Circular Orbits and Spin in Black-Hole Initial Data

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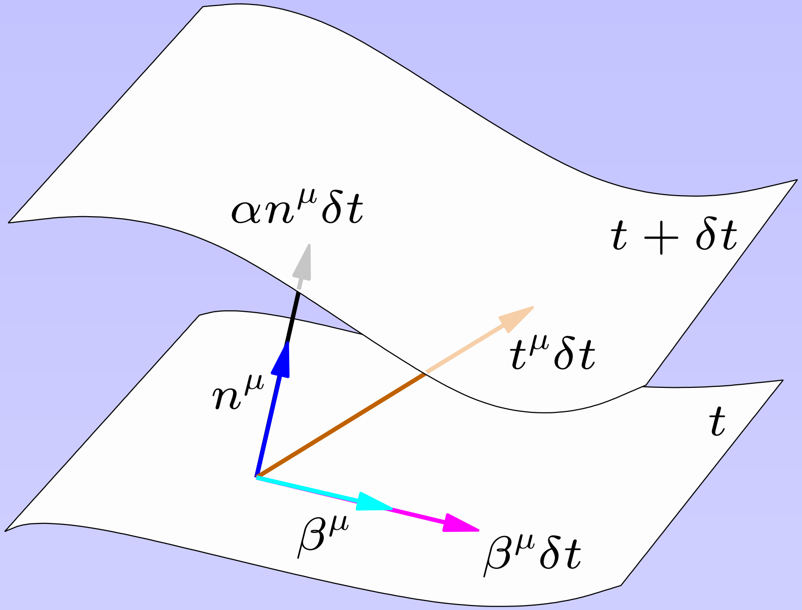
The $3+1$ Decomposition



Lapse : α Spatial metric : γ_{ij}

Shift vector : β^i Extrinsic Curvature : K_{ij}

Time vector : $t^\mu = \alpha n^\mu + \beta^\mu$

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \qquad K_{\mu\nu} = -\tfrac{1}{2}\gamma^\alpha_\mu \gamma^\beta_\nu \mathcal{L}_n g_{\alpha\beta}$$

Constraint equations

$$\bar{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$\bar{\nabla}_j\left(K^{ij} - \gamma^{ij}K\right) = 8\pi j^i$$

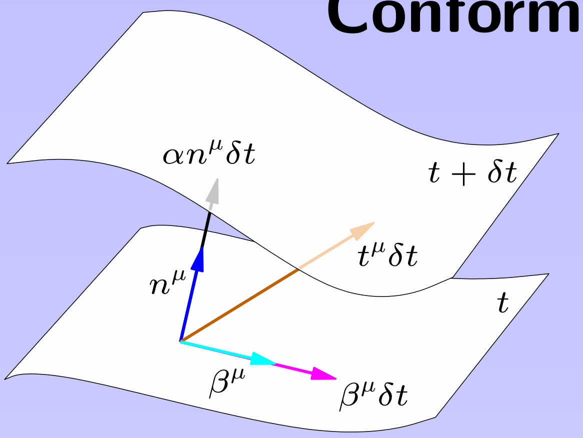
$$S_{\mu\nu} \equiv \gamma^\alpha_\mu \gamma^\beta_\nu T_{\alpha\beta}$$
$$j_\mu \equiv -\gamma^\nu_\mu n^\alpha T_{\nu\alpha}$$
$$\rho \equiv n^\mu n^\nu T_{\mu\nu}$$
$$T_{\mu\nu} = S_{\mu\nu} + 2n_{(\mu}j_{\nu)} + n_\mu n_\nu \rho$$

Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

$$\partial_t K_{ij} = -\bar{\nabla}_i \bar{\nabla}_j \alpha + \alpha\Big[\bar{R}_{ij} - 2K_{i\ell}K^\ell_j + KK_{ij} - 8\pi S_{ij} + 4\pi\gamma_{ij}(S-\rho)\Big] + \beta^\ell \bar{\nabla}_\ell K_{ij} + K_{i\ell}\bar{\nabla}_j \beta^\ell + K_{j\ell}\bar{\nabla}_i \beta^\ell$$

Conformal Thin-Sandwich Decomposition



$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

$$K^{ij} = \frac{\psi^{-10}}{2\tilde{\alpha}} \left[(\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij} \right] + \tfrac{1}{3}\psi^{-4}\tilde{\gamma}^{ij} K$$

Hamiltonian Const. $\tilde{\nabla}^2\psi - \frac{1}{8}\psi\tilde{R} - \frac{1}{12}\psi^5 K^2 + \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = -2\pi\psi^5\rho$

Momentum Const. $\tilde{\nabla}_j(\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij}\tilde{\nabla}_j\tilde{\alpha} = \frac{4}{3}\tilde{\alpha}\psi^6\tilde{\nabla}^i K + \tilde{\alpha}\tilde{\nabla}_j\left(\frac{1}{\tilde{\alpha}}\tilde{u}^{ij}\right) + 16\pi\tilde{\alpha}\psi^{10} j^i$

Const. Tr(K) eqn. $\tilde{\nabla}^2(\psi^7\tilde{\alpha}) - (\psi^7\tilde{\alpha})\left[\frac{1}{8}\psi\tilde{R} + \frac{5}{12}\psi^5 K^2 + \frac{7}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} - \psi^5\beta^i\tilde{\nabla}_i K\right]$
$= -2\pi\psi^5 K(\rho + 2S) - \psi^5\partial_t K$

$$\tilde{A}^{ij} \equiv \frac{1}{2\tilde{\alpha}}\left[(\tilde{\mathbb{L}}\beta)^{ij} - \tilde{u}^{ij}\right]$$

Constrained vars : ψ, β^i, and $\tilde{\alpha} \equiv \psi^{-6}\alpha$

Freely specified : $\tilde{\gamma}_{ij}$ $\quad \tilde{u}^{ij} \equiv -\partial_t\tilde{\gamma}^{ij}$
K *and* $\partial_t K$

Quasiequilibrium $\Rightarrow \begin{cases} \partial_t\tilde{\gamma}^{ij} = 0 \\ \partial_t K = 0 \end{cases}$

Equations of Quasiequilibrium

Ham. & Mom. const. eqns., & Const Tr(K) eqn. from Conf. TS $+\ \tilde{u}^{ij} = \partial_t K = 0$ $\Rightarrow$ **Eqns. of Quasiequilibrium**

With $\tilde{\gamma}_{ij} = f_{ij}$ and $K = 0$, these equations have been widely used to construct binary neutron star initial data[3, 11, 4, 14].

Binary neutron star initial data require:

- boundary conditions at infinity compatible with asymptotic flatness and corotation.
$$\psi|_{\mathrm{r}\to\infty} = 1 \quad \beta^i|_{\mathrm{r}\to\infty} = \Omega_0 \left(\frac{\partial}{\partial\phi}\right)^i \quad \alpha|_{\mathrm{r}\to\infty} = 1$$
- compatible solution of the equations of hydrostatic equilibrium. $(\Rightarrow \Omega_0)$

Binary black hole initial data require:

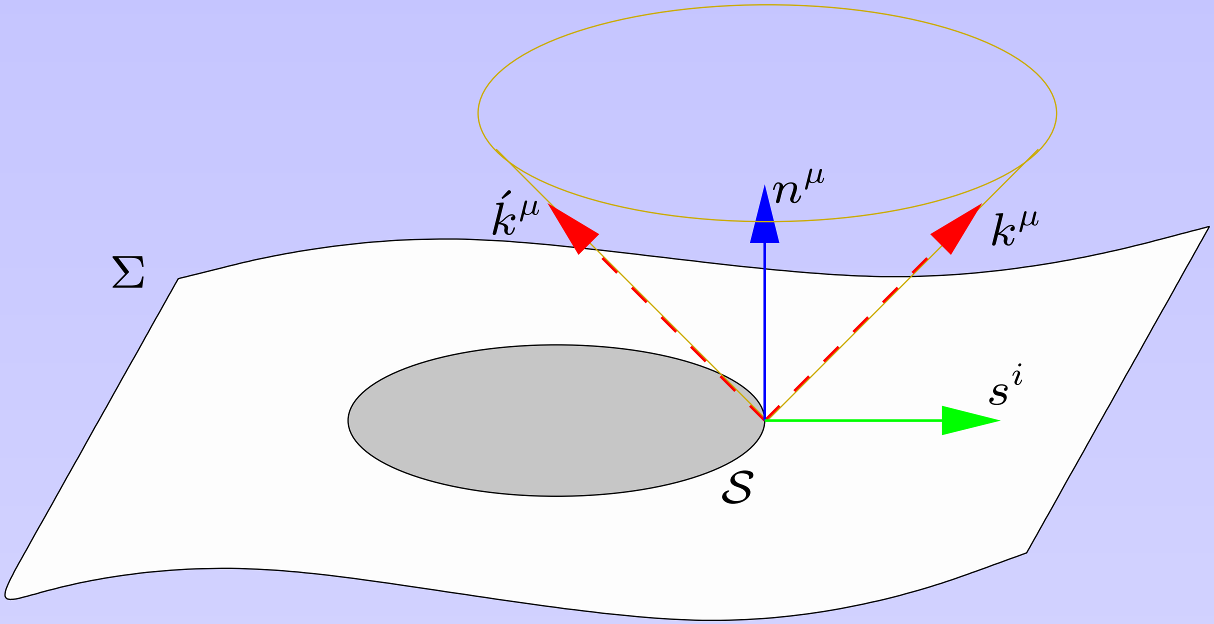
⋆ a means for choosing the angular velocity of the orbit Ω_0.

⋆ *with excision*, inner boundary conditions are needed for ψ, β^i, and $\tilde{\alpha}$.

Gourgoulhon, Grandclément, & Bonazzola[9, 10]: Black-hole binaries with $\tilde{\gamma}_{ij} = f_{ij}$ & $K = 0$, "inversion-symmetry", and "Killing-horizon" conditions on the excision boundaries.

"Solutions" require *constraint violating* regularity condition imposed on inner boundaries!

The Inner Boundary



$$s_i \equiv \frac{\bar{\nabla}_i \tau}{|\bar{\nabla}\tau|}$$

$$h_{ij} \equiv \gamma_{ij} - s_i s_j$$

$$k^\mu \equiv \frac{1}{\sqrt{2}}\left(n^\mu + s^\mu\right)$$

$$\acute{k}^\mu \equiv \frac{1}{\sqrt{2}}\left(n^\mu - s^\mu\right)$$

Extrinsic curvature of $\mathcal{S}$ embedded in spacetime

$$\Sigma_{\mu\nu} \equiv \tfrac{1}{2} h^\alpha_\mu h^\beta_\nu \mathcal{L}_k g_{\alpha\beta}$$

$$\acute{\Sigma}_{\mu\nu} \equiv \tfrac{1}{2} h^\alpha_\mu h^\beta_\nu \mathcal{L}_{\acute{k}} g_{\alpha\beta}$$

Extrinsic curvature of $\mathcal{S}$ embedded in Σ

$$H_{ij} \equiv \tfrac{1}{2} h^k_i h^\ell_j \mathcal{L}_s \gamma_{k\ell}$$

$$\Sigma_{ij} = \frac{1}{\sqrt{2}}\left(H_{ij} - J_{ij}\right)$$

$$\acute{\Sigma}_{ij} = -\frac{1}{\sqrt{2}}\left(H_{ij} + J_{ij}\right)$$

Projections of K_{ij} onto $\mathcal{S}$

$$J_{ij} \equiv h^k_i h^\ell_j K_{k\ell}$$

$$J_i \equiv h^k_i s^\ell K_{k\ell}$$

$$J \equiv h^{ij} J_{ij} = h^{ij} K_{ij}$$

Expansion of null rays

$$\theta \equiv h^{ij}\Sigma_{ij} = \frac{1}{\sqrt{2}}\left(H - J\right)$$

$$\acute{\theta} \equiv h^{ij}\acute{\Sigma}_{ij} = -\frac{1}{\sqrt{2}}\left(H + J\right)$$

Shear of null rays

$$\sigma_{ij} \equiv \Sigma_{ij} - \tfrac{1}{2} h_{ij}\theta$$

$$\acute{\sigma}_{ij} \equiv \acute{\Sigma}_{ij} - \tfrac{1}{2} h_{ij}\acute{\theta}$$

AH and QE Conditions on the Inner Boundary

The quasiequilibrium inner boundary conditions start with the following assumptions:

1. The inner boundary $\mathcal{S}$ is a (MOTS):
   marginally outer-trapped surface

   $\rightarrow \quad \theta = 0$

2. The horizons are in quasiequilibrium:
   $\rightarrow \quad \sigma_{ij} = 0$ and no matter is on $\mathcal{S}$

   Raychaudhuri's equation implies that MOTS initially evolves along k^μ.

   $$\mathcal{L}_k\theta = \tfrac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu = 0$$

