1-7. Consider a unit cube with one corner at the origin and three adjacent sides lying along the three axes of a rectangular coordinate system. Find the vectors describing the diagonals of the cube. What is the angle between any pair of diagonals?

1-8. Let **A** be a vector from the origin to a point P fixed in space. Let **r** be a vector from the origin to a variable point $Q(x_1, x_2, x_3)$. Show that

$$\mathbf{A} \cdot \mathbf{r} = A^2$$

is the equation of a plane perpendicular to **A** and passing through the point P.

1-9. For the two vectors

$$\mathbf{A} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \qquad \mathbf{B} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$

find

(a) $\mathbf{A} - \mathbf{B}$ and $|\mathbf{A} - \mathbf{B}|$ **(b)** component of **B** along **A** **(c)** angle between **A** and **B**
(d) $\mathbf{A} \times \mathbf{B}$ **(e)** $(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} + \mathbf{B})$

1-10. A particle moves in a plane elliptical orbit described by the position vector

$$\mathbf{r} = 2b \sin \omega t\, \mathbf{i} + b \cos \omega t\, \mathbf{j}$$

(a) Find **v**, **a**, and the particle speed.
(b) What is the angle between **v** and **a** at time $t = \pi/2\omega$?

1-11. Show that the *triple scalar product* $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ can be written as

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Show also that the product is unaffected by an interchange of the scalar and vector product operations or by a change in the order of **A**, **B**, **C**, as long as they are in cyclic order; that is,

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}, \qquad \text{etc.}$$

We may therefore use the notation **ABC** to denote the triple scalar product. Finally, give a geometric interpretation of **ABC** by computing the volume of the parallelepiped defined by the three vectors **A**, **B**, **C**.

1-12. Let **a**, **b**, **c** be three constant vectors drawn from the origin to the points A, B, C. What is the distance from the origin to the plane defined by the points A, B, C? What is the area of the triangle ABC?

1-13. If **X** is an unknown vector satisfying the following relations involving the known vectors **A** and **B** and the scalar ϕ,

$$\mathbf{A} \times \mathbf{X} = \mathbf{B}, \qquad \mathbf{A} \cdot \mathbf{X} = \phi$$

Express **X** in terms of **A**, **B**, ϕ, and the magnitude of **A**.

1-14. Consider the following matrices:

$$\mathsf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad \mathsf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \quad \mathsf{C} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}$$

Find the following
(a) $|\mathsf{AB}|$ **(b)** AC **(c)** ABC **(d)** $\mathsf{AB} - \mathsf{B}'\mathsf{A}'$