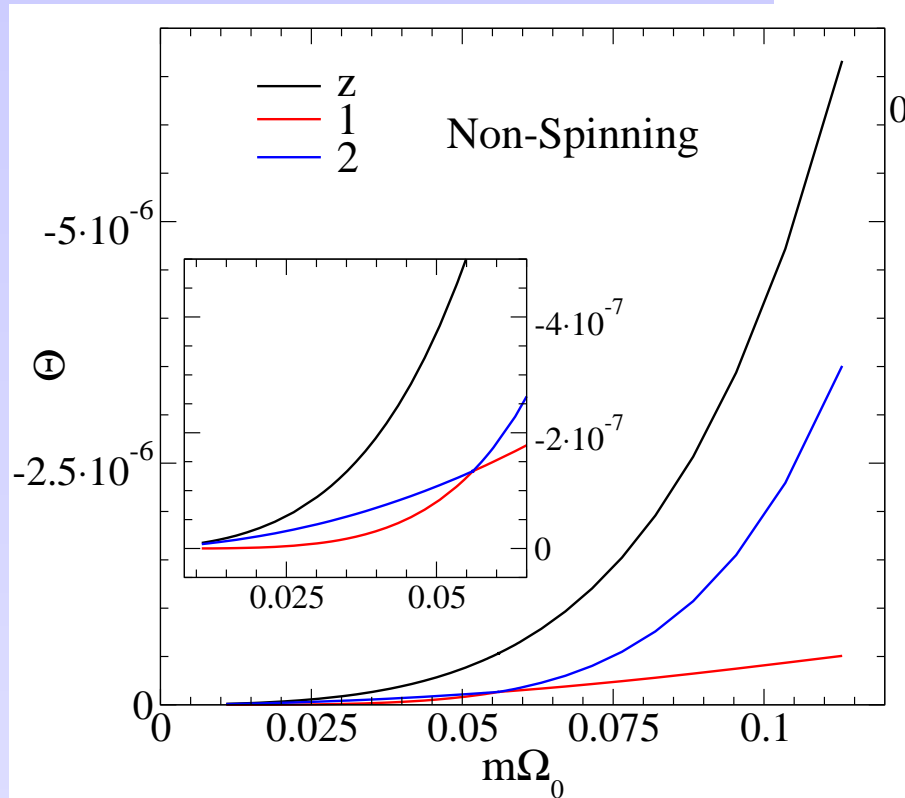
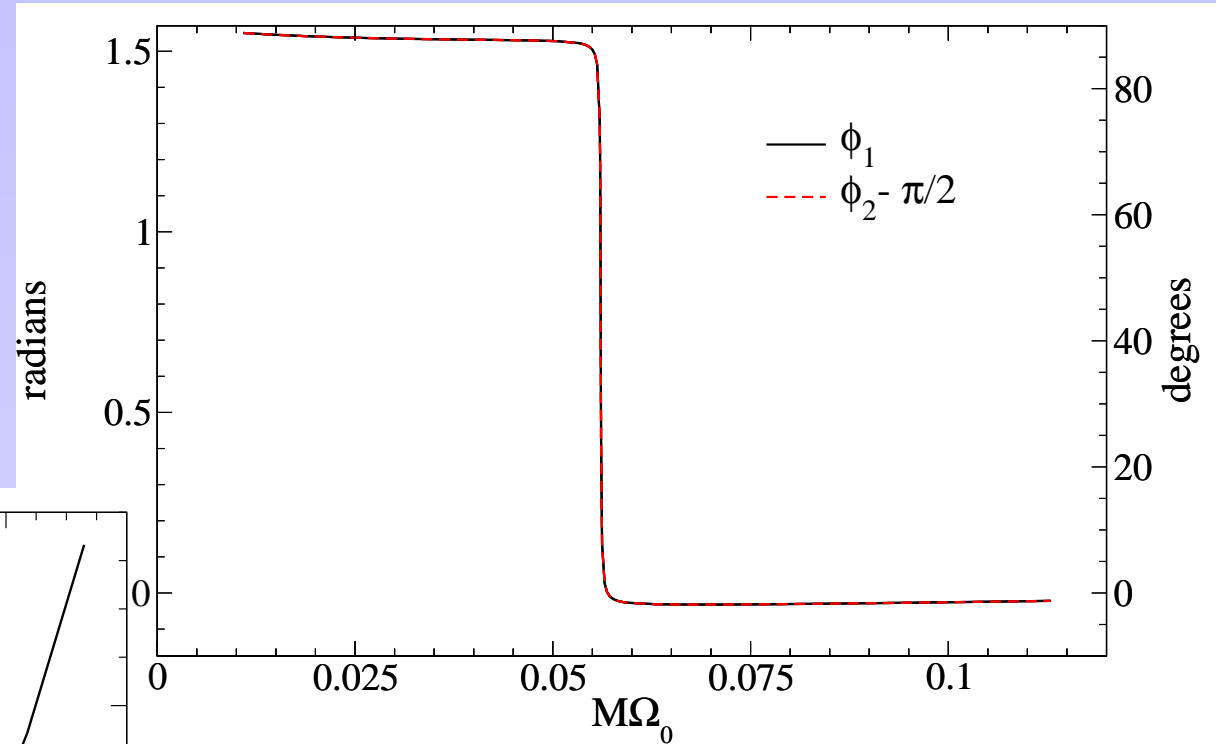
Corotating BBH AKVs



Corotating BBH AKVs



Non-Spinning BBH AKVs



Summary

- New method determines *best* AKV:

$$\text{smallest } S_{ij}S^{ij} : \xi^i = \epsilon^{ij}D_j v.$$

- Computed spin essentially the same as from Killing Transport for corotation & non-spinning equal-mass cases. Differences may be more significant when higher spin rates or greater BH distortion are considered.
- Only one AKV solution yields spin. Other AKV solutions provide new diagnostic tool for exploring horizon geometry of distorted BHs.
- Algorithm for “conformal spheres” implemented and tested. Algorithm for general S^2 nearly finished.

References

- [1] A. Ashtekar and B. Krishnan. Dynamical horizons and their properties. *Phys. Rev. D*, 68:104030/1–25, 2003. [1](#)
- [2] J. D. Brown and J. W. York, Jr. Quasilocal energy and conserved charges derived from the gravitational action. *Phys. Rev. D*, 47:1407–1419, 1993. [1](#)
- [3] O. Dreyer, B. Krishnan, D. Shoemaker, and E. Schnetter. Introduction to isolated horizons in numerical relativity. *Phys. Rev. D*, 67:024018/1–14, Jan. 2003. [2](#)