3. The time evolution vector lies in outgoing null surface through $\mathcal{S}$:
   $\rightarrow \quad t^\mu k_\mu|_\mathcal{S} = 0$

$$\left.\begin{aligned} k^\mu &\equiv \tfrac{1}{\sqrt{2}}(n^\mu + s^\mu) \\ t^\mu &= \alpha n^\mu + \beta^\mu \end{aligned}\right\} \Longrightarrow \quad \alpha|_\mathcal{S} = \beta^i s_i|_\mathcal{S} \equiv \beta_\perp|_\mathcal{S}$$

AH/Quasiequilibrium Boundary Conditions

$$\theta = \frac{\psi^{-2}}{\sqrt{2}}\left[\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j + 4\tilde{s}^k\tilde{\nabla}_k \ln\psi - \psi^2 J\right]$$

$$\begin{aligned}\sigma_{ij} \quad = \quad & \frac{1}{\sqrt{2}}(H_{ij} - \tfrac{1}{2}h_{ij}H)\left(1 - \frac{\beta_\perp}{\alpha}\right) \\ & - \frac{1}{\sqrt{2}}\frac{\psi^4}{\alpha}\left\{\tilde{D}_{(i}\beta_{\parallel j)} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k - \tfrac{1}{2}[\tilde{h}_{ik}\tilde{h}_{j\ell}\tilde{u}^{k\ell} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{h}_{k\ell}\tilde{u}^{k\ell}]\right\}\end{aligned}$$

AH/Quasiequilibrium Boundary Conditions

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$$\begin{aligned}\tilde{s}^k\tilde{\nabla}_k\ln\psi &= -\tfrac{1}{4}(\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j - \psi^2 J) \\ \beta^i &= \alpha\psi^{-2}\tilde{s}^i + \beta_\parallel{}^i \\ 0 &= \tilde{D}_{(i}\beta_{\parallel j)} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k\end{aligned}$$

AH/Quasiequilibrium Boundary Conditions

$$\theta = \frac{\psi^{-2}}{\sqrt{2}}\left[\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j + 4\tilde{s}^k\tilde{\nabla}_k \ln\psi - \psi^2 J\right]$$

$$\begin{aligned}\sigma_{ij} &= \frac{1}{\sqrt{2}}(H_{ij} - \tfrac{1}{2}h_{ij}H)\left(1 - \frac{\beta_\perp}{\alpha}\right) \\ &\quad - \frac{1}{\sqrt{2}}\frac{\psi^4}{\alpha}\left\{\tilde{D}_{(i}\beta_{\parallel\, j)} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k - \tfrac{1}{2}[\tilde{h}_{ik}\tilde{h}_{j\ell}\tilde{u}^{k\ell} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{h}_{k\ell}\tilde{u}^{k\ell}]\right\}\end{aligned}$$

$$\begin{aligned}\tilde{s}^k\tilde{\nabla}_k \ln\psi &= -\tfrac{1}{4}(\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j - \psi^2 J) \\ \beta^i &= \alpha\psi^{-2}\tilde{s}^i + \beta_\parallel{}^i \\ 0 &= \tilde{D}_{(i}\beta_{\parallel\, j)} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k\beta_\parallel{}^k\end{aligned}$$

The boundary condition on the lapse is not fixed by QE assumptions. It has been shown to be associated with the initial temporal gauge choice and is taken to be freely specifiable.

Defining the Spin of the Black Hole

The spin parameters ${\beta_{\|}}^{i}$ can be defined by demanding that the time vector associated with quasiequilibrium in the corotating frame must be null, forming the null generators of the horizon.

$$k^{\mu} \propto (n^{\mu} + s^{\mu}) \implies k^{\mu} = \left[1, \alpha s^{i} - \beta^{i}\right]$$

This vector k^{μ} is null for any choice of α & β^{i}.

In the frame where a black hole is not spinning, the null time vector has components $t^{\mu} = [1, \vec{0}]$.

Corotating Holes

Corotating holes are at rest in the corotating frame, where we must pose boundary conditions. So,

$$k^{\mu} = \left[1, \alpha s^{i} - \beta^{i}\right] = [1, \vec{0}]$$

Thus we find

$$\beta^{i} = \alpha s^{i} \ \Rightarrow \ {\beta_{\|}}^{i} = 0$$

Non-Spinning Holes

Non-spinning holes are at rest in the inertial frame. With the time vectors in the inertial and corotating frames related by

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \bar{t}} + \Omega_0 \frac{\partial}{\partial \phi}$$

$$k^{\mu} = \left[1, \alpha s^{i} - \beta^{i}\right] = [1, -\Omega_0 (\partial/\partial\phi)^{i}]$$

Thus we find

$$\beta^{i} = \alpha s^{i} + \Omega_0 \left(\frac{\partial}{\partial \phi}\right)^{i} \Rightarrow \ {\beta_{\|}}^{i} = \Omega_0 \xi^{i}$$

$$\xi^{i} \approx \left(\tfrac{\partial}{\partial\phi}\right)^{i} \quad \& \quad \tilde{D}_{(i}\xi_{j)} - \tfrac{1}{2}\tilde{h}_{ij}\tilde{D}_k \xi^{k} = 0$$

The Orbital Angular Velocity

- For a given choice of the Lapse BC, $\tilde{\gamma}_{ij}$ and K, we are still left with a family of solutions parameterized by the orbital angular velocity Ω_0.
- Except for the case of a single spinning black hole, it is not reasonable to expect more than one value of Ω_0 to correspond to a system in quasiequilibrium.

GGB[9, 10] have suggested a way to pick the quasiequilibrium value of Ω_0:

Ω_0 is chosen as the value for which the ADM energy $E_{\rm ADM}$ equals the Komar mass $M_{\rm K}$.

Komar mass	$M_{\rm K} = \frac{1}{4\pi}\oint_\infty \gamma^{ij}(\bar{\nabla}_i\alpha - \beta^k K_{ik})\mathrm{d}^2S_j$	Acceptable definition of the mass *only for stationary spacetimes*.
ADM energy	$E_{\rm ADM} = \frac{1}{16\pi}\oint_\infty \gamma^{ij}\bar{\nabla}_k(\mathcal{G}^k_i - \delta^k_i\mathcal{G})\mathrm{d}^2S_j$	Acceptable definition of the mass *for arbitrary spacetimes*. $\mathcal{G}_{ij} \equiv \gamma_{ij} - f_{ij}$

Summary of QE Formalism

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \qquad K^{ij} = \psi^{-10}\tilde{A}^{ij} + \tfrac{1}{3}\gamma^{ij}K \qquad \tilde{A}^{ij} = \frac{\psi^6}{2\alpha}(\tilde{\mathbb{L}}\beta)^{ij}$$

$$\partial_t \tilde{\gamma}_{ij} = 0$$

$$\tilde{\nabla}^2\psi - \tfrac{1}{8}\psi\tilde{R} - \tfrac{1}{12}\psi^5K^2 + \tfrac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = 0$$

$$\tilde{\nabla}_j(\tilde{\mathbb{L}}\beta)^{ij} - (\tilde{\mathbb{L}}\beta)^{ij}\tilde{\nabla}_j \ln \alpha\psi^{-6} = \tfrac{4}{3}\alpha\tilde{\nabla}^i K$$

$$\tilde{\nabla}^2(\alpha\psi) - (\alpha\psi)\left[\tfrac{1}{8}\tilde{R} + \tfrac{5}{12}\psi^4K^2 + \tfrac{7}{8}\psi^{-8}A_{ij}A^{ij}\right] = \psi^5\beta^i\tilde{\nabla}_i K$$

$$\partial_t K = 0$$

$$\tilde{s}^k\tilde{\nabla}_k \ln\psi|_{\mathcal{S}} = -\tfrac{1}{4}(\tilde{h}^{ij}\tilde{\nabla}_i\tilde{s}_j - \psi^2 J)|_{\mathcal{S}}$$

$$\theta = 0$$

$$\beta^i|_{\mathcal{S}} = \begin{cases} \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} & \text{corotation} \\ \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} + \Omega_0\xi^i|_{\mathcal{S}} & \text{irrotation} \end{cases}$$

$$\sigma_{ij} = 0 \qquad t^\mu k_\mu = 0$$

$$\alpha|_{\mathcal{S}} = \text{unspecified by QE}$$

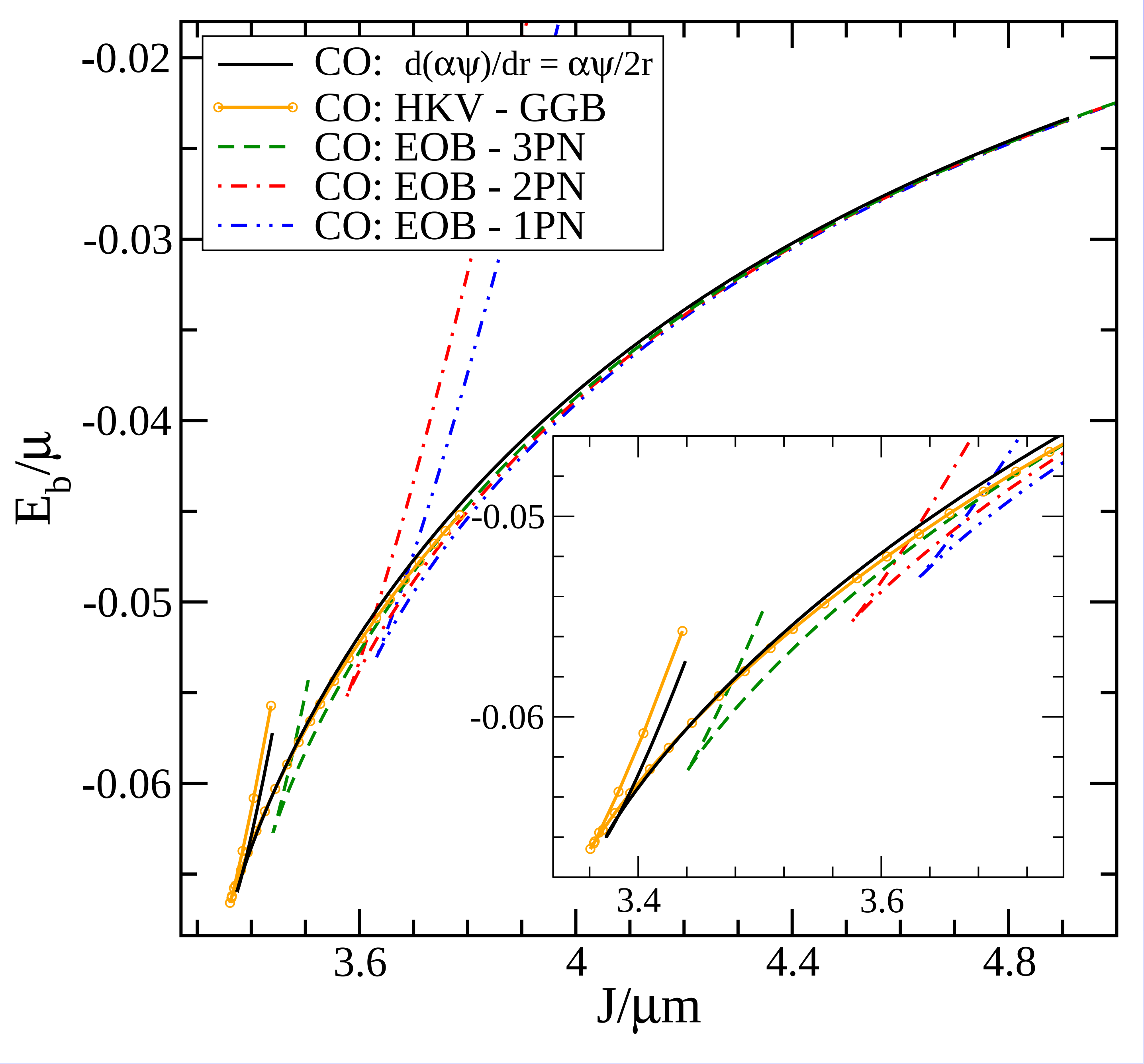
$$\psi|_{\mathrm{r}\to\infty} = 1$$

$$\beta^i|_{\mathrm{r}\to\infty} = \Omega_0\left(\frac{\partial}{\partial\phi}\right)^i$$

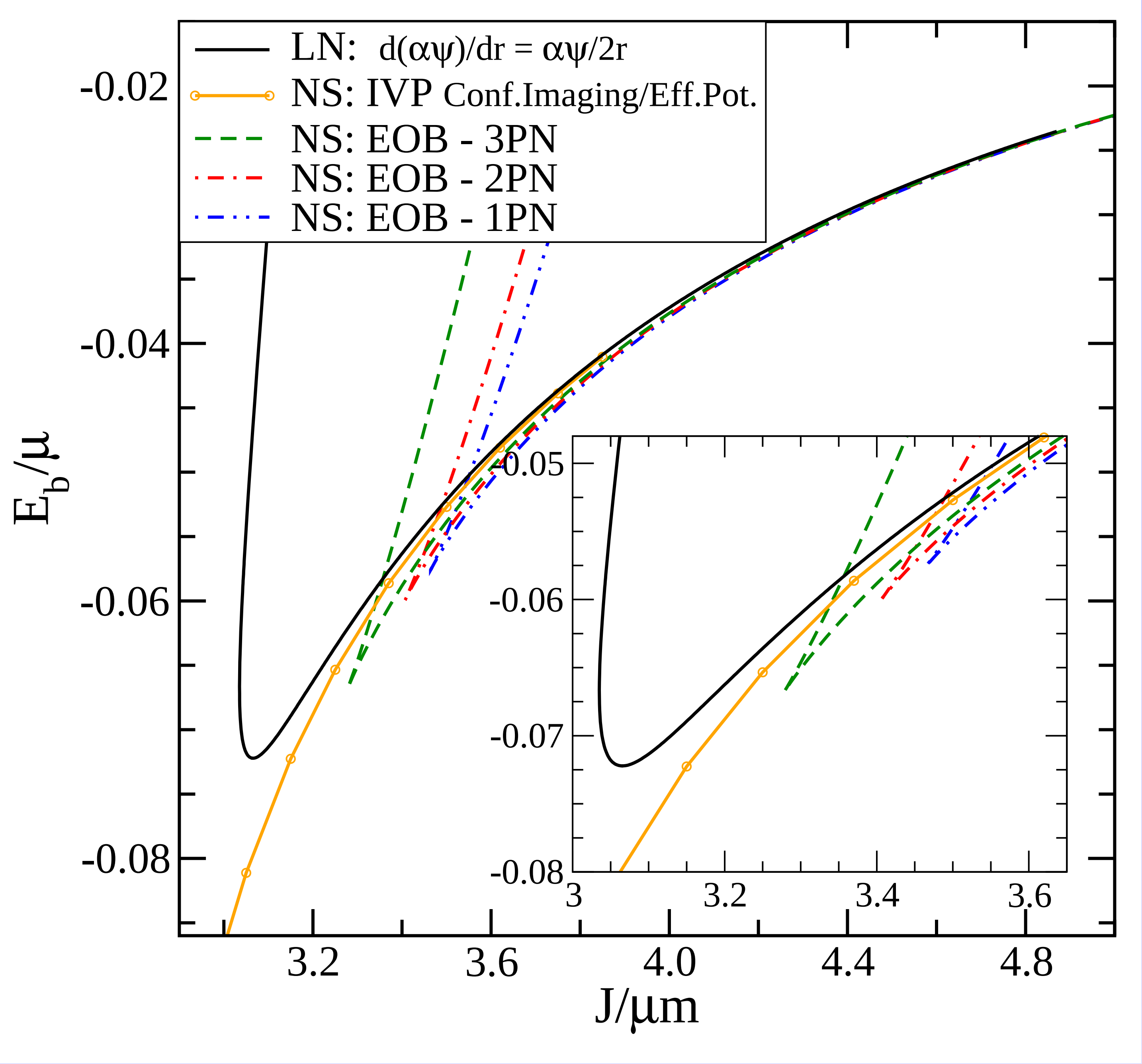
$$\alpha|_{\mathrm{r}\to\infty} = 1$$

The only remaining freedom in the system is the choice of the lapse boundary condition, the initial spatial and temporal gauge, and the initial dynamical("wave") content found in $\alpha|_{\mathcal{S}}$, $\tilde{\gamma}_{ij}$ and K.

Corotating; Comparison; E_b/μ vs $J/\mu m$



"Leading-Order" Non-Spinning; Comparison; E_b/μ vs $J/\mu m$



Are We Finding Quasicircular orbits?

- The Komar mass criteria is used to choose the quasiequilibrium model which implies quasicircular orbits.

- Can we find an independent way of verifying this?
  - Comparison with post-Newtonian:
    1. Compare plots as we have done already.
    2. Look at post-Newtonian definitions of ellipticity (eg. Mora & Will[13] or Memmesheimer, Gopakumar, & Schäfer[12]).
  - An effective potential approach[6, 7].

Measuring the Spin of a Black Hole

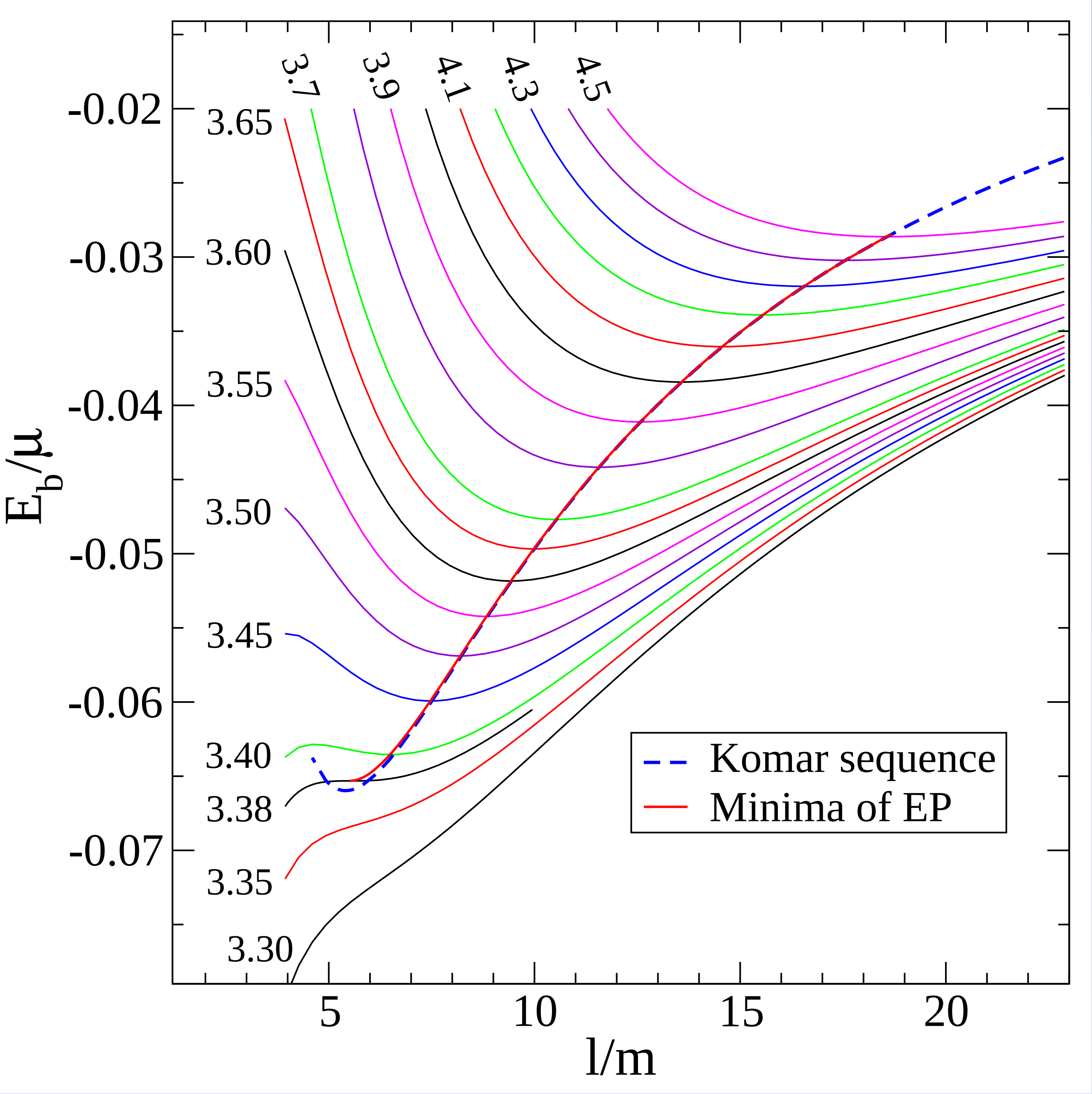
- Spin is only rigorously defined at spatial/null infinity.
- Must use *quasi-local* definition: e.g. Brown & York[5] or Ashtekar & Krishnan[2]

$$
\begin{aligned}
S &= -\frac{1}{8\pi}\oint_{BH} K_{ij}\xi^i s^j \sqrt{h}\, d^2x \\
&= -\frac{1}{8\pi}\oint_{BH} \tilde{A}_{ij}\xi^i \tilde{s}^j \sqrt{\tilde{h}}\, d^2x
\end{aligned}
$$

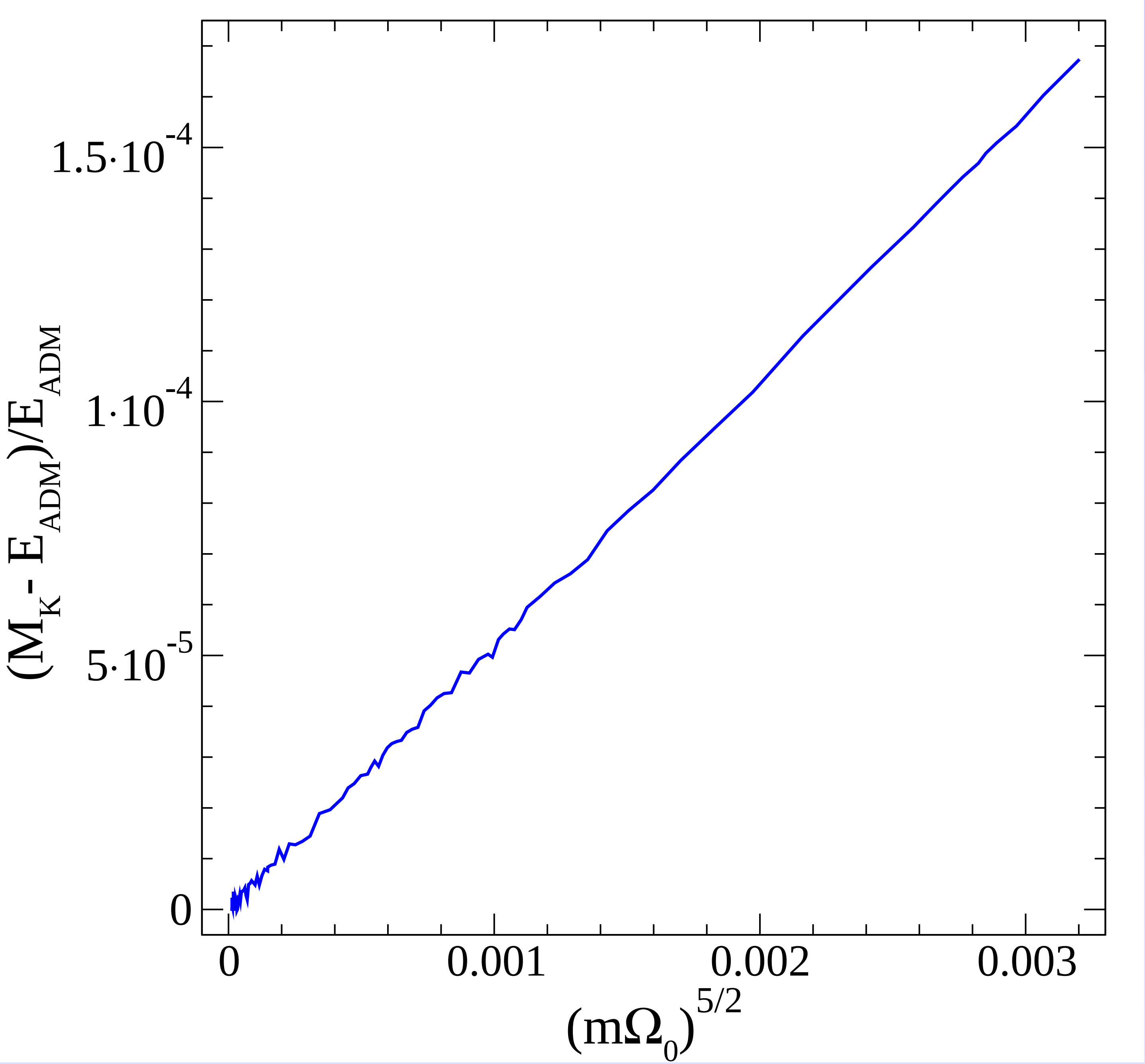
$\xi^i \tilde{s}_i = 0$

$$
\xi^i = \begin{cases} \xi^i_{\mathsf{CK}} & : \quad \text{Killing vector of } \tilde{h}_{ij} \Rightarrow \text{conformal Killing vector of } h_{ij} \\ \xi^i_{\mathsf{KV}} & : \quad \text{Killing vector of } h_{ij} \end{cases}
$$

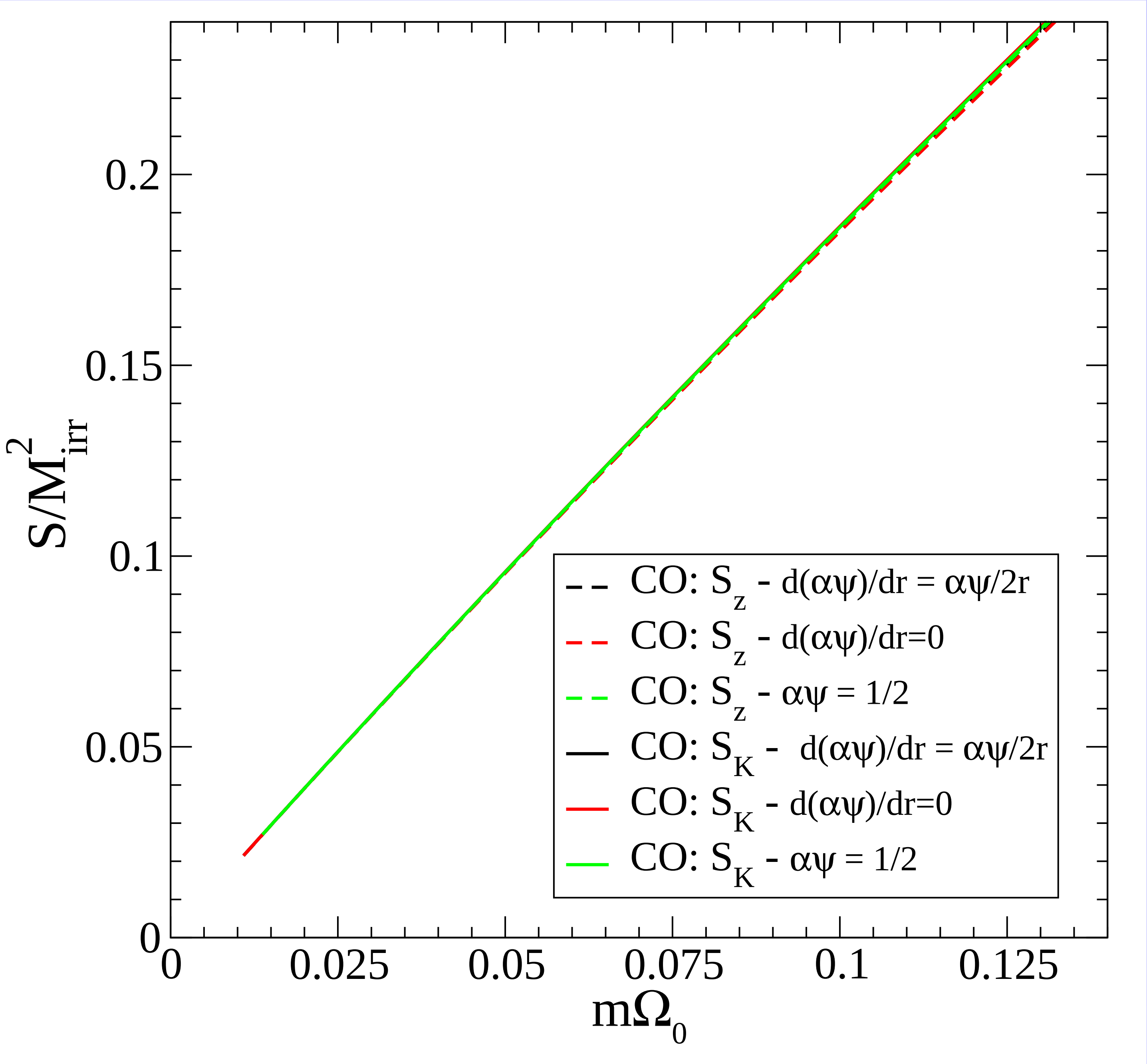
Corotation: Effective Potential; E_b/μ vs ℓ/m



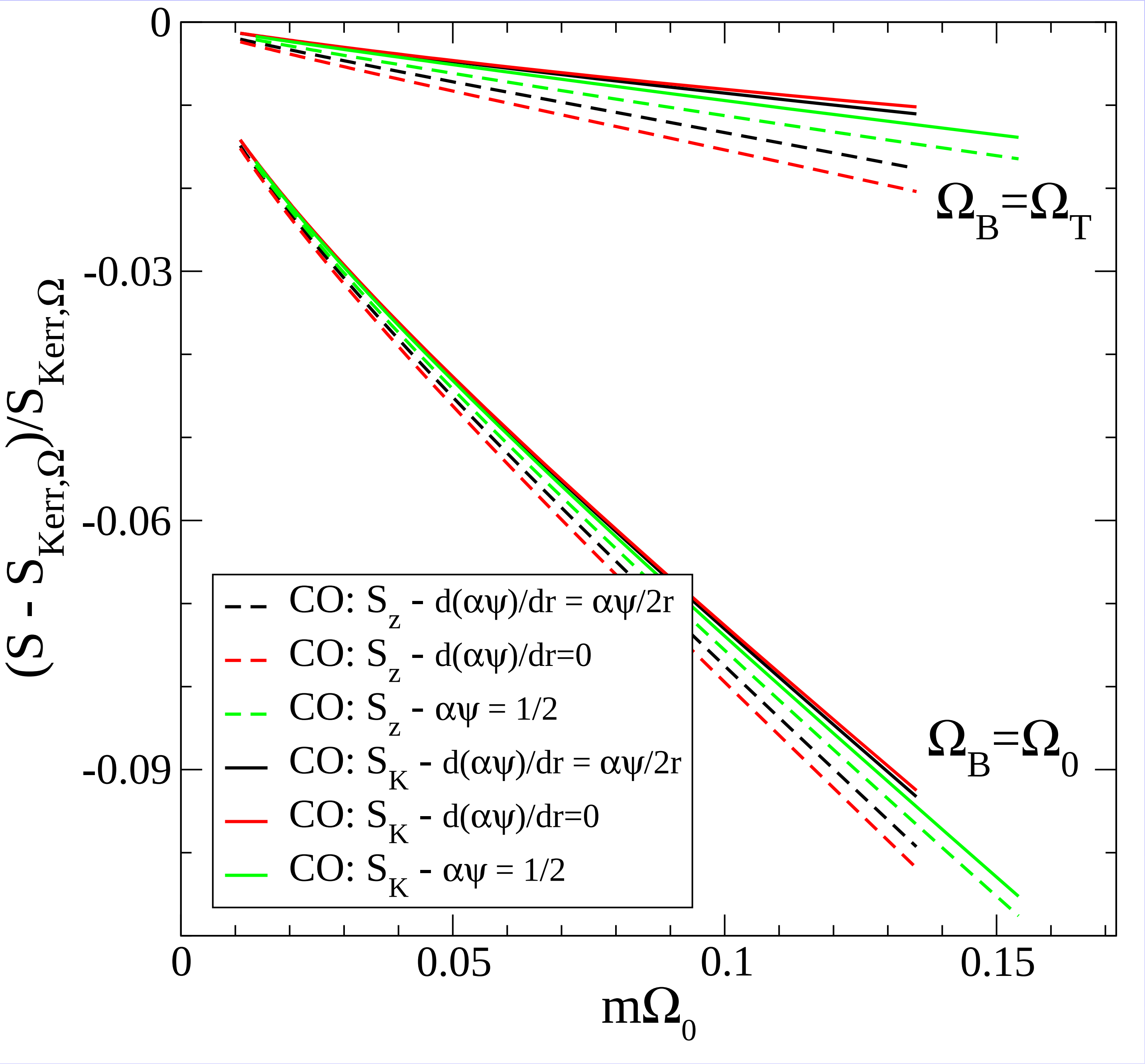
Corotation: Error in Komar-Condition; E_b/μ vs ℓ/m



Corotation: Flat & True KV Spin; S/M_{irr}^2 vs $m\Omega_0$



Corotation: Flat & True KV Spin Error; $(S - S_{Kerr})/S_{Kerr}$ vs $m\Omega_0$



Tidal Field Rotation Rate

From the local inertial frame of one black hole, the tidal field of the second black hole is rotating with an angular velocity Ω_T[1]

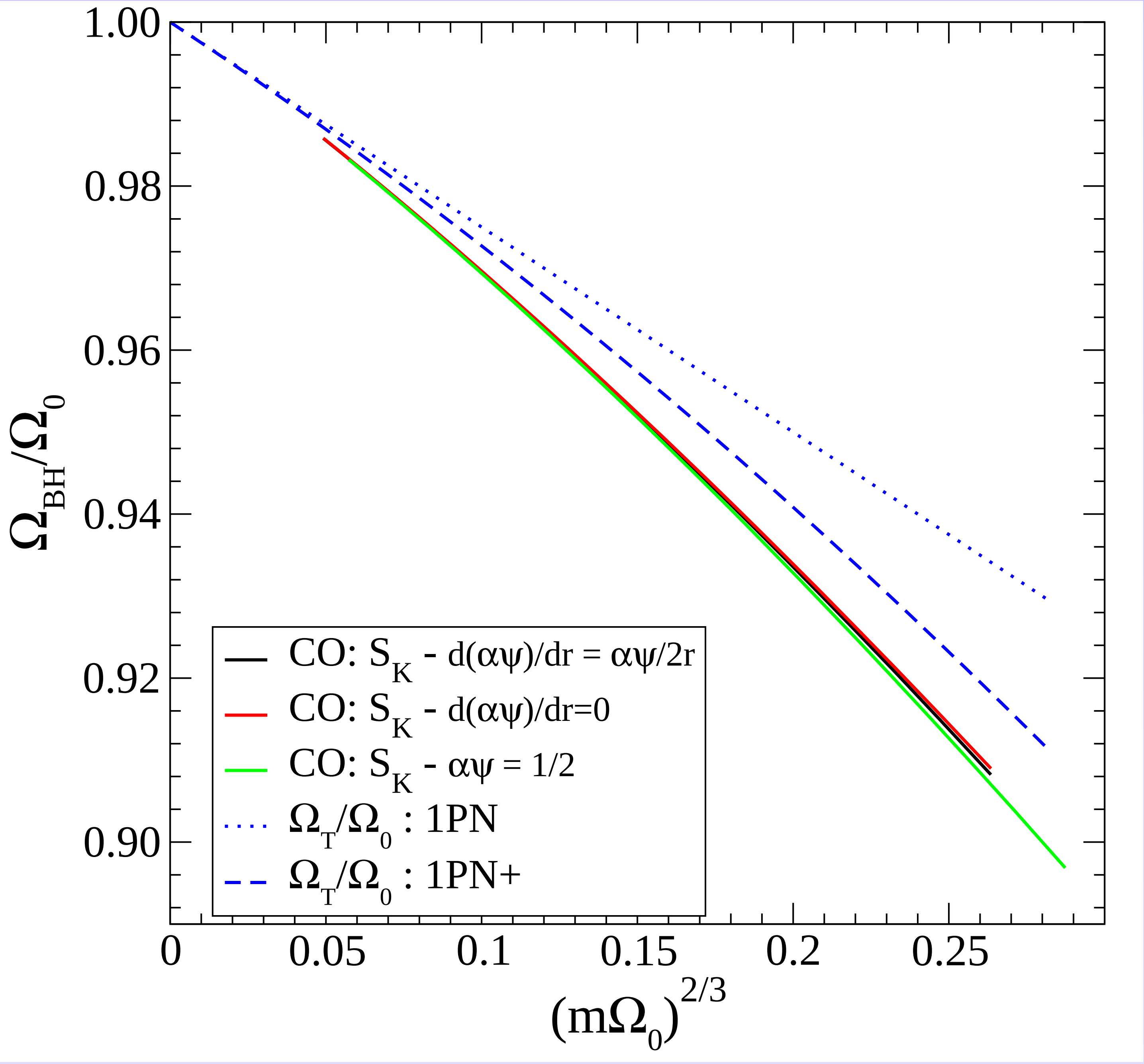
$$\Omega_T = \Omega_0 \left[1 - \eta \left(\frac{m}{b}\right) + O\left(\frac{m}{b}\right)^{3/2}\right]$$

To 1PN order

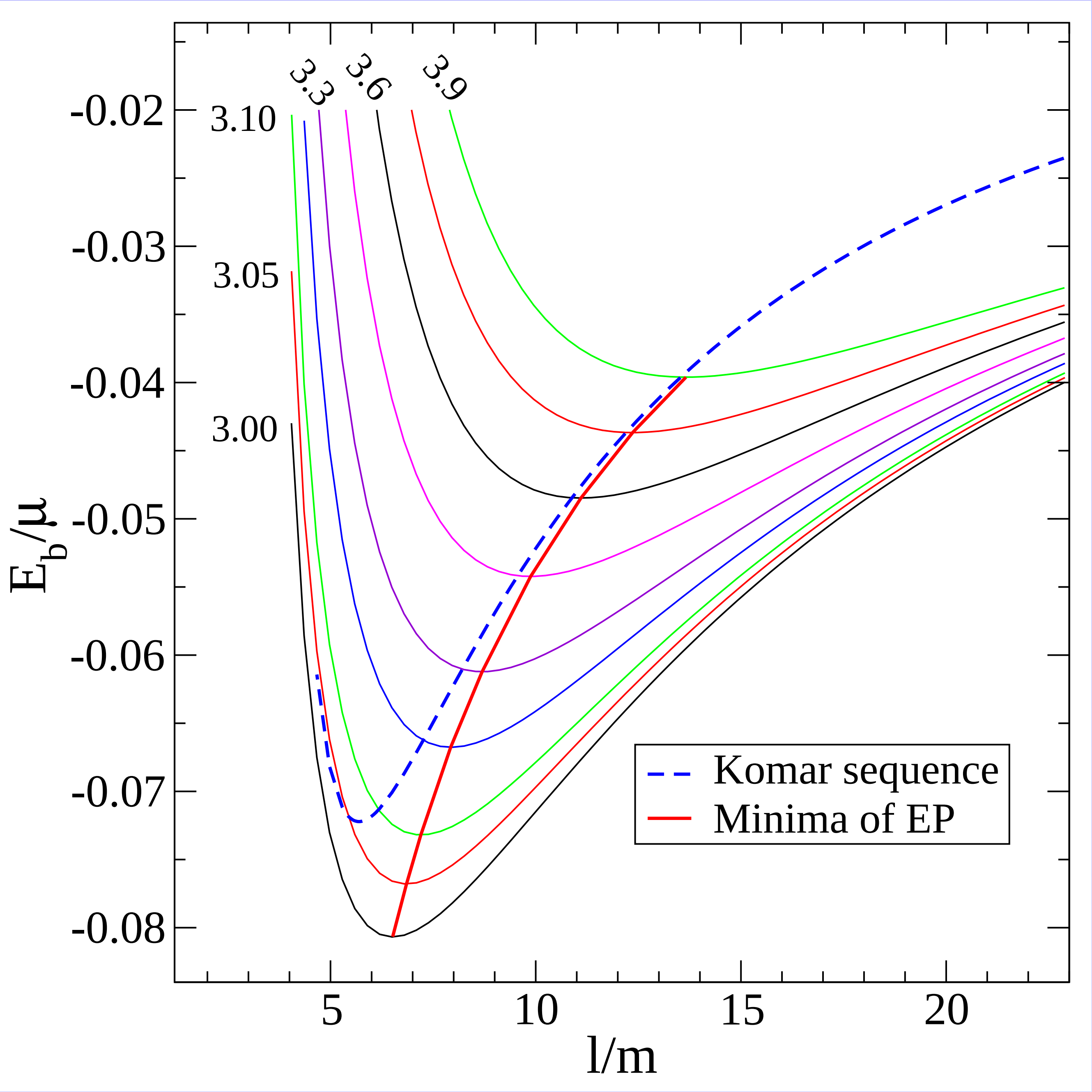
$$\frac{m}{b} = (m\Omega_0)^{2/3}\left[1 + \left(1 - \tfrac{1}{3}\eta\right)(m\Omega_0)^{2/3} + O\left(m\Omega_0\right)^{4/3}\right]$$

$$\frac{\Omega_T}{\Omega_0} = 1 - \eta\,(m\Omega_0)^{2/3} + \Lambda\,(m\Omega_0) - \left[\eta\left(1 - \tfrac{1}{3}\eta\right) - \Gamma\right](m\Omega_0)^{4/3} + \ldots$$

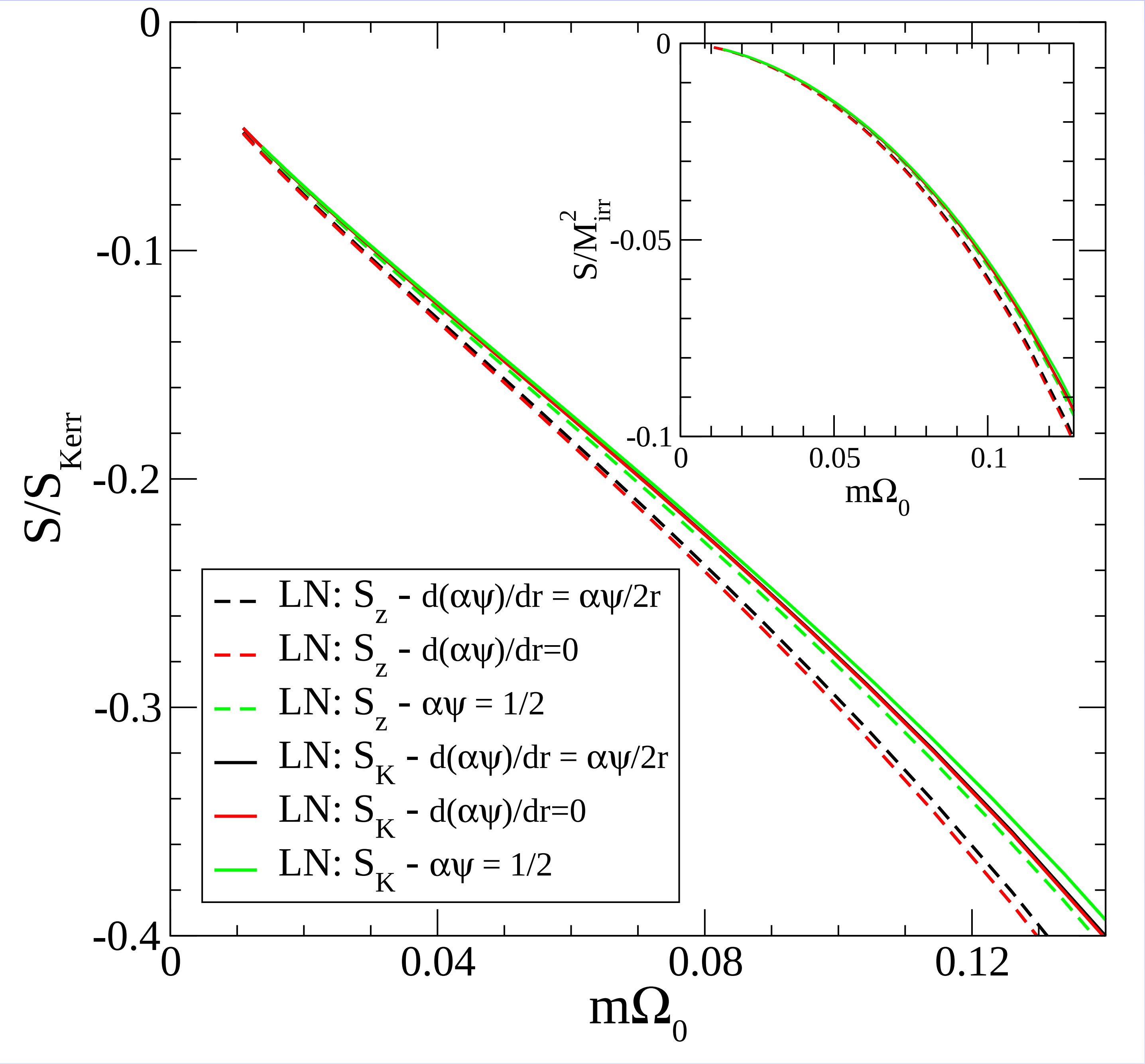
Ratio of Spin and Orbital Angular Velocities; Ω_{BH}/Ω_0 vs $m\Omega_0$



“Leading-Order” Non-spinning : Effective Potential; E_b/μ vs ℓ/m



LN: Flat & True KV Spin Error; S/S_{Kerr} vs $m\Omega_0$



Non-Spinning Black Holes

Clearly the leading-order notion of defining non-spinning black holes is not adequate

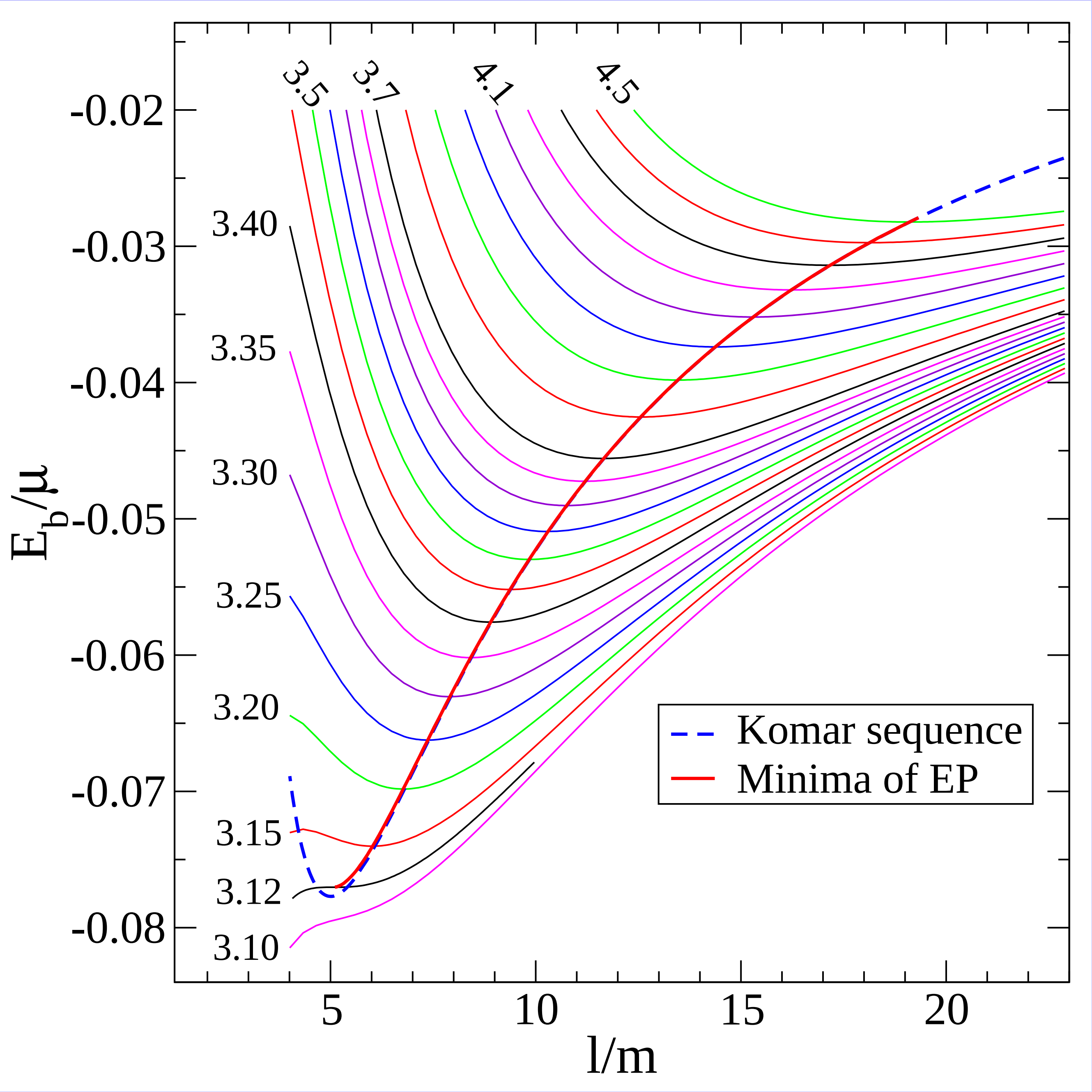
$$\beta^i|_{\mathcal{S}} = \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} + \Omega_0\xi^i|_{\mathcal{S}}.$$

Instead, choose

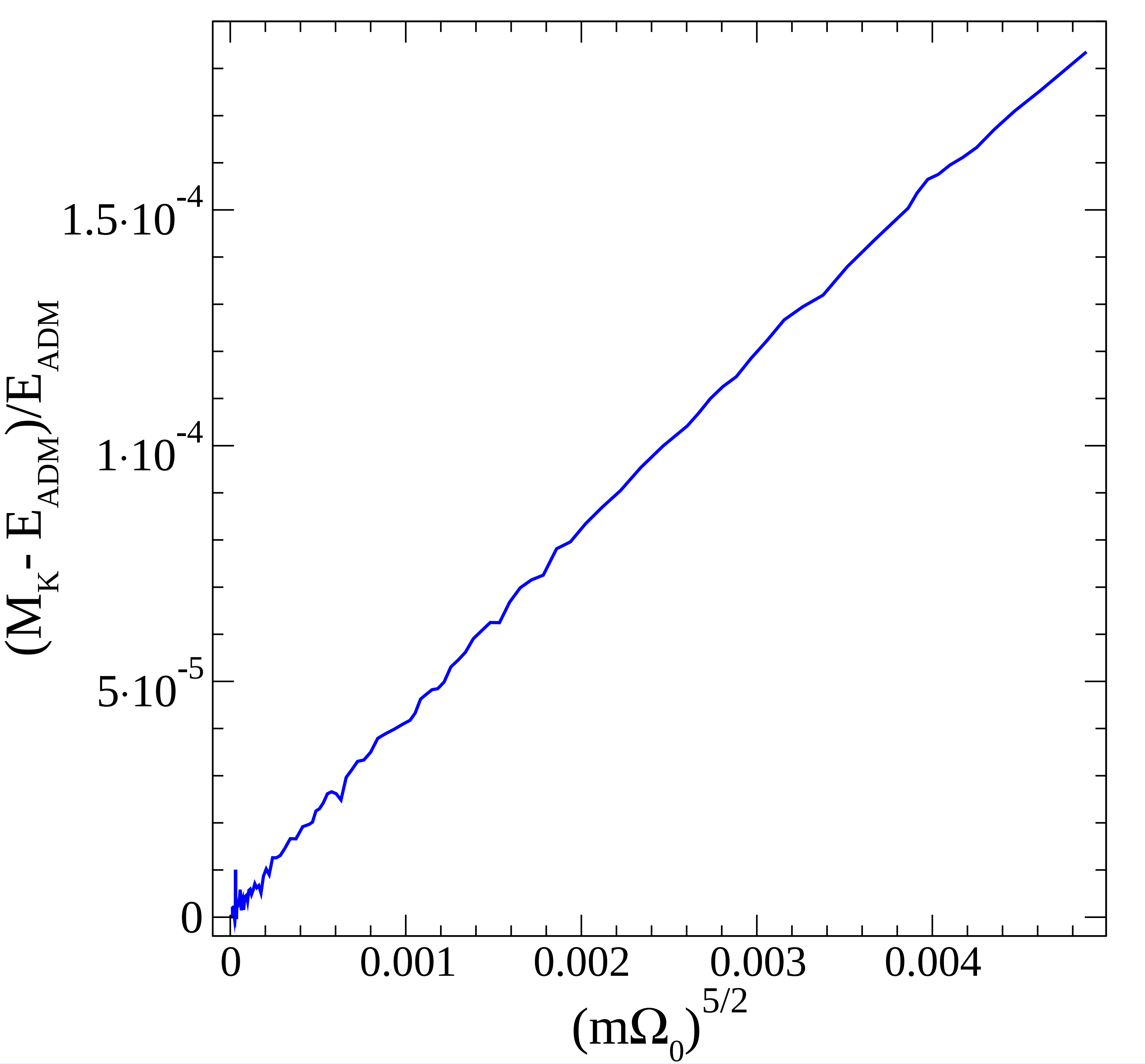
$$\beta^i|_{\mathcal{S}} = \alpha\psi^{-2}\tilde{s}^i|_{\mathcal{S}} + \Omega_{\text{BH}}\xi^i|_{\mathcal{S}}$$

and choose Ω_{BH} so that the quasilocal spin vanishes.

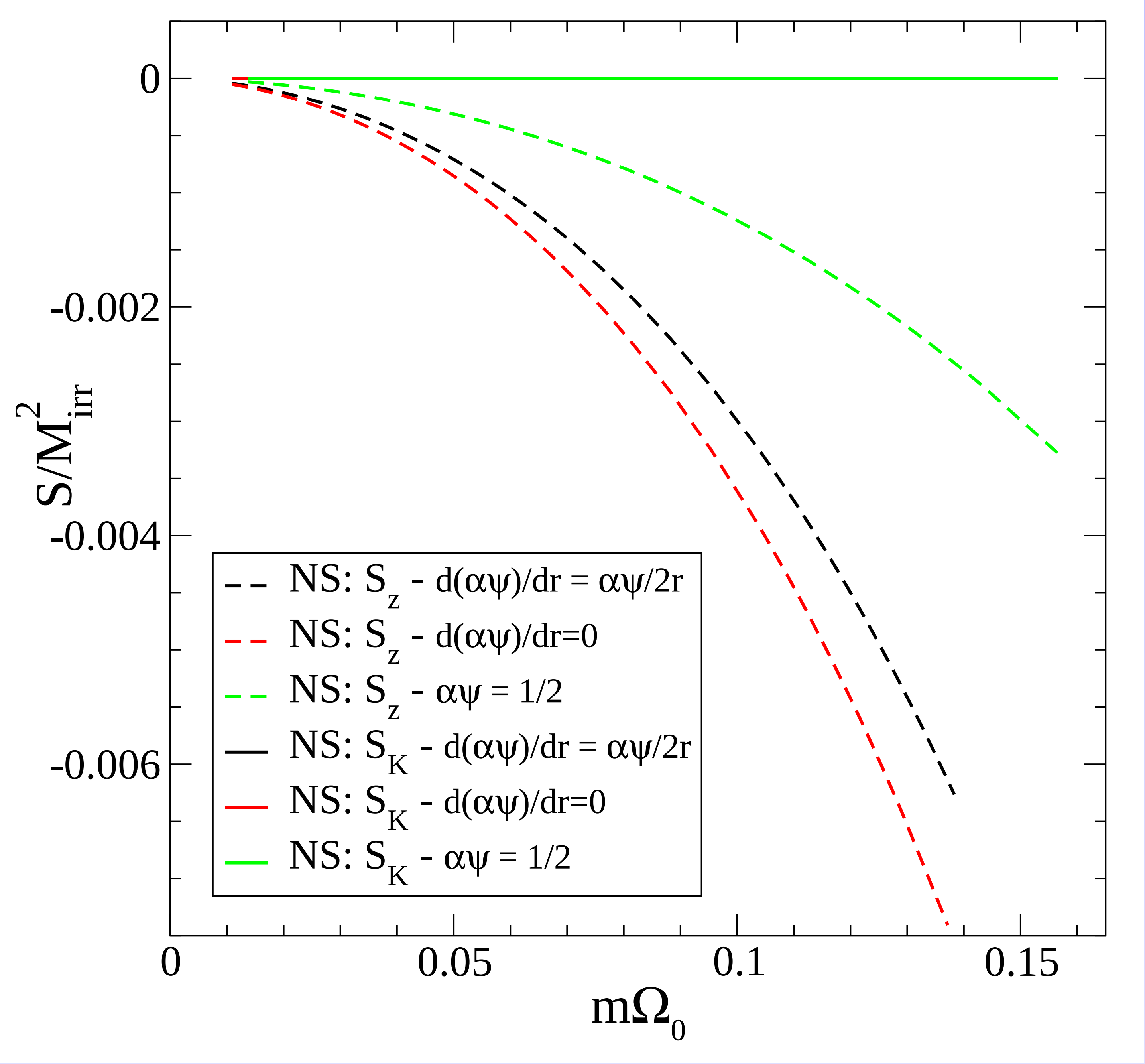
Non-Spinning: Effective Potential; E_b/μ vs ℓ/m



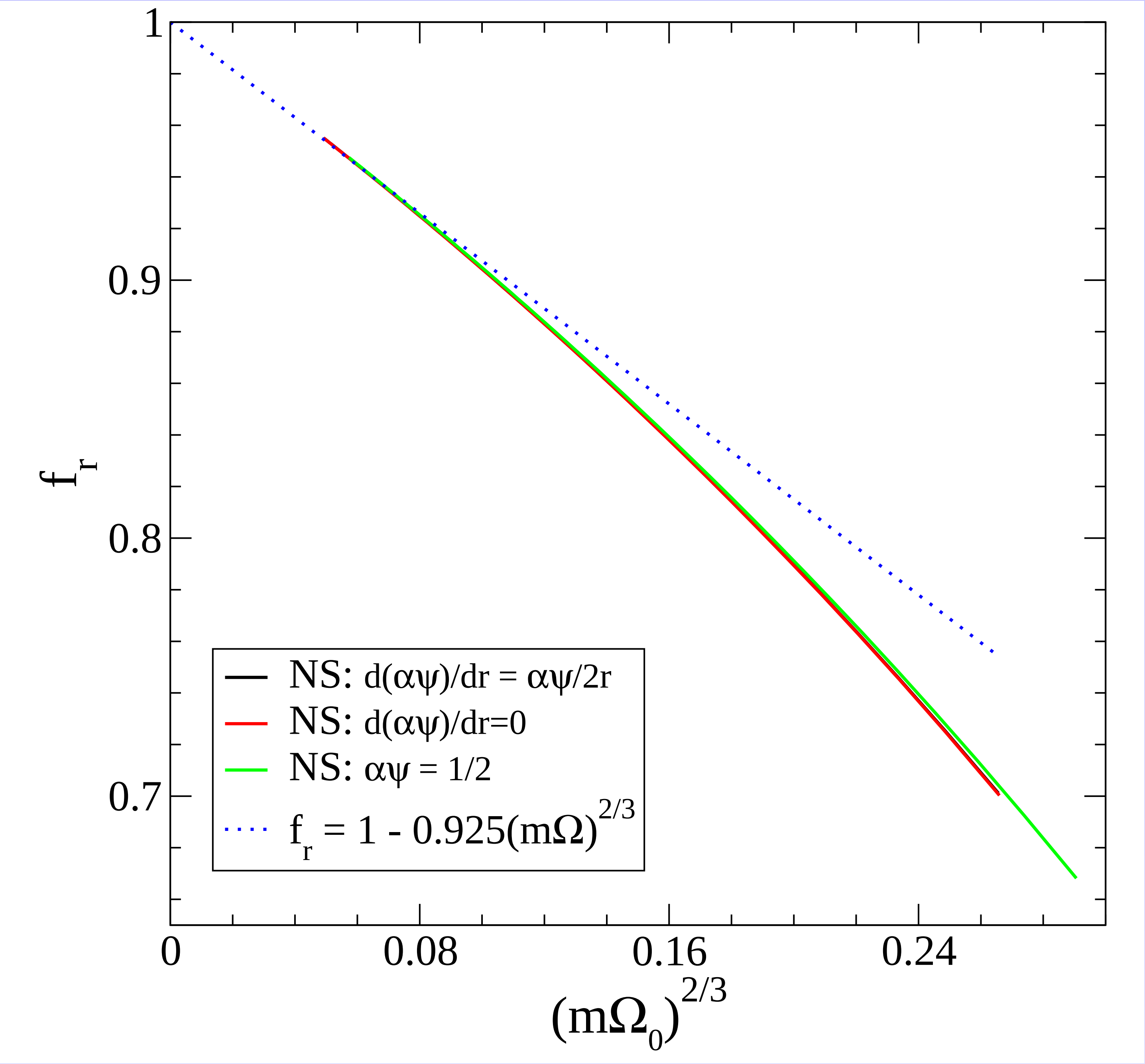
Non-Spinning: Error in Komar-Condition; E_b/μ vs ℓ/m



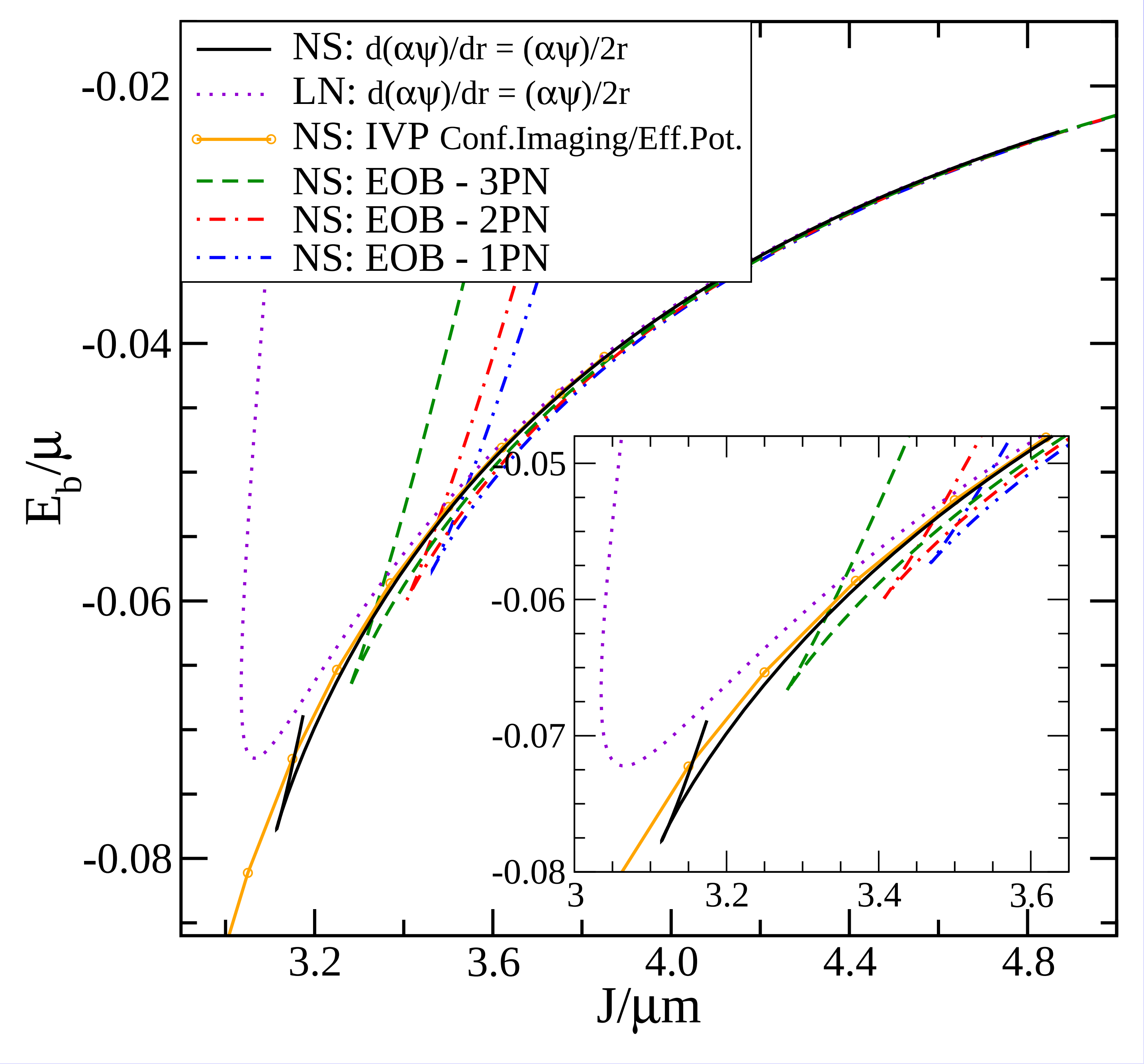
Non-Spinning: Flat & True KV Spin Error; S/S_{Kerr} vs $m\Omega_0$



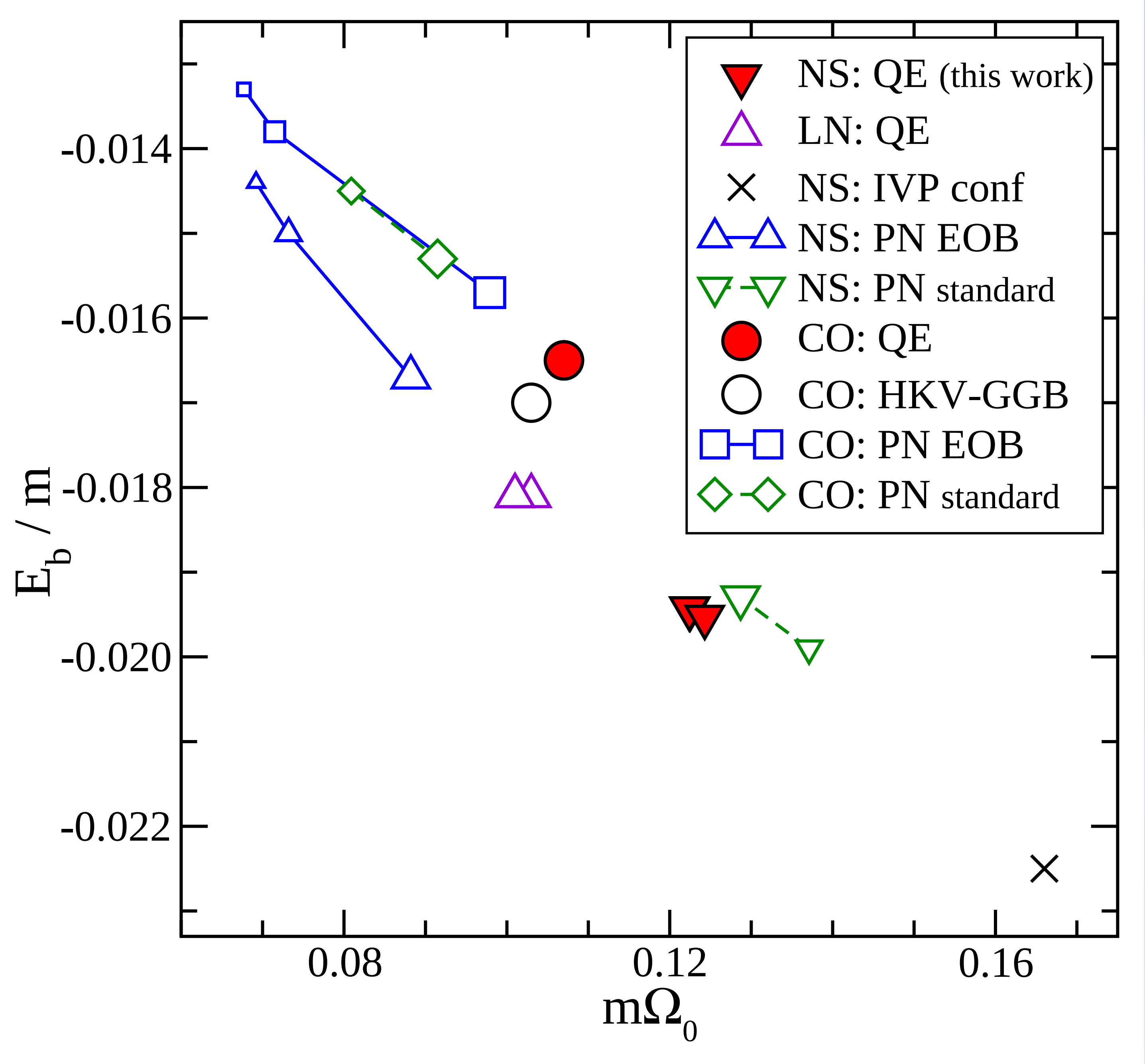
Non-Spinning Rotation Fraction; $f_r \equiv \Omega_{BH}/\Omega_0$ vs $m\Omega_0$



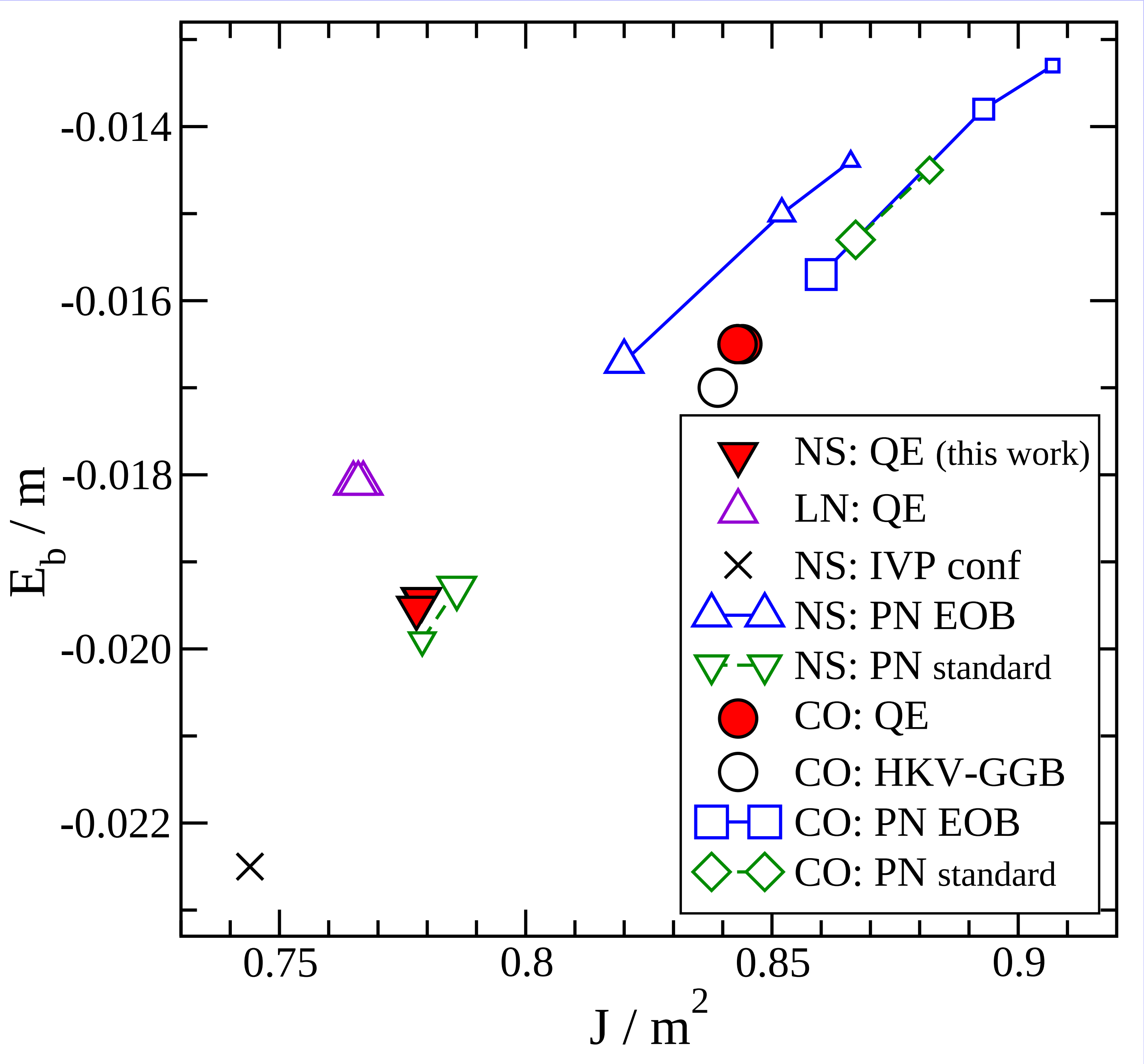
Non-Spinning; Comparison; E_b/μ vs $J/\mu m$



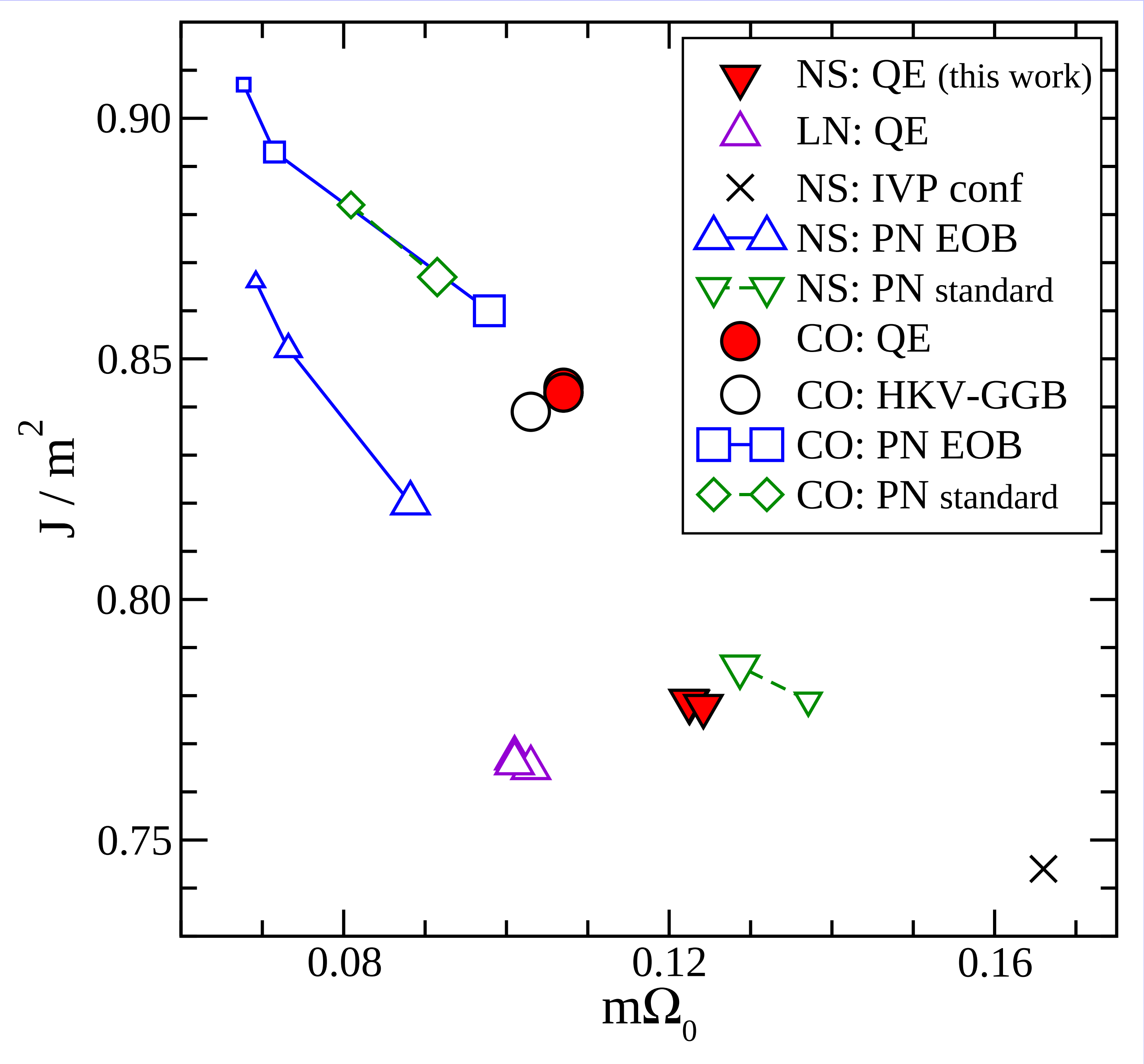
Comparison of ISCO; E_b/M_{irr} vs ΩM_{irr}



Comparison of ISCO; E_b/M_{irr} vs J/M_{irr}^2



Comparison of ISCO; J/M_{irr}^2 vs ΩM_{irr}



Thermodynamic Identity

For a binary system with a true helical Killing vector, the following thermodynamic identity applies to conformally-flat data[8]:

$$\delta E_{\mathrm{ADM}} = \Omega_0 \delta J_{\mathrm{ADM}} + \sum \kappa_i \delta \mathcal{A}_i$$

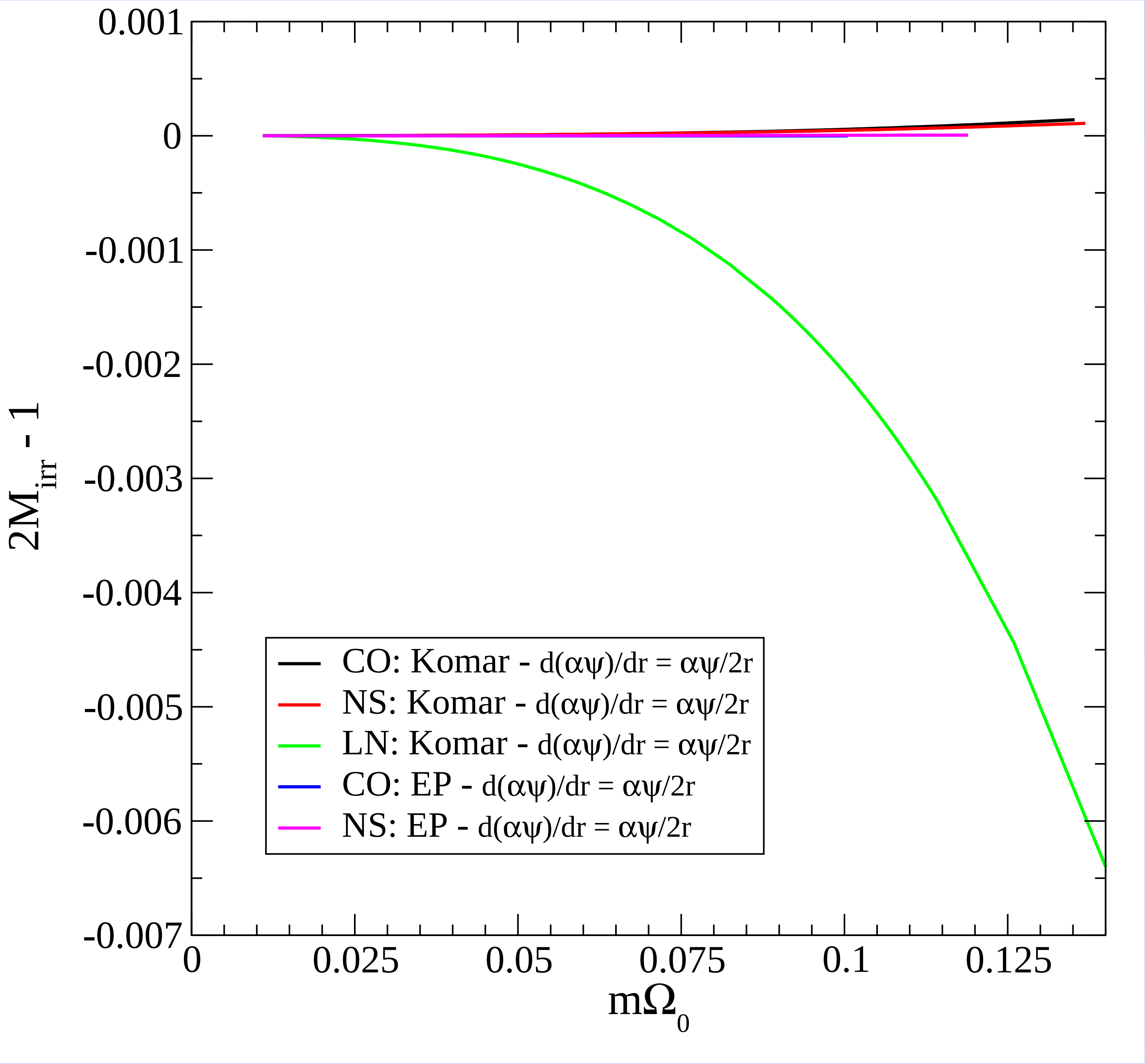
We can use the freedom to define a fundamental length scale along a sequence of BBH ID to enforce

$$\delta E_{\mathrm{ADM}} = \Omega_0 \delta J_{\mathrm{ADM}}$$

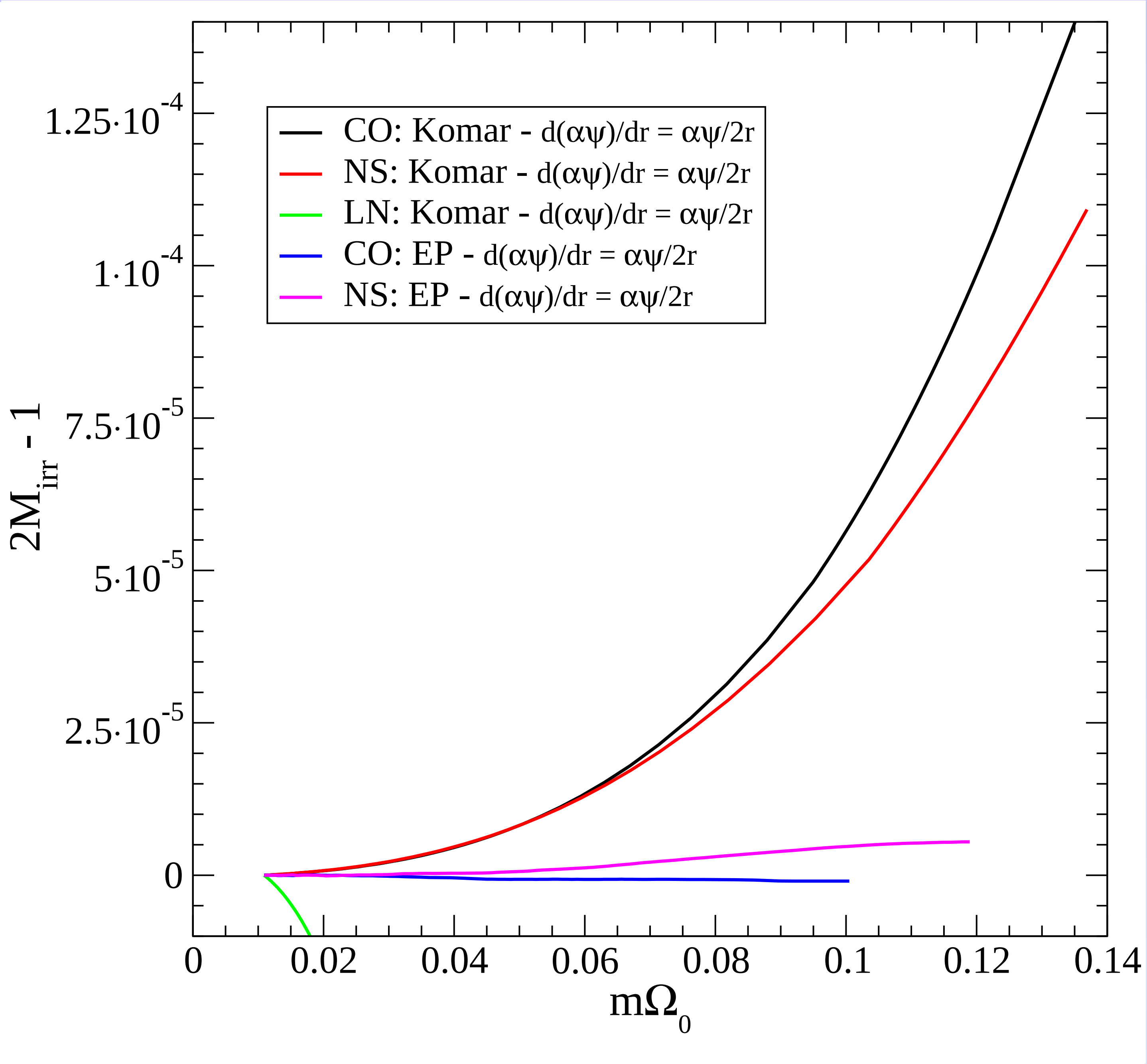
Scaled data satisfying the thermodynamic identity should now satisfy

$$\sum (\delta M_{irr})_i \approx 0$$

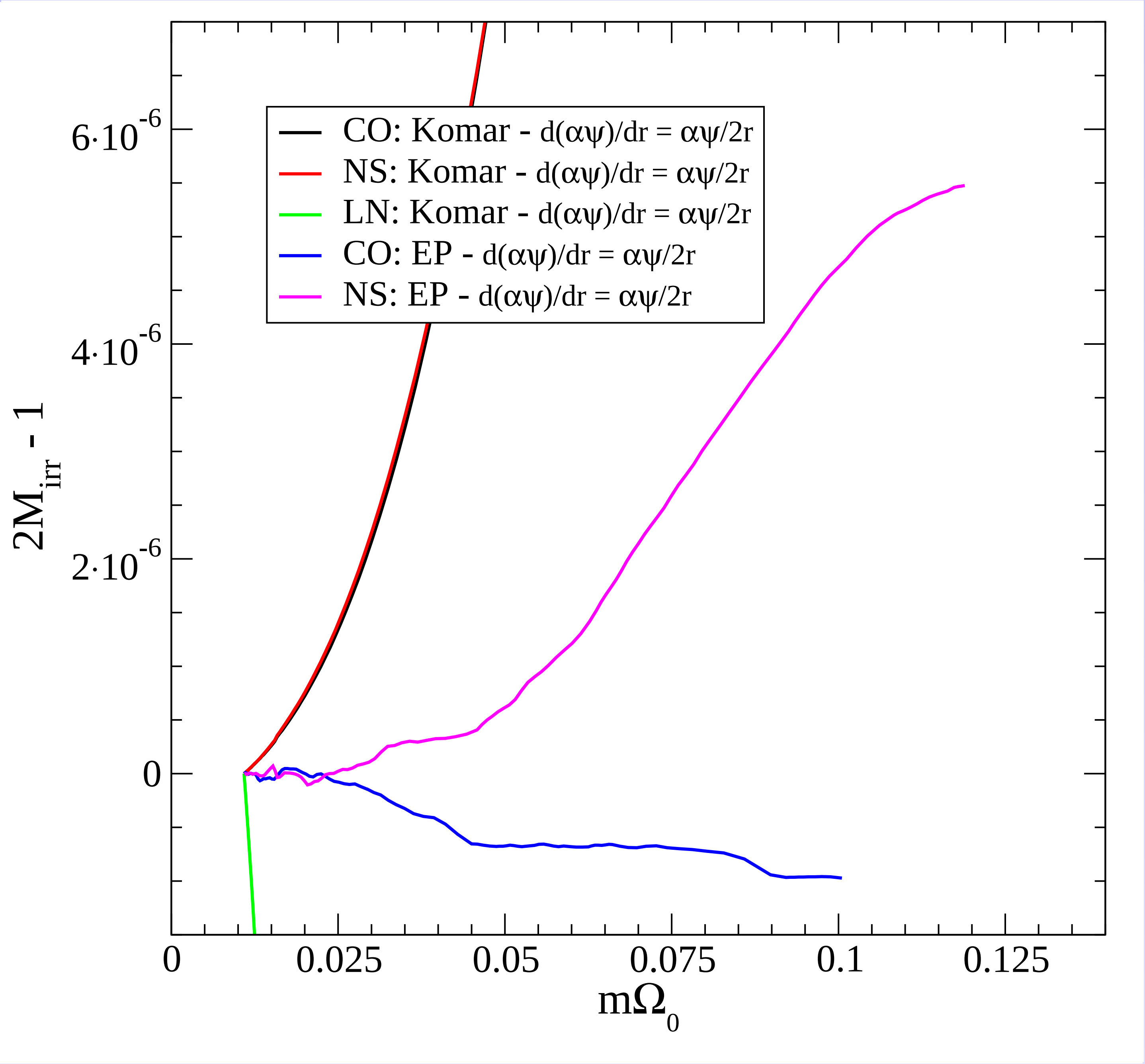
Circular Orbits and the Thermodynamic Identity



Circular Orbits and the Thermodynamic Identity



Circular Orbits and the Thermodynamic Identity



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